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þÿ D E V E L O P I N G S T U D E N T S UNDERSTANDING OF LINEAR EQU

þÿ D E V E L O P I N G S T U D E N T S UNDERSTANDING OF LINEAR EQU WITH ONE VARIABLE THROUGH WITH ONE VARIABLE THROUGH BALANCING MODEL OF TEACING: GRADE FANA PRIMARY SCHOOL GRADE FIVE STUDENTS OF EWKET FANA PRIMARY SCHOOL IN FOCUS

þÿ D E V E L O P I N G S T U D E N T S UNDERSTANDING OF LINEAR EQUATION BALANCING MODEL OF EWKET

Belaynesh, Kassanew

DEVELOPING STUDENTS' UNDERSTANDING OF LINEAR EQUATION WITH ONE VARIABLE THROUGH BALANCING MODEL OF TEACING: GRADE FIVE STUDENTS OF EWKET FANA PRIMARY SCHOOL IN FOCUS

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DECLARATION

I confirmed that this thesis entitled "Developing students' understanding of linear equation with one variable through balancing model of teaching: grade five students of Ewket Fana Primary School in Focus" is my original work, and it has not been submitted and presented to any other university for any consideration and that all sources of material used for the thesis have been dully acknowledged.

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ABSTRACT

The purpose of this quasi-experimental single group study design was to develop the students' understanding of solving linear equation with one variable through balancing model of teaching: the case of grade five students in Ewket Fana Primary School. To achieve the research objectives, design-based research was chosen as an approach to conduct this study. The target population of the study was five sections of grade five students and their mathematics teachers. From this population, the participants of the study were grade five section A students and their mathematics teacher. The students were selected using simple random sampling technique and their mathematics teacher was selected using comprehensive sampling technique. To answer the basic research questions of this study, the data were gathered through observation, test and interview. The conceptual understanding test results of the students were analyzed using paired sample t-test, and the interview and classroom observation data were analyzed through data driven thematic method. Quantitative findings showed that most students scored performed better in the posttest as compared to the pretest result, and qualitative results indicated that, after a series of learning attempts through balancing model method most students have developed more flexible strategies to solve linear equation. Moreover, grade five section A students and their mathematics teacher showed a positive attitude towards learning through the balancing model teaching method. Therefore, the researcher concluded that teaching through balancing model method has developed students' conceptual understanding of linear equation and helped both students and their respective teacher to bring a positive attitude towards the learning process. In order to develop students understanding and sustain an innovative approach for the teaching of mathematics subject, the researcher recommended that educational officials of Ewket Fana Primary School, in particular, and officials in Amhara region, in general, should design and implement continuous professional development program for primary school teachers in order to improve their pedagogical skills. They are also expected to re-design primary school curricular materials in such a way that they could smoothly be integrated into the student's real-life situations.

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CHAPTER ONE

INTRODUCTION

1.1. Background of the study

Mathematics is an aspect of personal activity and culture artifact that can produce pleasure, satisfaction, and attraction. According to Peel (1971), mathematics is a form of reasoning formulating testing conjectures and making sense of things. Additionally, Jaworski (2006) attest that mathematics is a culturally shared study of patterns and language for everyday life, a central part of human communication and a means of articulating patterns, relationships, rationality and aesthetic. For all these reasons, mathematics concepts are connecting one to his or her universe in many ways by incorporating language, culture and practices of daily living. The goal of mathematics education is to produce students who are skilled in resolving the problem, fostering attitudes, interests and a high motivation towards mathematics. As to Anthony and Walshaw (2014), the main purpose of teaching and learning mathematics subject is to develop the ability of students and connect any mathematical problems to the students' real-world situations. On the other hand, mathematical ability involves effective thinking with conceptual learning, thus students need to be taught to think logically along with practicing numerical problems (Mahmood, et al., 2012).

The major component of the mathematics curriculum in all countries around the world is an algebra concept (Bal, 2016). Understanding algebra in school mathematics is one of the most important goals for mathematics education. Therefore, algebra concepts include unknowns, variables, expressions, equations and mathematical operations as well as algebra deals with expressions with symbols and the extended numbers beyond whole numbers in order to solve equations, to analyze functional relations and to determine the structure of the representational system which consists of expressions and relations. Bal (2016) states that primary school algebra, which involves arithmetical skills and numerical patterns carries great importance to enhance the student's algebraic understanding.

In the primary school, students learn algebra concepts starting from arithmetical operations after that solving linear equation. Linear equation is one of the most important topics in algebra and also it is played a central role in the development of other mathematics concepts.

However, students from many countries displayed relatively negative attitudes towards mathematics particularly algebra concepts (Hescovics & Linchevski, 1994). This also holds true in the Ethiopian context. The study conducted by the Ministry of Education (2003), for example, disclosed that mathematics teaching and learning is not promising in Ethiopian schools. Some of the mathematics instruction problems include the traditional teaching approaches, lack of reference materials, lack of professional development programs, and the absence of encouraging students' prior experiences (Ministry of Education, 2003, p.27). Moreover, researchers advocate that alternative teaching style and strategies could promote a positive attitude among students and bridging the transition from concrete to abstract mathematics. Therefore, the selection of relevant teaching methods is one of the primary principles in mathematics education by taking into account the nature of the subject matter, the objectives of the lesson and the nature of the learners. According to Bal (2016), applying different teaching approaches to mathematics lesson helps the students to value the lesson more, learning with fun and able to solve various examples with different methods.

Many studies stated that balancing model teaching method is an effective way to develop students understanding of the relational view of the equal sign and their ability to operate on equations. According to Vlassis (2002) Balance model provides a very mental picture for solving linear equation in one variable and the equation is viewed, by analogy, as a set of balance scale and the equal sign in the equation is represented by the scale being 'in balance'. And again Caglayan and Olive (2010) stets that, balancing model is useful to the students to interrelate environment and mathematics concepts and that they both operate on rules, employ experiences, drills and practical applications. Regarding this finding, Warren and Cooper (2005), for example, noted that balancing model is an innovative tool that makes algebra more accessible for students. By working in cooperative groups, the students can improve communication skills as they explore algebraic concepts together. In algebra, the use of a balance model can be a powerful tool to understand the idea of an equation. Atteh et al., (2017), also describe that balancing model helps to develop:

- Understanding of relational view of equal sign.
- Ability to operate on equation.
- The view of an equation as an object.

- Understanding of eliminations in algebraic operation.
- Representational fluency.

Therefore, this study aims to build up student's conceptual understanding by integrating the use of contexts and models on algebra, particularly on the topic of linear equation with one variable. To address this objective, the researcher designed learning materials and learning activities infused with balancing model to promote the students understanding and reasoning on a linear equation. The design makes most of the algebraic notions within balancing activity to facilitate students learning on linear equation with one variable. So the output of this study is hoped to provide a practical solution that teachers can use to reform their algebra teaching.

1.2. Statement of the problem

Many researcher findings on mathematics education have emphasized the learning difficulties of students related to algebra concepts. Different research findings, as well as the experience in teaching mathematics, show multiple difficulties students encounter in learning algebra concepts (Kilpatrick & Izsak, 2008). The difficulties of the students are caused by several factors including the way algebra is taught which remains very traditional. Moreover, algebra involves using letters along with formal rules for operating the letters that are really abstract for students and also the students develop a formula approach often used in its application (Hescovics & Linchevski 1994). The challenges of the students in learning algebra concept were caused by mathematics teachers teaching style as well as most students have not experienced practical learning; instead, their attempt to solving problems merely depends on memorizing the formula and procedures (Eichhorn, et al., 2018). As a result, primary classroom algebra particularly linear equation is the major mathematical challenge in the operational stage to most students. Regarding this, Cai and Moyer (2008) noted that solving linear equations is a particularly important concept in algebra that causes confusion for students. There are many reasons why solving linear equations is a challenging skill for students to understand. According to Kieran (2003), students learning difficulties are centered on the meaning of the equal sign, lack of symbolic understanding, the change from arithmetic to algebraic conventions and use of a structure. So students without a relational view of the equal sign cannot adequately interpret the linear equation. As a result, many students do not have the capacity to construct a good intuitive

basis for the concept of an equation and they fail to construct meaning for the new symbolism and reduced to perform meaningless operations on symbols they do not understand.

In our country, mathematics teachers are not necessarily good at teaching in schools by using active learning methods for exploiting the advantages of coming through promoting learners' learning styles and strategies (MoE, 2003). In addition, the researcher's practical experience as a mathematics teacher at various levels for five years has shown that the alternative learning styles and strategies are not considered in the learning of mathematics subject. Therefore, most Ethiopian students consider mathematics as one of the most difficult subject and the Ethiopian mathematics textbooks failed to include innovative approaches (MoE, 2003).

The researcher's preliminary observation confirmed that the theoretical and practical experience gaps mentioned above are also observed in Ewket Fana Primary School. The researcher's discussion with grade five mathematics teacher showed that he had not used the balancing model teaching method during his teaching-learning process and also his students did not experience the balancing model teaching method for solving any linear equation. The main reasons for not applying an innovative approach, i.e., balancing model teaching approach, in this school were lack of pedagogical knowledge of the teacher, textbook writers' failure to include innovative approaches while designing the textbooks and lack of resources.

Therefore, to develop a relational and conceptual understanding of the equal sign and to support students' mathematical thinking and algebraic reasoning in elementary schools, teachers should apply appropriate or innovative teaching method in order to realize the desired objectives of the lesson. In this regard, balancing model teaching method is considered to be an appropriate teaching method in solving linear equation problems in the current mathematics literature. That is, different studies (e.g., Vlassis, 2002; Atteh et al., 2017) have emphasized the effectiveness of balancing model method of teaching in learning about the concept of solving a linear equation. According to Vlassis (2002), learning linear equation by using the balance model or scale has motivated the students to always relate mathematical concepts with observation and exploration data and it can be used for bridging the gaps in understanding the abstract concepts in algebra. Similarly, Atteh et al., (2017), unfold that the balance model also promoted the active participation and intellectual involvement of learners in solving a linear equation. Based on this,

balancing model teaching method plays an important role in developing active learning approach and improve the conceptual understanding of students towards linear equation.

1.3. Theoretical and Conceptual Frameworks of the Study

1.3.1. Theoretical framework

This study drew its theoretical framework from constructivism which has two belief systems, individual constructivism and social constructivism. The theory of individual constructivism viewed learning as a process in which the learner actively constructs or builds new ideas or concepts based on current and past knowledge and experience (Von Glasersfeld, 1990). However, social constructivism is considered as an extension of the traditional focus on individual learning to address the collaborative and social dimension of learning (Ernest 1994). Social constructivists posit that knowledge is constructed when individuals engage socially in talk and activity about shared problems or tasks. From these conceptualizations, one can see that these theories suggest that learners mind is not like an empty vessel that has to be filled-in with knowledge, but that learners are active beings who are capable of servicing of new knowledge from known and related experience and also through social interaction with other learners in a group. The implication behind these theories is creating active learning rather than passive learning in the collaborative and cooperative small-group and large-group discussion environment.

An important aspect of solving equation involves having both procedural and conceptual understanding of the concept. Star (2005) provided a definition of procedural understanding, focusing on understanding symbols and rules, and conceptual understanding as making a connection and creating a network within the information. In addition, there are a limited number of procedures necessary for solving linear equation. For this reason, conceptual understanding may be more important when students solve linear equations (Star, 2005). If students understand concepts such as the meaning of the equal sign, inverse operations, and the rule of constants and coefficients, they may be able to use this conceptual understanding to accurately implement correct procedures while solving an equation. On the basis of these background explanations, the researcher considered individual constructivist and social constructivist theories of learning as the basic theoretical frameworks of this study.

1.3.2. Conceptual framework

The study was guided by a conceptual framework which depicted a representation of dependent and independent variables and the relationships between them as shown by an arrow in figure 1.1.

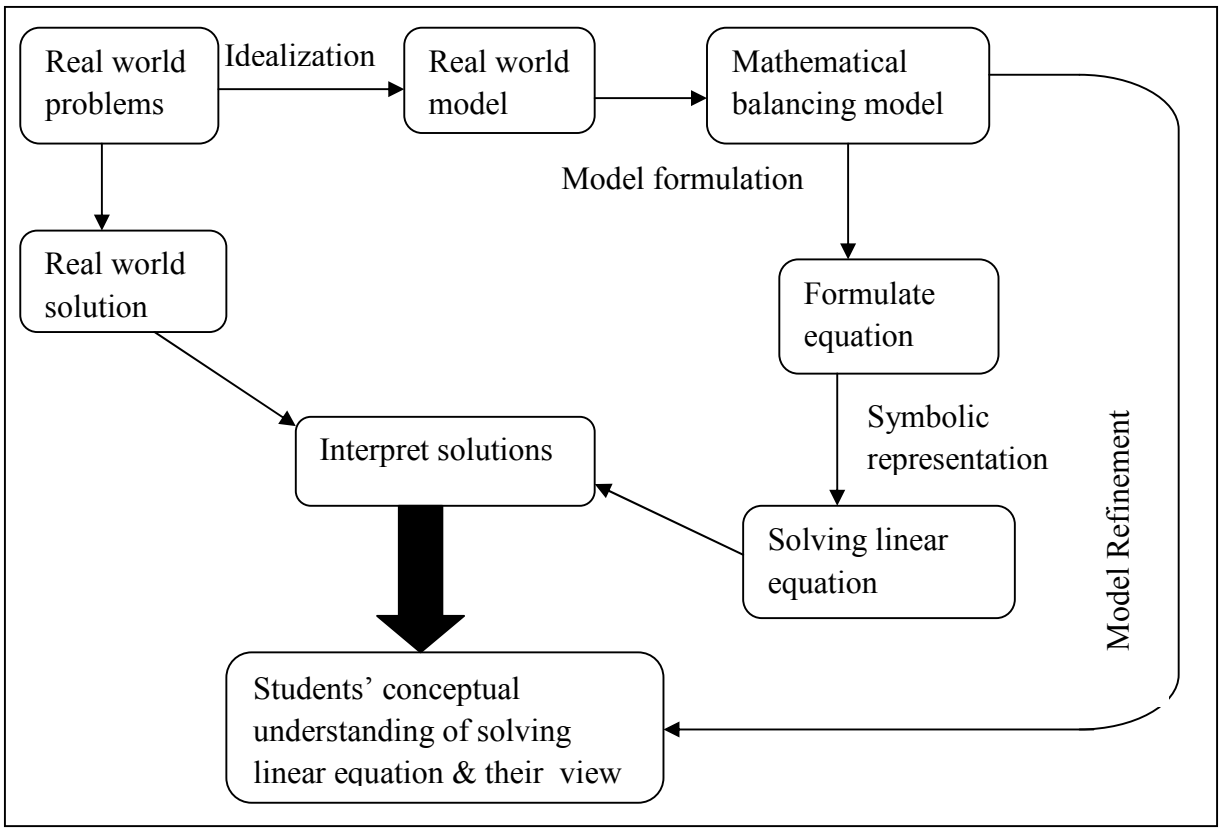


Figure 1.1: Conceptual framework of the study

The central thesis of the study was to check how far the selected teaching method (i.e., balance model teaching method or innovative method) contributed to grade five students' conceptual understanding of solving linear equation, which results in improved their conceptual understanding of solving linear equation, and how far the method contributed to grade five learners views about the nature of learning of linear equation, which resulted in making a positive impact on their views about the nature of learning of linear equation.

1.4. Objective of the Study

In this study, the problem was what must be done to assist the learner to improve their understanding and knowledge as well as acquiring new principles to solve linear equation in one

variable. Therefore, this research aims to use the balance model as an effective tool to improve student's conceptual understanding of effectively solving mathematical problems relating to linear equations. To the researcher's best knowledge, students can develop firm conceptual understanding in linear equations when the balance model approach is employed in the implementation of mathematics lessons. So, the main objectives of this study are:

- To develop students' conceptual understanding of solving linear equation with one variable through balancing model method of teaching in Ewket Fana Primary School.
- To establish and sustain the balancing model method of teaching approach on the concept of solving any linear equation with one variable in Ewket Fana Primary School.

1.5. Research Questions of the Study

The basic research questions in this study include the following:

- To what extent do students' show better conceptual understanding of solving linear equation when they are taught using the balancing model method?
- How can I establish and sustain balancing model teaching method while teaching the concept of solving any linear equation with one variable in Ewket Fana Primary School?

1.6. Significance of the Study

The findings of this study will help mathematics educators, mathematics teachers' as well as future researchers by enhancing their professional know-how. Specifically, this study is hoped to provide the following significances:

- The findings of the study will be used as a reference point for mathematics curriculum experts to train or re-train the new approach for mathematics teachers and even re-design all mathematics textbooks.
- The findings of the study will be used as a guideline for mathematics teachers to design effective instructional activities for teaching the concept of solving linear equation and engaging their students in a sequence of instructional activities in order to support their conceptual understanding about linear equation.

- The findings of this study will serve as baseline data for future researchers who want to do large scale investigation in the contribution of balancing model on students' conceptual understanding of linear equation.

1.7. The Scope of the Study

In order to make the study more manageable, this research was delimited to both geographically and conceptually. Geographically the scope of the study was delimited to one government primary school at Bahir Dar Town, that is, in Ewket Fana Primary School, grade five, section A students. The researcher chose the Ewket Fana Primary School in a convenient way. The reason why this school was selected using the convenience approach was based on the direction of the researcher (i.e. distance, time) and also five grade students were purposely selected, Because in Ethiopia context elementary school learners learned equation concept starting in five grade. Conceptually, this study was delimited to the development of students' understanding in the area of solving linear equation. This topic is chosen for purposely, that means linear equation is one of the most important topics in algebra and also it is played a central role in the development of other mathematics concepts.

1.8. Limitation of the Study

As this study focuses on checking how far balancing model teaching method contributed to students' conceptual understanding in solving linear equation, yet the study faced different limitations in its progress. For one thing, it was constrained by instructional materials like balancing model teaching aid. For the other, the way the textbook is designed for also obstructed the proper implementation of the method. Moreover, the study was constrained by students' lack of awareness of the balancing model teaching method while doing different activities through their own procedure. However, despite these restrictions, the researcher has tried hard to make the study as complete as possible.

1.9. The operational definition of key terms

Linear equation: - a linear equation is an equation in which variables appear as separate terms and neither variable contains an exponent other than one.

Balancing model: - a balancing model is a unique tool that represents a situation or problem with equal weight to represent equation for helping students makes sense of algebraic concepts and provides manipulative experimental basis for the development of an abstract idea.

Conceptual understanding: - conceptual understanding means knowing what to do and knowing the reason why doing it.

Students' view: - is a mental or neutral state of readiness, organized through experience, exerting a direct or dynamic influence upon the individual's response to all objects and situations with which it is related.

CHAPTER TWO

REVIEW OF THE RELATED LITERATURE

2.1. Constructivism Learning Theory

This section explored the constructivist learning theory by defining constructivism and illustrating how constructivism relates to independent learning methods of mathematics subject in elementary education. Katic, et al., (2009) defined as Constructivism is a learning theory that attempts to explain how students learn by constructing for themselves. Also, the study shows that, in the constructivist classroom, students are required to be active learners do not passively receive information but constantly create new knowledge based on prior knowledge in conjunction with new experiences (Katic, et al., 2009). Many constructivists theorize that learners have different needs at different stages of development. A mathematics teacher that uses a constructivist approach relates representations and explanations of new information which will meaningfully connect with student's prior knowledge (Hendry, et al., 1999). In applying constructivist theories of learning a mathematics teacher should understand the student's status and implement multiple instructional strategies to address multiple learning styles (Felder & Brent, 2001; Kim, 2005).

Constructivist learning theory emphasized two salient beliefs; students need to learn by doing and they need to understand mathematics in terms of real life (Gordon, 2009). According to this idea, all learning is intimately tied to experience and through practice (Katic, et al., 2009). On the other hand, constructivist views the student's interpreted new knowledge from social interaction and collaboration in society (Corden, 2001). In general, in a psychological constructivism perspective, learners construct meaning socially as well as individually. So based on this learning theory, in this study, the researcher intention is to develop students' understanding of solving linear equation by connecting student's real-life situation and their previous experience.

2.1.1. Individual constructivism learning theory

Individual constructivism is a theory of knowledge argues that humans generate knowledge and meaning from an interaction between their experiences and their ideas (Glasersfeld, 1974). Similarly, Von Glasersfeld (1990) suggest that individual constructivists

learning theory emphasized that, learner constructs or build new knowledge or concepts based on current and previous experience. The basic idea of this learning theory is an active and constructive process with the learner viewed as an information constructor. According to Fletcher (2005) argues that "constructivist learning paradigm should be considered as an alternative to transmission view since a fundamental goal of mathematics instruction is to help learners build structures that are more complex, powerful and abstract than those learners possess before instruction"(p. 31). This individual constructivist learning that promotes deeper construction the meaning of knowledge in elementary learners during the teaching and learning of mathematics. Based on this, Individual constructivists take a strong position that children have mathematical realities that do not overlap an adult's mathematics (Steffe & Kiern, 1994). So Individual constructivist concentrate on understanding learner's mathematical realities and the internal mechanisms by which they change.

2.1.2. Social constructivism learning theory

Social constructivism focuses on an individual's learning that takes place as a result of discussions and interaction in the group and among groups (Corden, 2001; Reznitskaya, et al., 2007). Discussion plays a vital role in increasing students' ability to test their ideas synthesize the ideas of others and build a deeper understanding of what they are learning. Moreover, discussion increases student's motivation, collaborative skills and the ability to solve the problem (Matsumura, et al., 2008). Increasing learner's opportunities to talk with one another and discuss their ideas increase their ability to support their thinking, develop reasoning skills and to argue their opinions persuasively and respectfully (Reznitskaya, et al., 2007). So learning is a social context that involves active participation and inquiry. Moreover, small-group learning in mathematics has revealed some insights into how learners think and relate the concept of their real-life situation. This insight includes cooperative and collaborative learning strategies. Based on this Pewewardy (2002) illustrated that mathematical knowledge is a social construction, a cultural product and associated with the social-economic situation of the community and it helps to enhance the development of the learners. Additionally, Pewewardy (2002) stated that mathematics concepts are connected one to his or her universe in many ways by incorporating language, culture and daily living practices. So in this study, the researcher intends to develop social group learning will have on grade five students and change students' conceptual

understanding in the linear equation when they are engaged in learning through balancing model methods.

2.2. Algebraic Thinking

Algebraic thinking is important for students to achieve in higher level mathematics courses, so many students difficult and struggles with algebraic understanding (Witzel, et al., 2003). Many students are confused change from arithmetic to algebraic conventions and use of structure; some of these problems are amplified by teaching approaches because elementary mathematics classrooms often do not prepare insufficient for helping students understand the abstract, structural concepts necessary for supporting the demonstrated procedural activities for algebraic thinking (Cai & Moyer, 2008). Too often, students without understanding the meaning of important concepts such as variables, coefficients, and equal signs they have to learn to reduce performing meaningless operations on algebraic symbols. As a result, students are lack of conceptual understanding in algebra and it is understood procedures or formulas. Students without making connections skilled procedure-oriented learning, algebra are frequently not easy. "algebra is difficult for students because the representations are abstract and the required operations, especially those relating quantities in word-problem situations, conflict with operations students have learned to use through modeling with an arithmetic" (Kilpatrick & Izsak 2008, p.12). So, elementary school learners focused on reasoning about expressions and equal signs they are successful in solving linear equations with one variable. The main point of this study is the progress of elementary learner's algebraic thinking.

2.3. Solving Linear Equations

A linear equation is a particularly important concept in algebra and an important component of elementary school mathematics education (Cai & Moyer, 2008). Students face various challenges as they study algebra particularly the concept of linear equation. This several researchers (e.g., Kilpatrick & Izsak 2008; Poon & Leung 2010; Knuth, et al., 2006; Capraro & Joffrion, 2006; Siegler, 2003; Star, 2005) have identified some common challenges that students frequently faced when attempting to solve equations, including the following:

- A lack of a symbolic understanding of constant terns and coefficients within an equation (Kilpatrick & Izsak, 2008; Poon & Leung, 2010; Vlassis, 2008).

- A lack of understanding of the meaning of the equal sign (Knuth, et al., 2006).
- A reliance on procedural knowledge without conceptual understanding (Capraro & Joffrion, 2006; Siegler, 2003; Star, 2005).

2.3.1. Lack of symbolic understanding

The first challenges of elementary school learners to solving any linear equation as symbolic understanding (Kilpatrick & Izsak, 2008; Poon & Leung, 2010). According to Poon and Leung (2010) described that many learners without understanding the concept of the equation simply accept or understand the formula, rules, and techniques of algebraic concepts. For example, in Poon and Leung studies, many elementary learners do not know the roles of 3 in the two expressions, 3 and 3n. In the first example, 3 is a constant term of the equation and in the second 3 is the leading coefficient of the equation but most students are given the same meaning. In this case, most students have experienced rules and formulas without understanding the concept.

Additionally, Vlassis (2008) described that the challenges of students practice with symbolic knowledge and emphasizing that students have complexity with symbolic understanding, because of the multiple meanings that mathematical symbols hold. For example, the minus sign, many students cannot further simplify ($3 - x = 5$). According to this Vlassis (2008) examining eight grade student's symbolic understanding of minus sign. Vlassis interviewed 17 students related to the meaning of the minus sign in algebra concepts. Students experienced difficulties with solving $-2x = 4$ because the leading coefficient of the equation is negative. And some students do not solve $3 - x = 5$ because the negative sign is before the variable and not the constant. In this case, most students wrongly transformed the equation to $x = 5 - 3$. Although the Vlassis study provided insight into students challenges the meaning of the minus sign, but the small sample size makes generalization difficult. So, in this study, the researcher is to teach linear equation by using a balancing model to enhance students understanding of symbolic representation.

2.3.2. Meaning of equal sign

The other mean challenges for students solving linear equations involve interpreting the equal sign (Knuth, et al., 2006). Many researchers (e.g. Knuth, et al., 2006) described that, a lack

of understanding of equal sign as basic difficulties to students connected with equation concepts. The equal sign is everywhere at all levels of mathematics courses, but small instructional changes the meaning of the equation or solution (Knuth, et al., 2006). So students are without understanding the meaning of equal sign or equality, they have difficulties in solving any equations. Within Knuth, et al., (2006) study, middle school students (n = 177) completed a written assessment of algebraic understanding in the quantitative study. In the equation students were asked to solve multiple equations with one variable, $4x + 1 = 6 - x$. When the question included variables on both side of the equation, students were often unable to understand how to proceed. Student's responses were coded as an answer only, no response, and guess. Results indicated that students who defined the equal sign did not understand the meaning of equality and not used an algebraic strategy to solve the equations. According to Kilpatrick, et al., (2001) many learners either conceptualizes the equal sign as a separation of the problem and the solution or as left to a right directional symbol for working out problems. Both of these misconceptions of the equal sign are challenges for students solving linear equations because equations frequently contain constant terms leading coefficients and variables on both sides of the equation. The study also showed that students did not understand that the equal sign represented a relationship between two quantities; they have difficulty manipulating equations in order to find a solution (Knuth, et al., 2006). A limitation of this study was that researchers did not investigate how students developed their conception of an equal sign. Description of curriculum or classroom activities may have provided insight into the ways students developed these conceptions. So in this study based on the gap of middle school students understanding of the meaning of the equal sign, the researcher intention is using a different representation of models to full-file student's misconception of the meaning of equality.

2.3.3. A Reliance on procedural knowledge without conceptual understanding

A reliance on procedural understandings of an equation is the third important aspect of solving linear equations (Capraro & Joffrion, 2006; Star, 2005). Star (2005) provided the meaning of procedural understanding, focusing on understanding formulas, symbols and rules; and conceptual understanding as making connections and creating networks within the information. Although Star (2005) maintained that procedural understanding is necessary, other researchers, For example, Siegler (2003) discussed conceptual understanding are necessary for

students to develop mathematics learning. Rittle-Johnson and Alibali (1999) grade five students randomly selected (n=60) and assessed understanding of equivalence before and after instruction. Each of three group students received different treatments as they learned about addition and subtraction equivalence. The control group received no instruction, while one treatment group received procedure-oriented instruction and the other group received conceptual- oriented instruction. Rittle-Johnson and Alibali (1999) concluded that gains made by procedural group students did not transfer to improvements in conceptual understanding, "In contrast, gains in conceptual understanding led to fairly consistent improvements in procedural knowledge in this study" (p.186).

Although researchers and educators agree that both procedural and conceptual knowledge are important, there is disagreement about the emphasis that should be placed on each type of knowledge within the study of mathematics. Star (2005) stated that flexibility can be derived from deep procedural knowledge. Contrastingly, other researchers emphasized the depth of knowledge that results of conceptual understanding. Kilpatrick, et al., (2001) stated conceptual understanding rather than procedural understanding claiming that an emphasis on rules and procedures does not help students create new idea and meaning. Finally, Kilpatrick, et al., (2001) cautioned that reliance on procedural understanding results in an over-reliance on visual cues, such as manipulative. But procedural and conceptual understanding are both valuable as students learn mathematics. Conceptual understanding allows students to make connections and develop understandings, not just memorize a set of facts or procedures. Because as Star (2005) suggested, there are a limited number of procedures necessary for solving equations; conceptual understanding may be more important as students solve equations. If students understand concepts such as the meaning of an equal sign, inverse operations and the role of constants and coefficients, they may be able to use this conceptual understanding to accurately implement correct procedures while solving equations. Concrete manipulative assist students in developing conceptual understanding. Therefore in this study, the researcher using a balancing model method of teaching to teach the concept of a linear equation to develop students' conceptual understanding of the symbolic representation of the equation and the meaning of the equal sign.

2.4. Effect of Balancing Model Teaching Method in Understanding Linear Equation

In the teaching and learning process balancing model is actually not a new concept. Several authors are discussed some of its applications in physics and mathematics subject (Siegler & Chen, 2002; Dawson, et al., 2010). The benefit of employing using a balanced model involves the use of physical material, where students can directly observe and experience how the balance really works. In algebra, the use of a balanced model might be a powerful tool to know the concept of equations simply. Several researchers found many advantages of balancing model, especially in bridging students' procedural and conceptual knowledge to more structural understanding (Dawson, et al., 2010). Other advantages offered by the balance model are discussed in the following sections.

2.4.1. Promoting the view of the equal sign as a symbolic representation

Balancing models help students understand the equal sign. According to Polly (2011) to examine third-grade students related to solving linear equation problems using a beam balance model are important to understand the meaning of equality. within the study, Polly noted that beam balance models help students deeper understanding of equal sign because as boxes are placed on the two sides of the equation, the heavier side drops until the equation are balanced. A limitation of including Polly's study is generalizability between third grade and middle school students. In the teaching and learning process, teachers usually give a direct implication that an equation is indeed like a balance pivoted about the equal sign without any visualization (Sfard, 1991). This would hardly encourage students to imagine how it could happen and why they need to maintain a balance. Such questions need an indirect answer with visual proof that can be done by presenting and experimenting with a real balance scale in the classroom. This idea is important because transferring the idea of transformation in algebra would require seeing an equation as an object to be acted on (Sfard, 1991).

2.4.2. Developing student's symbolic understanding

The second advantage of balancing activity is related to the first one that is to promote the symbolic representation of the equation. Vlassis (2002) found that a balanced model is an effective tool for conveying the principles of transformation because the principles applied to create a balance situation on a balance scale really suit the process of solving equations.

Additionally Caglayan and Olive (2010) conducted a qualitative study in which eighth-grade students (n = 24) solving linear equation through balancing model using cups and tiles to represent variables and constants respectively. So, Caglayan and Olive (2010) concluded that models and representations are developing student's symbolic understanding for solving a linear equation. One limitation of this study was the inability to exhibit subtracting integers with this balancing model. In this case, the balance scale gives meaningful insights on eliminating the same terms from both sides of an equation to obtain the value of the unknown.

2.4.3. Increasing conceptual understanding

Mathematical representation or models are used to represent abstract concepts, such as algebra, explicitly and concretely; learners understand the concept by acting in a hands-on manner (Moyer, 2002). So students to learn mathematical concepts by using representations or models increase their conceptual understanding. Suth and Moyer-Pockenham (2007) explained that providing students with the balance model gives them the opportunity to come up with multiple representations. Having experience with real balancing stimulates students to show their understanding of drawings, verbal explanations, or even formal representation freely. Their experiences become sources of reflections when they are going to represent their ideas.

Besides the advantages, some studies also reported limitations of the balance scale to solve linear equations. This limitation is revealed by Surber and Gzesh (1984), the balance model seems incapable, and even confuses students, to work with reversible operations and difficult to solve linear equation with subtraction. Those limitations imply the need to present other supporting activities to cover what the balance scale cannot.

2.5. Educational Sustainability Development

Educational sustainable development is to empower students in developing competencies through a holistic interdisciplinary perspective of content pluralistic active learning teaching strategies. According to Hopkins (2012), education for sustainable development means including key sustainable development issues into teaching and learning; it also requires participatory teaching and learning methods that motivate and empower learners to change their behavior and take action for sustainable development. Pauw, et al., (2015) studies stated that educational sustainable continues to grow both in content and pedagogy and its visibility and respect have

grown in parallel. The study also showed that educational sustainable contents cover diverse disciplines and pedagogical approach develops students' critical evaluation of alternative perspectives and calls for learner-centered teaching strategies (e.g. critical thinking, participatory decision making, value-based learning, and multi-method approach) (Pauw, et al., 2015). Essentially, educational sustainable development aims to facilitate learning in such a way that learners understand the concept based on their own observation and develop competencies to take action for sustainability.

Mathematical representation or models is the process of formulating and improving a mathematical concept to represent and solve a real-world problem. Mathematical representation or models are plays useful roles towards sustainable development in arriving the understanding, prediction, and control development process (Teacher, et al., 2018). So the mathematical model is the application of mathematics to solve real-life problems. Also, mathematics teachers need to use mathematical models or teaching aids in the mathematics classroom to enhance and sustain alternative strategies to teach mathematics. As Warschauer, et al., (2014) has suggested mathematical models or teaching aids make teachers take easier, effective and motivate students' attention to be active learners. So the instruction and learning of mathematics improve when mathematics teachers use alternative teaching strategies in coincidence with the learning style preferences of their students (Dyer & Osborne, 1999).

Several researchers' findings indicate that learners can explore problem situations and alternative ways to solve problems. For example, Carpenter, et al., (1998) found that many elementary students were able to use alternative strategies to solve a problem. They also found that 65% of the learners in their research sample used an alternative strategy before standard algorithms were taught. By the end of their study, 88% of their sample had used alternative strategies at some point. They also found that learners who used alternative strategies to solve the problem it better conceptual understanding in the subject area of mathematics.

In order to develop the sustainability of balancing model teaching method at Ewket Fana Primary School, the intention of this study is to design learning activities which offer a new approach and to give some insight to mathematics teachers and students in a short time the effectiveness of balancing model teaching method in the concept area of linear equation with one variable.

2.6. Summary of the Literature Review

This chapter reviewed pertinent literature regarding learning theories, algebraic thinking, solving linear equations, and the effect of balancing model teaching approach and the sustainability of balancing model teaching method. Firstly this study uses a constructivist paradigm to develop students understanding of solving linear equation through balancing model teaching method. This approach is strongly supported by how students learn constructivist views of knowing and learning that learners construct their own knowledge of learning by through practice and interacting cooperatively and collaboratively in small or large social groups through engaging actively in problem-based activities.

Several researchers are providing algebraic thinking is struggles in middle school students, as well as many researchers, are giving that some guidelines and standards which middle school curriculum should include representations and models. While solving equations, students should not just develop procedural understanding, but conceptual understanding must be fostered as related to symbolic understanding and the meaning of the equal sign. Different researchers have emphasized the effectiveness of balancing model teaching method on the concept of solving linear equation, but not all researches agree with this idea. So in the present study, the researcher hobs identified the benefits of using balancing model teaching method for the teaching of solving linear equations including distinct representations of equation elements. This study will contribute insight into the use of a balancing model method of teaching in the concept of linear equation.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.1. Research Design

This study generally aims to contribute an innovation approach on algebra teaching in grade five, particularly the topic of linear equations with one variable. This study used for the quasi-experimental single-group design-based research. This single-group design research allows the same group to be compared over time by considering the trend of the data before and after the treatment. Moreover, this study employed both quantitative and qualitative approaches.

3.2. Design of the Intervention Plan

In order to support students understanding of solving linear equation with one variable, the researcher designed the lesson based on constructivism theory of learning by integrating the use of contexts and balancing model. To reach the aim and answer the basic research questions, the researcher makes a design of learning sequences equipped with teaching and learning materials and design the lessons into twelve days lesson. Therefore, the twelve days intervention plans have been designed as in the following table.

Table 3.1: Intervention plan for each topic on solving linear equation

Day	**Topic**	**Learning outcome: at the end of this lesson, students will be able to:**
One	Pre-test	Take the pre-test
Two & three	Meaning of equation and equation formulation	- Define the meaning of equal relation. - Establish an equality statement.
Four & five	Addition of equation in the form of; x + b =c.	- Find the value of the unknown number. - Find the general formula of the equation; x + b = c.
Six & seven	Subtraction of equation in the form of; x - b = c	- Find the value of the unknown number. - Find the general formula of the equation; x – b = c.
Eight & nine	Multiplication equation in the form of ax = c	- Find the value of the unknown number. - Find the general formula of the equation; ax = c.
Ten & eleven	Multi-step equation	- Find the value of the unknown number. - Find the general formula of the equation; ax + b=c.
Twelve	Post-test	- Take the post-test.

According to table 3.1, the researcher conducted the teaching experiment process for ten days by preparing an intervention plan (see Appendix E). Before the implementation of the teaching experiment, the researcher administered the pre-test for the participant students. After administering the students pre-test, the researcher implemented the intervention process by preparing the lesson plan delivered using balancing model teaching method. While walking around and instructing with students during the lesson, the researcher asked students to explain their work related to balancing model teaching method. In design-based research, therefore, the theory and the practice are intertwined to develop theories about the learning process and the activities or tools that can support the students learning.

3.3. A Procedure of Research Design

In order to achieve the aims of the study, the researcher designed instructional activities based on the constructivist theory of learning, tried and revised each lesson. This approach has three main phases including 1) preparing for the experiment 2) teaching experiment in the classroom and 3) conducting a retrospective analysis.

3.3.1. Phase 1: Preparation and design

The purpose of the preparation phase of design-based research is the analysis of the practical problems. In this study, the researcher reviewed some studies related to reported problems that usually happened in algebra classes, particularly in the Ethiopian classroom. This would be followed by studying the Ethiopian curriculum and its components, such as textbooks, a list of topics and teachers guide released by the ministry of education of the country. In order to do that the researcher collected preliminary data about the characteristics and prior knowledge related to linear equation concepts of the participant students. Combining that information, the researcher comes up with critical causes of the students' difficulties in learning linear equation. Then after, the researcher reviewed other studies that focus on how to teach linear equation, to figure out what the students will have during their linear equation studies. Finally, learning materials developed. This included lesson plans, teacher guides, manipulative and students worksheets. The next step was to test these conjectures in teaching experiment phase.

3.3.2. Phase 2: Teaching experiment

The main goal of the second phase was to test and improve the conjecture that is already made in the preparation phase and to develop an understanding of how it works. In this phase, the researcher tested the sequence of instructional activities designed in the preparation phase. Due to the cyclic character of the design based research, the teaching experiment phase should have one macrocycle containing a full series of learning. The macrocycles consist of micro cycles. During this teaching experiment, the researcher collected data through observation, students' worksheet and students' reflection. After the implementation of one microcycle, the learning process was analyzed in each micro cycle, to adjust the continuity of the learning sequence. As in the pilot study, this learning was also analyzed during the micro cycle and later

after the macrocycle. At the closing session of the teaching experiment phase, the researcher gave post-test to the students.

3.3.3. Phase 3: Retrospective analysis

In the design based research, the retrospective analysis phase aims to review, observation and interpretation of what was happening in the classroom during the teaching experiment phase. This phase was done by comparing the hypothesized students' learning with the actual conditions during the teaching experiment. In this study, the researcher used qualitative analysis from classroom observation and interview used to get an insight about how the students understanding in the concept of linear equation and students and teachers point of view about the innovative approach during the learning process. In addition, the teacher made tests examined the student's prior knowledge and conceptual change after an intervention. The researcher compared the result from pre-test and post-test to obtain a quantitative and qualitative explanation of how the innovative teaching approach support students' conceptual understanding of solving a linear equation.

3.4. Population of the Study

The aim of the study was to develop students understanding of solving linear equation with one variable through balancing model teaching method. Based on the purpose of the study, this study was conducted at Ewket Fana Primary School, grade five students and their mathematics teachers. In Ewket Fana Primary School, there were five sections grade five students (with a total number of 223, M = 109 & F = 114) and one grade five mathematics teacher. These were considered as populations of the study. Therefore, the target populations of the study were Ewket Fana primary school grade five students and their mathematics teachers.

3.5. Sample Size and Sampling Technique

To determine the appropriate sample size for this study, one section students (with a total of 48) out of five sections of grade five students were randomly selected and their respective mathematics teacher was selected using comprehensive sampling technique. The reason for selecting this teacher using comprehensive sampling technique is that in this school there is only

one grade five mathematics teacher. Therefore, the participants of the study were grade five section A students and their respective mathematics teacher.

3.6. Data Gathering Instruments and Procedures

The data collection instruments of the study were classroom observation, test and interview. In this study, the researcher gathered data based on the study design procedure charted out in the preparation phase, teaching experiment phase and retrospective phase

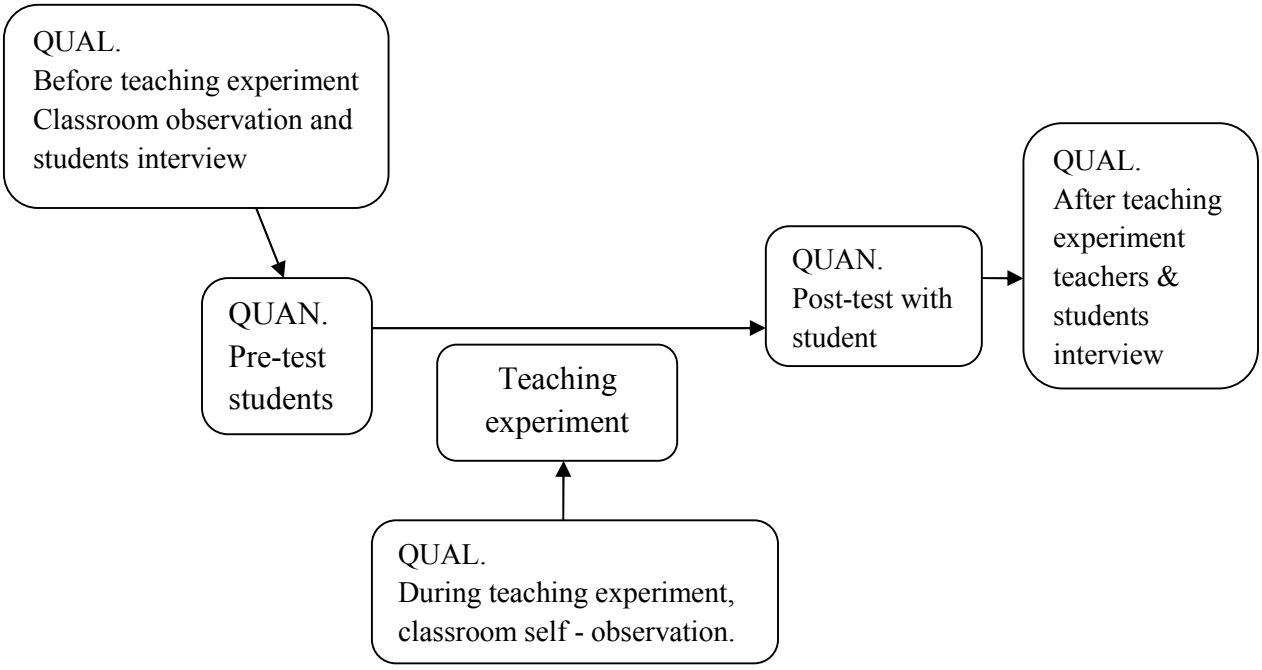


Figure 3.1: general data gathering instruments of the study

3.6.1. Preparation phase

Collecting data in this phase mainly aims to provide sufficient information about students' prior background. This is important to reveal the starting points of the students and the applicability of the design in the classroom, and to make an adjustment with the initial step in our hypothetical learning theory. There are three instruments used that the researcher gathered data in the first phase, such as classroom observation, pre-test and interview with students.

3.6.1.1. Classroom observation before teaching experiment

In the preparation phase, the researcher collects data through observation. This data gathering tool helps to get a general overview of student's motivation and participation in the previous lesson and to get a general overview about whether the student mastered the previous lesson (see Appendix A). Generally, the purpose of this observation was to find out the students status about the previous topic.

3.6.1.2. Pre-test

After observing the participant students' teaching-learning process on mathematics subject, the researcher administrated pre-test prior to the teaching experiment. The pre-test questions manly aims to reveal two things, that is, to know the students' understanding of the prerequisite knowledge and to find out the students' understanding of the topic of the lesson (see Appendix B). This test organized in the form of a written test.

3.6.1.3. Students' pre-intervention interview

After conducted the classroom observation and administrated the student's pre-test, the researcher interviewed five students. The researcher selected 3 students they were low middle and high achieving on the pre-test and chose two students who were low and high participants in the classroom activities. The purpose of this interview was to collect data related to students' attitude towards mathematics subject in general and linear equation in particular.

3.6.2. Data collection during the teaching experiment phase

The data gathered during this phase was expected to describe how the students learned during five series of lessons and how far the design helped students to learn. In this phase, the researcher collected data from classroom observation and students self-evaluation reflection. Then after, the researcher collected data at the end of the teaching experiment using post-test to analyze the students' post-test result.

3.6.2.1. Classroom self-observation

In the teaching experiment phase, the researcher evaluated the students' teaching-learning process in each lesson. During that time, the researcher collected data using observation to determine the students' motivation and attitude in the learning process. The observation checklist

(see Appendix C) was modified to match the initial observation checklists items. Moreover, this data gathering tool was intended to describe preconceived notions related to benefits and challenges attributed to each treatment, to describe concerns related to using balancing model which resulted from the few times and to identify the strength and limitation of this teaching method.

3.6.2.2. Post-test

After conducting the teaching experiment, the researcher administrated the post-test to participant students. The post-test consists of linear equation problems, as it is used to examine the students' conceptual change after the teaching experiment in the area of solving linear equation with one variable. The purpose of this post-test was to investigate the effect of the learning process and to figure out the extent to which students understanding about linear equations after joining the actual lessons (see Appendix B).

3.6.2.3. Post-interview with teachers and students

At the end of the lesson, the researcher conducted an interview with grade five mathematics teacher and representative students. The students' interview questions were modified to match the initial interview questions and the teacher interview was related to the importance of the balancing model teaching method. Generally, the purpose of this interview was to find out the perceived benefits attributed to each treatment and teacher and students view about the balancing model method of teaching approach.

3.7. Pilot study

A pilot study was conducted to establish the suitability of the instrument. In this study, to test the appropriateness of the conceptual test items twenty grade five section C students of Ewket Fana Primary School were randomly selected for the pilot study. The purpose of the pilot study was to determine the reliability coefficient of the instrument of the study through a trial run. The validity and reliability of the scores were then established.

3.8. Validity and Reliability of the instruments

3.8.1. Validity of the instruments

The instrument used in this study was self-developed by the researcher based on students' misconception and difficulties in learning algebra concept in the current mathematics literature and practical experience. To bring the study near scientific rigors, the researcher pays attention to the validity and reliability issues during the data collection and data analysis process. In gathering information during the preparation and implementation phases, the researcher employed data triangulation, in which more than one method used to gather the students algebraic thinking. To guarantee the validity of the instruments, the test and interview questions were validated by the use of none statistical approaches including peers' and my research advisor's critical evaluation such as a member of the department, grade five mathematics teachers and school moderating committee and thereby edited and selected relevant items on the basis of the comments.

3.8.2. Reliability of the instruments

To ensure the reliability of the conceptual test the researcher has test the reliability of the conceptual test through Cronbach coefficient. Therefore, the reliability coefficient of the test was 0.76. This reliability coefficient was considered adequate for the internal consistencies of the conceptual test.

3.9. Data Analysis

To answer research question number one, which reads '*to what extent students' show better conceptual understanding of solving linear equation when they are taught using the balancing model method of teaching'?* The researcher utilized the quantitative data collected from the teacher made test result, and qualitative data collected from classroom observation. In this study, the researcher analyzed the quantitative data by comparing the pre- and post-test results for each student using paired samples t-test analysis method to determine whether there exists a statistically significant difference in the performance of each student before and after the teaching experiment. In addition, the researcher analyzed qualitative data based on classroom self-observation data using data-driven thematic analysis method. The very purpose of the

qualitative analysis was to explain the difference between the student's conceptual change before and after the intervention.

To answer research question number two, which reads '*How can I make a balancing model of teaching approach be established and sustained on solving any linear equation with one variable at Ewket Fana Primary School*?' To answer this question, the researcher analyzed qualitative data obtained from classroom observation and interview made with a teacher and selected students. Based on the teacher and the students' response, data-driven thematic analysis method was done to check the sustainability of balancing model teaching method on the concept of solving any linear equation with one variable.

3.10. Ethical Considerations

In research, ethical consideration is one of the most important points that deserve attention. This is mainly due to:

- ✓ The necessity to strictly respect the consent of the participant whether they are willing to participate in the research or not. Likewise, it is to assure to the subjects of the study that they are free to withdraw from participating in the study whenever they found it necessary.
- ✓ The necessity to protect subjects of the study from possible dangers that might be encountered;
- ✓ Confidentiality was confirmed, i.e., the actual names of participants in the study are kept a secret while the sex or age of the respondents might be used where it seems appropriate.

Thus, the researcher assured the participant students and their mathematics teacher that the research is strictly governed by the above ethical principles and they have also agreed. This was done during data collection and implementation time.

CHAPTER FOUR

FINDINGS OF THE STUDY

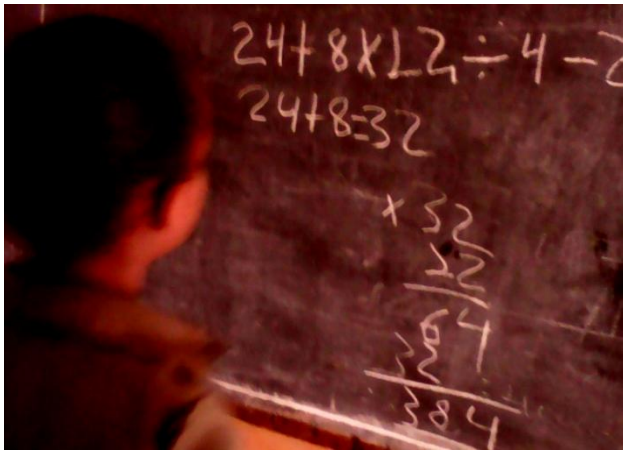
In this chapter, results found in the study are presented. The results are presented in three broad sections in line with the basic research questions of the study. In the first section, the participant students' prior understanding is presented next to this the development of students understanding about solving linear equation is outlined, and the sustainability of balancing model teaching method on the concept of solving any linear equation is reflected in the third part.

4.1. Students Understanding about Algebraic Expression

To find out the status of the students' understanding of the previous lesson, the researcher gathered data through observation, pre-test and interview before the beginning of the teaching experiment. These data serve as the baseline data for this study. Moreover, these data served as a piece of evidence to gain insights about the students' prior understanding of the previous topic, i.e., the students' understanding related to equation concept, the students' degree of classroom participation and teacher's teaching method.

4.1.1. Classroom observation before the teaching experiment

Before the teaching experiment, the researcher observed grade five students teaching-learning process for two consecutive lessons. At that time, the researcher was a non-participant observer. When the researcher joined to the target classroom for the purpose of pre-intervention observation, the students were learning about 'simplification of an algebraic expression'. In order to give an overview of what was happening during the pre-intervention lessons, one question each from the two lessons were brought to this presentation. The questions include number 1 and 2 which clearly show the conversation between the teacher and students during the two lessons observed at the preparation phase. In the first day, the students were learning about how to simplify the expression: $24 + 8 \times 12 \div 4 - 2$. One of the students' answers is given as follows:



24 + 8 x 12 ÷ 4 -2=

32 x 12 ÷ 4 -2……..add 24 and 8

384 ÷ 4 – 2……multiplied 32 by 12

96 – 2 ……divided 384 by 4

94……..subtracted from 94 to 2.

Figure 4.1: An example of the student's misconception for the order of operation.

As it is shown in figure 4.1, the student's answer was 94. In contrast, the correct answer was 46. This indicates that students did not have a good understanding of the order of operation rule. Instead, the students level of understanding was to perform multiple-operations in any order which appears possible for them, they attempted to calculate any part of an expression that they think they can do, and in any order.

During the lesson observation time, the researcher observed the second lesson of the preparation phase. In that lesson, the students were learning about simplifying the following algebraic expression: 'simplify the expression $x^2 - 5x$; given the value of x is equal to 4'. The students solved this question by using the following procedure:

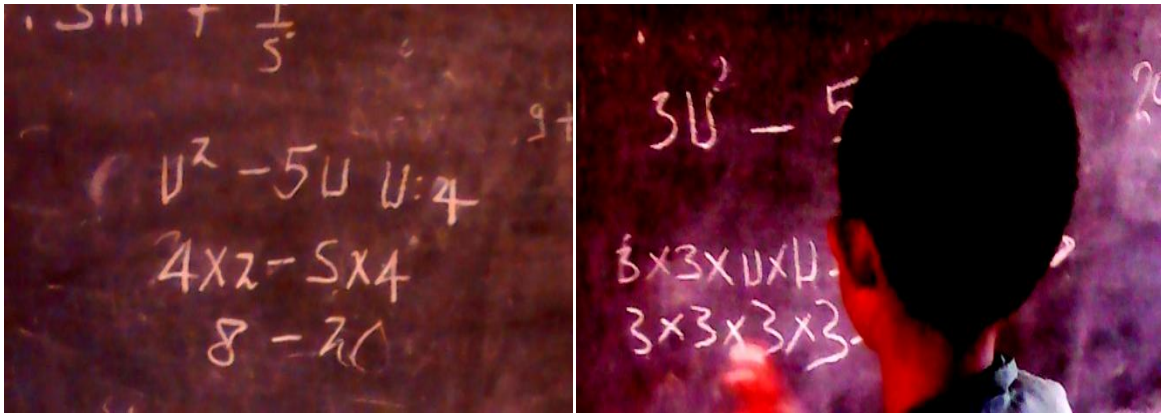


Figure 2.2: An example of the students' misconception for the rule of coefficients and exponents

Generally, the preparation phase observation data showed that the classroom teacher used traditional teaching methods like talk and chalk strategies and failed to use different teaching aids or alternative strategies. During this time, most of the students were passive listeners, i.e., they did not participate actively in the teaching-learning process. During this phase, the instructional message flows only from the teacher to the students. Moreover, the students did not justify their work as the teacher failed to use varieties of teaching methods, and he was unable to encourage the students by giving different activities that could help them to develop conceptual understanding during the lesson.

4.1.2. Pre-test results

As explained in the methodology part, the pre-test had two main purposes. They were identifying the students' understanding of the prerequisite knowledge and checking their understanding of equation concepts. In the present study, the students' prerequisite knowledge that was observed in the pre-test covers their understanding of algebraic expression and the topic of the lesson assessed in the pre-test related to the meaning of variables and basic steps to solving linear equation problem. To achieve these two purposes, six items were developed (see Appendix A). The first three items measured the students' level of understanding of the prerequisite knowledge, while the other three items were used to check the students understanding related to linear equation problem. (The pre-test was conducted on 7/6/2011). On this background, the researcher tried to elaborate on the results of the students' pre-test items.

Problem 1: 'Write the following word problem within a mathematical algebraic expression, i.e., the difference between twice of one unknown number and four.'

The way the textbook is organized and the preparation phase observation data indicated that the students had learned about the concept of algebraic expressions and the formulation of an algebraic expression statement in the previous lesson. In line with what they had learned, the result of the pre-test indicated that only 5 (10.5 %) of the students understood the concept of the problem and correctly changed the given word problem into the mathematical formula. Most of the students (43 (89.5 %)) failed to change this word problem into a mathematical formula. In an attempt to formulate algebraic expressions, only 5 students did the task by using mathematical

symbols whereas most of the students were deficient in competence in abstract reasoning, language acquisition and mathematical structure within the learning of algebraic expression.

Problem 2: 'In the expression; 3 + y x 2, given the value of y = 4, find the value of the given expression?'

The aim of this problem was to check the students' understanding of the order of operation. The order of operation is very important when simplifying algebraic expressions and equations. The result from problem two indicated that 6 (12.5 %) of the students were able to accurately understand the order of operation rules and write the correct answer to the question. the remaining 42 (87.5 %) of the students could not simplify the problem using a proper algorithm. This indicates that most of the students could not know the rules of the order of operation.

Problem 3: 'In the algebraic expression $x^2 - 5x$, given the value of x = 5, find the value of the given expression?'

The aim of this problem is to check students' understanding about the role of mathematical symbols (coefficients, constant terms and exponents). In the process of simplifying algebraic expression or solving any linear equation, the students were required to apply a succession of transformation rules in their multiplication of symbols (2n), operation signs (+, -, x & ÷), exponents (x^2) and variables which may appear as constants, exponents and coefficients or other rules in the expression or equation. So, looking at the result from problem three only 2 (4 %) out of 48 (100%) students were able to solve the problem correctly, that is, $x^2 - 5x = 5^2 - 5x5 = 25 - 25 = 0$. And the other 46 (96%) of the students did not solve the problem correctly. They rather attempted to solve like $x^2 - 5x = (5 \times 2) - (5 \times 5) = 10 - 25 = -15$. So this shows that most of the students did not know the rules of exponents and coefficients.

Problem 4: 'Fill in certain numbers to equal the two expressions; such as:☐ + 5 =☐ + 2.

The aim of this problem is to check whether or not the students understand the equal sign or equal relation. An incomplete or incorrect understanding of the meaning of the equal sign will be detrimental to students' performance and learning linear equation problems. In this problem, an attempt was made to ask students to find a number that satisfies an equivalent equation. Moreover, this problem allowed more than one solution. Based on this, the researcher examined

the students' conception of the meaning of equality or equal relations. So, 31 (64.5 %) of the students were not aware of the meaning of equal relation as they could not write the correct answer. The other 17 (35.5 %) of the students used certain notations to represent the unknown number. An example of the students' possible answer could be $\boxed{2} + 5 = \boxed{5} + 2$. This representation shows that a good arithmetical understanding of students in the idea of equal relation. But, most of the students did not use this strategy.

Problem 5: 'In the equation; x + 4 = 12, find the value of x?'

The aim of this problem was to examine the students' understanding of the inverse operation. In this case, in order to solve the problem the students should remove 4 from both sides of the equation, this means creating zero pairs on the constant term of the given equation. However, the result of problem five indicated that only 37 (77 %) of the students answered the problem using a proper algorithm by selecting 8. But, out of 37 students, 29 students answered this problem using their memorization skill or reality. This means that the students worksheet shows that 8 + 4 = 12, so the value of x = 8, whereas the other 8 students solved this question by using inverse operation rule; x + 4 = 12; x + 4 – 4 = 12 – 4 = x = 8. The remaining 11 (23 %) students did not solve the problem correctly. Therefore, based on students answer on the worksheet most of the students did not master the undoing process and creating zero pairs or emphasized memorization skills without its meaning.

Problem 6: 'The cost of one cloth is 93 birr; this cost is three times the cost of another cloth. Find the cost of the second cloth?'

The aim of this problem is to observe students conceptual and procedural understanding of solving linear equations. Therefore, problem 6 was designed to test the students' mathematical representation skill and conceptual understanding. The result of problem 6 indicated that there were 24 (50 %) students who failed to understand the concept of the problem and they solved as the cost of the second cloth is; 93 x 3 = 279 birr. The remaining 24 (50 %) of the students understood the concept of the problem and wrote the correct answer. But out of 24 students, 11 students did not use certain notions to represent the unknown number; instead they used directly guess-and-check strategies.

4.1.3. Students' pre-intervention interview

After collecting data through classroom observation and students pre-test, the researcher gathered data through student interview. The essence of this pre-intervention interview was to get the opinion of the students about the current traditional, teacher-center method of learning mathematics in Ewket Fana Primary School and the students view about the mathematics subject. The researcher used the following pseudo names to represent the five students that participated in the interview. Student 1: Meseret, student 2: Adane, student 3: Almaze, student 4: Yared and student 5: Abebech. All of the names were made to represent names in the Amhara region culture in general and names in Bahir Dar town.

When asked student's opinion about mathematics as a subject, the students tried to disclose the following:

> For me mathematics subject is very difficult. It is very difficult for me as compared to other subjects. I don't like mathematics as it made me to memorize many formulas to solve mathematics questions (Meseret, pre-intervention interview data taken on February 8/ 2011).
>
> In my view, mathematics subject is a very difficult subject. It is difficult because the subject requires me to remember many formulas to solve mathematics problems. So, I personally felt that mathematics is a very difficult subject. (Adane, pre-intervention interview data on February 8/2011)
>
> I don't like mathematics subject because it is a very difficult subject. It always requires me to do a lot of thinking and memorizing the formula or the procedures. (Almaze, pre-intervention interview data on February 8/ 2011).
>
> Mathematics is my favorite subject. This subject is simple to understand the concept and I like mathematics. (Yared, pre-intervention interview data on February 8/2011).
>
> I do not enjoy studying mathematics (Abebech, pre-intervention interview data on February 8/2011).

Similarly, when the researcher asked the students' views about the connection between the mathematics concepts learnt in school with the outside world, the students stated as follows:

> No, I think there is no connection between the two. (Meseret, pre-intervention interview data on February 8/ 2011).
>
> I don't know. I think they have no connection (Adane, pre-intervention interview data on February 8/2011).

> Yes, for me I only see some arithmetic like the one involving buying and selling. (Almaze pre-intervention interview data on February 8/2011).
>
> Yes, because all people use mathematics concepts by any means. (Yared, pre-intervention interview data on February 8/2011).
>
> No, it is hard to see. For me, I see buying and selling or collecting change, but I really do not think that is mathematics. (Abebech, pre-intervention interview data on February 8/2011).

When the researcher asked the students opinion about currently learning mathematics in school, the students attest that:

> Studying mathematics is very difficult; it's very hard for me to use many formulas to do mathematics problems (Meseret, pre-intervention interview data on February 8/2011).
>
> I see mathematics subject as a hard subject. Because mathematics subject is very difficult for me as compared to other subjects. (Adane, pre-intervention interview data on February 8/2011).
>
> I think it is very difficult to learn mathematics because it is difficult to memorize the formulas and procedures therein. (Almaze, pre-intervention interview data on February 8/2011).
>
> In the current situation, mathematics teaching and learning as well. I have a relatively better interest to mathematics subject as compared to other subjects. (Yared, pre-intervention interview data on February 8/2011).
>
> Studying mathematics subject is difficult as it requires remembering many formulas. (Abebech, pre-intervention interview data on February 8/2011).

Table 4.1: summary of the pre-intervention interview

Questions	**Number of students that gave a positive response**	**Number of students that gave a negative response**
What is your opinion about mathematics as a subject?	1	4
Can you see any connection between the mathematics you learned in school and in a real-world situation?	2	3
What is your opinion about the ways you are currently learning mathematics in school?	1	4

Table 4.1 shows that most of the students do not have a positive impression of mathematics as a subject. Moreover, only one of the interviewed students had a positive opinion about mathematics as a subject and the way they are currently learning mathematics in Ewket Fana Primary school. On the other hand, only two of the interviewed students were able to manage to draw some connection between the mathematics they are learning in school and their real-life situation.

Generally, the preparation phase data shows that in the previous lesson most of the students did not achieve the lesson study and they failed to actively perform the learning activities. Moreover, these data emphasized that most of the students solved mathematical problems using guess-and-check strategies and they did not have a good attitude towards studying mathematics.

4.2. The Development of Students' Understanding of Solving Linear Equation

In the preparation phase, the researcher reflected the status of the students in the teaching-learning of mathematics subject particularly the concept area of simplifying an algebraic expression. As briefly noted in the result of the preparation phase activity, many students did rely on a guess-and-check strategy to simplify any algebraic expressions. In order to minimize these challenges, the researcher gave the students a series of interventional activities using the balancing model teaching method for the concept of solving linear equation with one

variable. This intervention involves 5 series of lessons, with a pre-test and post-test conducted before and after the implementation of the 5 lessons. In this part, therefore, the researcher tried to reflect the development of the students' equation formulation skill, finding unknowns and how these two activities related to the balancing context during the learning process. During this time, the researcher was a participant observer. So, the following reflections are the result of the researcher's classroom self-observation data during the teaching-learning process of solving linear equation problems.

4.2.1. Lesson one: the meaning of an equation and equation formulation

Before attending this lesson, the students learned about simplifying an algebraic expression, such as the introduction part of linear equation. It is also important that the students knew about the meaning of variables, constant terms and mathematical operations. Although the students might have not used variables and constants, they might have found it in a form of balance or label in an algebraic expression. Therefore, the purposes of this lesson were to:

- Make sense of equal sign and equal relations.
- Formulate the equivalent equation using mathematical algorithms.
- Define the meaning of linear equation.

To achieve the above objectives, two main activities were presented. They are defining equal relation and formulation of the equivalence equation. So, the researcher started the lesson by asking students thinking about the meaning of equality based on their cultural event. After the students engaged with the context, the researcher asked the students in a group to measure the weight of 8 tiles on a real balance scale. This activity would require students just to give the concept of equal relation of two things they had performed and to make the generalization about the meaning of equality. During that time, all of the students put the same amount of tiles on both sides of the balancing scale. So this activity would be made simple to the students to measure two equal things.

The second main activity is also the formulation of the equivalence equation. In this part, the students would be challenged to formulate equivalence equation with their own experiences. So, the researcher asked the students to explain and discuss the mathematical structure of the sum of seven and one unknown number is nine. At the onset, the students were confused to write

this way in mathematical equation form. Some students answered that the unknown number is 2 because the sum of two and seven is nine. This indicates that the students understood facts or rules without its concepts. In order to develop students understanding about the formulation of the equivalence equation, the researcher gave some instruction to the students using a balancing model to formulate the above problem in mathematical equation form. To answer this question, the students might do the right side of the balance scale to put seven tiles and one cups and the left side of the balance to put nine tiles. For this context, most of the students may come up with the idea of equation formulation. Thus are; $7 + x = 9$. Therefore, after conducting this lesson many students were able to formulate an equation using mathematical symbols.

4.2.2. Lesson two: finding the unknowns

Based on what the students have done in the first lesson, the researcher assumed that the students would have been able to formulate an equation with mathematical structure. The formulation could be that the students understood the generalizations of numbers or representations of certain objects. However, during the first lesson, the students knew about only formulating a representation of mathematical equations. But, they did not see built-in relationships of the concepts with quantities. Therefore, in this lesson an attempt was made to realize the following lesson objectives:

- Use the rules of balancing to find the unknowns.
- Find the general formula of solving linear equation.

To achieve the above objectives, four sub-lessons were conducted. These are:

- Solving linear equation in the form of; $x + b = c$.
- Solving linear equation in the form of; $x – b = c$.
- Solving linear equation in the form of; $xb = c$ and
- Solving multi-step linear equation when x is a variable and b and c are constant terms.

4.2.2.1. Solving linear equation in the form of; x + b = c

Based on the analysis of the first lesson, the researcher concluded that most of the students had achieved the objectives of the first lesson. Therefore, the researcher revised lesson one and moved to lesson two. In this lesson, there were two main activities that would be

conducted; namely, finding the unknown number and generalizing the mathematical formula of solving for unknowns with addition. The researcher started the lesson by asked the students' prior understanding relating to equation formulation. The researcher told them in this lesson they investigate the unknown number of the equivalence equation and formula generalization. The researcher asked the students to remove the weight on one side of the balance, and what they could do on the other side to keep the balance is balanced? Based on this question, the researcher expected that students come to the idea of the addition of an equation with this activity. The conjectures and expectations toward this activity were the students would remove the same number of tiles with various strategies. If they removed the same amount of tile with both sides of the balance, the learning process was continuing to the idea that addition of equation in balancing context. In this case, the students would put tiles and cups on the balancing scale (tiles represent constant numbers and cups represent unknown numbers) and removed both sides of the balance scale the same amount of tiles. After that they easily got the value of the unknown number. In this activity, therefore, most of the students had removed the same number of tiles with different strategies and to find out the value of the variable by using a beam balance model. The aim of this activity is to encourage the students to think one step further.

After the students solved some contextual problems using a beam balance model, they moved to more mathematical situations, especially in their strategy to find the unknown in a given linear equation problem, that was related to this context in a mathematical equation. Then, the learning activity was continuing to solving linear equation in the form of; $x + b = c$. So the researcher asked the students in the group to find the solution of the equivalence equation $x + 3 = 9$. However, most of the students did not solve this problem using mathematical formulas, but the students respond this activity saying that the value of $x = 6$, because $6 + 3 = 9$. It implies that the students emphasized memorization skill or realities without its concept. Based on this, the researcher gave some instruction to the students to solve this problem by using balancing models and related this way in mathematical aspects. As a result, most students express the idea in the following way.

Figure 4.3: The students answer and argument for solving linear equation with addition

Figure 4.3 shows that after conducting solving addition of linear equation through balancing model, most students developed flexibility strategies to solve any linear equation problems.

4.2.2.2. Solving linear equation in the form of x – b = c.

In this lesson, the context still related to the previous lesson which is solving linear equation with addition. So, the researcher started the lesson based on the previous lesson concept by simply asking the students understanding about the inverse operation. This idea was helpful to the students who would involve in the preparation of solving linear equation with subtraction. The students had informally experienced the inverse operation of subtraction, but they did not connect this experience with mathematical aspects. After the students' responses, the researcher asked the students additional problems, which reads 'find the value of the unknown number in the equation; the difference between one unknown number and five is two'. As a result, the students' response was that the value of the unknown number is seven because the difference between seven and five is two. This indicates that most students were mastering memorization skill without its concept (see figure 4.4).

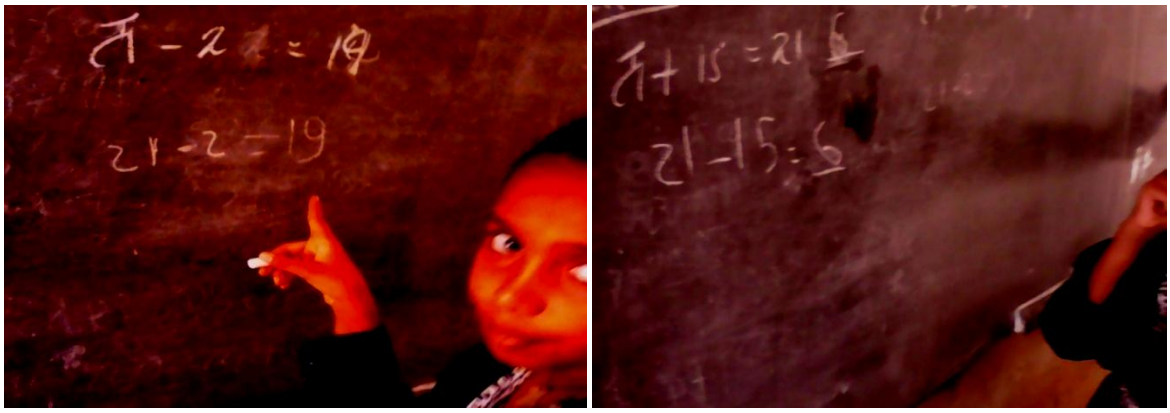


Figure 4.4: The students' gap in solving linear equation with addition and subtraction

In order to minimize the students' memorization skill, the researcher gave some instruction to the students to represent the above equivalence equation using different mathematical models and to figure out the value of the unknown number. After that, most students solved the problem as in the Figure 4.5 below.

Figure 4.5: The students answer and argument for solving linear equation with subtraction

In this activity, the students develop an understanding of the concept of "zero pair" while solving any linear equations. For example, to solve an equation, x – 5 = 2, the students placed a yellow x and five negative unit pieces on the right side of the equation. In order to solve this equation, the students added five positive pieces for both sides of the equation. In so doing, the students created a zero pair on the left side of the equation which could be removed, leaving x = 8 as the solution to the equation.

4.2.2.3. Solving linear equation in the form of bx = c.

The researcher began the lesson by bringing up the topic of inverse operations again. The discussion, therefore, would be the students come up to find the inverse operations of multiplication and to introduce solving linear equation with multiplication. In this case, the students list the inverse operation of addition, subtraction, multiplication and division. Based on the students' response, the researcher gave the other additional activities to the students, such as find the value of the unknown number in the equation 2x = 6 and 5m = 2. These problems aimed to observe the students understanding of the inverse operation. After the students' discussion, some students solved those problems based on informal strategies, used guess-and-check strategies. It also assumes that the students did not know the undoing process of solving linear equation with multiplication frequently. Then after, the researcher asked students to represent and solve the first problem by using a beam balance model. For this occasion, the students were simply representing and solve the problem in the following ways.

Figure 4.6: The students answer and argument for solving for unknowns with multiplication

Based on figure 4.6, the researcher observed that the students developed different strategies to solve any linear equation. But, when students learned how to solve multiplication equations, the idea of "undoing" process, some students struggled to create the general formula of solving linear equation and some students' emphasized memorization skill.

So, to enhance the students understanding of solving for unknowns with multiplication, the researcher repeatedly taught the lesson by using balance model after solving multiplication of

linear equation in the context of balancing scale. In the end, everyone is more experienced of the idea of undoing process and sensitive to the teaching-learning process.

4.2.2.4. Solving linear equation in the form of; ax + b = c

Solve for unknowns using balancing strategies were given to students in the first four lessons. Thus, the researcher assumed that the students would have been able to solve simple problems involving real balance scale. In this case, students' ability to manipulate the equations would still need to be developed. However, the students did not experience solving linear equations without balancing scale. In addition during the first four lessons, the students cannot recognize a linear equation problem given in a non- balancing context. Therefore, the students could move to a more mathematical situation, especially in their strategy to find the unknown in a given multi-step equation. So, the purposes of the present lesson were to:

- Build students flexibly using strategies in solving any linear equation.
- Develop students understanding of solving linear equation problem without balancing context.

To achieve the above objectives, the researcher started the lesson by communicating the previous lesson that they experienced equation representation and formula generalization, asked the students the general formula of solving for unknowns in a linear equation problem with one operation. Thus, most students had explained the general formula of solving for unknowns in a linear equation with one operation. So, the lesson five would be focused on more mathematical aspects rather than mathematical models. This would guarantee the applicability of the students' knowledge in a more general situation.

In this lesson, the researcher asked students two problems related to multi-step equation ($3x + 1 = 7$ and $2y – 2 = 4$). In those activities, the researcher expected that the students find the value of the variable by using mathematical formulas. In this occasion, the students were challenged to solve the problem by using mathematical formulas. Like the first time, most students listed the value of the variable without formula. In that case, the researcher gave the students some instruction related to represent this multi-step equation in different representations and after that to find out the value of the variable. After the researcher's instruction, most of the

students represent this mathematical equation in different models and persisted in their understanding of zero pairs and undoing process to solve in the following ways; 3x + 1 = 7.

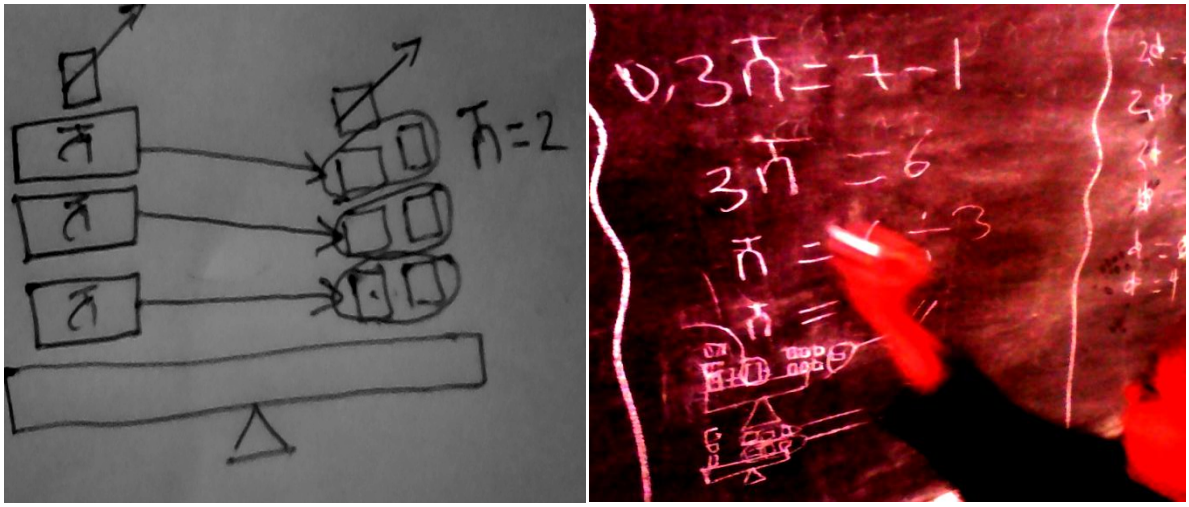


Figure 4.7: The students' answer and argument for solving multi-step linear equation

Figure 4.7 shows that the students seemed to be more flexible in selecting various strategies when dealing with solving linear equation problems after conducting the teaching experiment.

Generally, during the teaching experiment phase the classroom observation data indicated that, at the beginning of the teaching experiment, most students did not used a proper algorism to solve the given problems; instead they reflected guess-and-checked strategies that were adapted previously because of the students were unfamiliar with this teaching method. Thus, when students learned about solving linear equation through balancing model repeatedly they used different strategies to solve any linear equation problems and they had more interested to the lesson study.

4.2.3. Post-test results

After conducting the teaching experiment lesson, the researcher gave the post-test to the students on February 25, 2011. The post-test contains six questions about the five lessons of the teaching experiment lessons. The first problem of the post-test was to check the students understanding of the equation formulation and the other five problems were to check the students

understanding of finding a solution. The following presentation would discuss about each of the post-test items.

Problem 1: 'The difference of one unknown number and seven is ten. Write this word problem in the form of equivalence equation?'

The first problem of the post-test is developed to measure the students' equation formulation strategies. On the pre-test, most students did not change word problems into mathematical structure. After conducting the innovative approach (balancing model method of teaching), the students post-test result indicates that 36 (75 %) of the students were able to change this word problem into mathematical structure, such as $x - 7 = 10$. However, the remaining 12 (25%) of the students failed to understand the concept and provided an incorrect answer to the question raised. Based on the post-test result, therefore, most students have developed understanding of equation formulation.

Problem 2: 'In the question, the sum of three and one unknown number is twelve. Write the problem in the form of equivalence equation and solve the value of the variable number step by step?'

The main purpose of this problem was to know the students conceptual and procedural understanding of solving for unknowns with addition. In this problem, the students identified the meaning of the variables and constant terms. From the students' answer, 25 (52 %) of the students answered the question by using a proper algorithm to find the value of the unknown number. However, 8 (16.5 %) of the students answered this question based on their memorization skill (guess-and-check strategy), that is, the value of the unknown number is 9 because the sum of three and nine gives twelve. The remaining 15 (31 %) of the students did not understand the meaning of inverse operation and failed to write the correct answer.

Problem 3: 'In the problem, the difference between one unknown number and five gives twenty-four. Write the problem in the form of the equivalent equation and solve the value of the variable step by step?'

The main purpose of this problem was to know the students conceptual and procedural understanding about solving for unknowns with subtraction. In this problem, the researcher tried

to measure the students understanding about the representation of symbols, constant terms, mathematical operations and meaning of inverse operation. So, the students answer shows that 28 (58 %) of the students perfectly converted this word problem to mathematical context and solved the problem correctly. However, 6(12%) of the students answered the question without any mathematical representation by using guess-and-checked strategies, saying that the value of the unknown number is 29 because of 29 – 5 = 24. The remaining 14 (30 %) of the students did not understand the concept of the problem and, therefore, failed to provide the correct answer to this question.

Problem 4: 'In the equation, x + 3 = 8, find the value of the variable using balancing model and mathematical formulas?'

The purpose of this problem was to check the students' movements from balancing context into mathematical formulas. This problem demands the students an understanding of manipulating and maintaining a balanced situation on a balancing scale, which is indeed a concept of an equivalent equation. Based on the students answer in the worksheets, 42 (87.5 %) of the students were able to solve the problem by using balancing model and were able to change this balancing context in a mathematical situation. And the other 6 (12.5 %) of the students failed to understand the concept of the problem and, therefore, did not solve the problem correctly. Based on this, most of the students developed various strategies to solve any linear equation.

Problem 5: 'In the problem, the product of one unknown number and six are thirteen. Write the problem in the form of equivalence equation and solve the value of the variable step by step?'

The aim of this problem was to check the students understanding of solving for unknowns with multiplication. This problem was intended to measure the students understanding of the undoing process. So, the result of problem 5 indicated that 23 (48 %) of the students were able to represent and solve the problem, and 5 (10.5 %) of the students solved the problem merely based on their memorization skills. The other 20 (41.5 %) of the students solved the equation incorrectly.

Problem 6: 'Solve the following equivalence equation using a mathematical formula, such as; 4x + 5 = 17?'

The purpose of this problem is to examine the students' conception of solving a linear equation without balancing context. Moreover, this activity measures the students' flexibility in relating quantities. This problem would be presented in a formula context that explicitly shows a mathematical formula to solve. So, results from problem 6 showed that a high proportion of the students, i.e., 33 (69 %) of the students could not write the correct answer. The other 15 (31 %) of the students were able to solve the problem correctly. Based on this result, one can imply that most students were not able to solve a multi-step linear equation.

Table 4.2: Paired sample t-test comparison of the students' pre- and post-test result

	Paired Differences				**t**	**DF**	**Sig (2-tailed)**
	Mean	**Std. Deviation**	**95% confidence interval of the difference**				
			Lower	**Upper**			
Posttest-pretest	2.1042	1.6143	1.6354	2.5729	9.031	47	.000

From table 4.2, the mean difference of the two tests was 2.1042 (t = 9.031, p = 0.0001). In this data, it improved marks on average by approximately 2 points. The 95% confidence interval for the difference is (1.6354 from 2.5729). The result shows that there was a statistically significant difference between the pre-test and post-test result of the students.

Generally, prior to the implementation of the lessons, it was noticed that the students' test result and observation data revealed that many students only relied on guess-and-check strategy to simplify any algebraic expression problems. However, after conducting the lesson study through balancing model, most of the students were able to use different strategies to solve any linear equation problems and they were reasonable for their work. Since, the activities proposed in this study helped the students to make sense of some possible ways of solving problems that make them not too strictly limiting themselves to one fixed strategy.

4.3. The Sustainability of Using Balancing Model Teaching Method on Solving Linear Equation

To identify the students' and the teacher's perception of the balancing model teaching method, the researcher conducted an interview with five students and their classroom mathematics teacher after the teaching experiment has been accomplished. The possible findings are presented below.

4.3.1. The students' post-intervention interview

After conducting the teaching experiment and administered the post-test, the researcher conducted an interview with the students. Moreover, the researcher interviewed them so as to get their views and opinions about the learning process and the types of balancing model problems they had been going through in the past two weeks, i.e, lessons conducted during the intervention period.

When the researcher asked the students opinion about the balancing model teaching method, the students stated that:

> Balancing model teaching method is important to understand the meaning of equality. This is because rather than working in text, it is better to work in practice. For example, in the equation $2x = 6$, the expression $2x$ and 6 as in the mathematics aspect are equal and in the balance model context, they are balanced. So, this method made me to like mathematics subject (Meseret, post-intervention interview data on February 26/2011).
>
> Balancing model teaching method is very good because it helped me simply understand the principle of linear equation and other it helped me understand the concept clearly. So I will use this method for the future. (Adane, post-intervention interview data on February 26/2011).
>
> Balancing model teaching method is really important because it supports simply to find the value of the unknown numbers and the knowledge from this will help me solve other real-life problems and I will use it for the future. (Almaze, post-intervention interview data on February 26/2011).
>
> Balancing model method of teaching is useful because this teaching method is easy to find the solution of the equivalent equation (Yared, post-intervention interview data on February 26/2011).
>
> Balancing scale is helpful because it helps simply understand the concept of inverse operation and the process of finding the solution of an equation. So, I will use it for the future. (Abebech, post-intervention interview data on February 26/2011).

When the researcher asked the students understanding about the connection between mathematics concepts learned in school and the real-world situation, the students stated that:

> Yes. I can see. After learning linear equation concepts through balancing model in the school, I came to realize the connection between the two. (Meseret, post-intervention interview data on February 26/2011).
>
> I did not think they had a connection before I learned linear equation concept. But, now I came to understand that they have a strong connection, and I came to see that all marketing process outside the school employs mathematics concept of any sort. (Adane, post-intervention interview data on February 26/2011).
>
> Yes, the knowledge from this will help me solve other real-life problems. (Almaze, post-intervention interview data on February 26/2011).
>
> Yes, I can see the connection. (Yared, post-intervention interview data on February 26/2011).
>
> After having learned linear equation through balancing model, I came to see the connection between mathematics concepts learn in school and realities outside the school. (Abebech, post-intervention interview data on February 26/2011).

Table 4.3: Summary of Post-Intervention Interview Result

Questions	**Number of students that gave a positive response**	**Number of students that gave a negative response**
What is your view about the balancing model tasks you have been going through in the last two weeks?	5	0
Can you see any connection between the mathematics you learn in school and their real-world situation?	5	0

Table 4.3 showed that all of the interviewed students had developed a positive attitude about the balancing model teaching method that they went through in the entire process. In addition, the post-intervention interview also revealed that the students were able to have a better connection between their classroom mathematics and what they see in real life. Therefore, balancing model activities helped almost all the interviewed students see mathematics subject from a different

perspective and thus assisting them to connect their classroom mathematics emphasis with real-world activities.

4.3.2. Classroom teacher's interview data

As elaborated in the methodology section, the researcher conducted an interview with grade five students mathematics teacher after the teaching experiment. The purpose of this interview was to find out the teacher's points of view about the balancing model teaching method. During the final interview, the teacher disclosed that:

> Balancing model teaching method is a very important tool. It allows the concept to be sustainable and connected to the students' real-life situation. Moreover, the method addresses equation concepts to the students clearly by relating the mathematics concepts with students' real-life situation. It is true that using multiple representations appeared to provide students with opportunities to construct a deep understanding of linear equations, require students to link mathematical models to realities in the outside world, and encouraged the students to construct rational ways of explanations that incorporated multiple representations. While the students learn solving linear equation through balancing model, I also observed that most students had developed a good attitude about the lesson and actively participated in the classroom activities. Last, the model helped the students to likely the subject (Classroom teacher's personal interview, February 26/2011)

As the above vignette tried to confirm, the teacher had developed a positive attitude towards using the balancing model teaching method and he further asserted that he will use the balancing model in teaching the concept of linear equation for the future, and he promised to share his experience with other mathematics teachers in the school and those in the cluster center as well.

Generally, after having passed through a series of the learning process using the balancing model teaching method thereby to check the sustainability of the innovative approach, the classroom observation and interview data showed that grade five section A students' and their mathematics teacher's attitude towards studying mathematics in general and linear equation, in particular, has shown dramatic changes. That is, they developed a positive view, and agree to use this innovative approach (balancing model) to solve any linear equation problems for the future.

CHAPTER FIVE

DISCUSSION AND IMPLICATION OF THE RESULTS

5.1. Discussion of the Findings

This chapter contains a discussion of the findings obtained via quantitative and qualitative research approaches used to answer the basic research questions raised at the on-set. The discussion is grouped into two sections based on the basic research questions of this study. While the results of the study confirmed a number of findings of other researchers, the findings go beyond those discussed in the literature and some are presented in this part.

5.1.1. The development of students' conceptual understanding of solving linear equation

In order to complete and conclude the explanation about the students' signs of progress in and after the teaching experiment, the classroom observation was conducted before and during the implementation sessions and also a test was given before and after the classroom sessions. The results of those data are described in this part in comparison with the students' prior knowledge, i.e., before the learning implementation.

The students' pre and post-test result confirmed that the performance of the students before the teaching experiment was not encouraging. That is, the students' pre-test indicated that most of the students did not solve the given problems by using proper algorism. In contrast, the students' post-test result indicated that the students' performance becomes much better compared to the pre-test results and also most of them were solving the post-test items using more flexible strategies as well as achieved a good result. In addition, the results in table 4.3 (M = 2.1 & SD = 1.6) revealed that the students' level of understanding was improved at a statistically significant level and the teaching interventions improved the students' result in linear equation.

On the other hand, classroom observation data revealed that, during the preparation phase observation time, most of the students were passive listeners, i.e., they did not participate actively in the teaching-learning process as well as they did not justify their work; instead during intervention observation time most of the students used different strategies to solve any equivalence equation problems and also they were becoming very reasonable for their work. Moreover, most students had actively participated in the classroom activities and also interested

in the lesson study intervention. This situation shows that most of the students developed more flexible strategies to solve any linear equation problems and more interested to the lesson study after the teaching experiment. Therefore, the implementation of the balancing model teaching method was more effective in developing the conceptual understanding of students in the concept of solving linear equations. This finding agrees with the findings of Caglayan and Olive (2010) who conducted a qualitative study in which grade eight students (n = 24) solving linear equation through balancing model using cups and tiles to represent variables and constants, respectively. So, Caglayan and Olive (2010) concluded that models and representations have developed the students' symbolic understanding for solving a linear equation. In addition, supported this finding Polly (2011) who examined grade three students understanding related to solving linear equation using a beam balance model. The author concluded that using the balancing model to solve any linear equation, the students have developed a better conceptual understanding of the lesson study and it develops the students' creative skill.

The other result obtained from observation data showed that the students who were taught through balancing model teaching method were becoming more and more confident from time to time. This finding lined with the finding of Warren and Cooper (2005) and Johnson (1993). They described the principles of the balance model and students seesaw game activities thus making them to see a connection between what is taught in school and students everyday life activities. Furthermore, the students' attempt of solving linear equations through the balancing context during the teaching experiment time made the students to engage into the lesson, allowed them to reflect on their learning and they were made to reform their errors in their thinking. This result is consistent with the findings of Brizuela and Schliemann (2004). They noted that the teaching methods employed by the model ease understanding of the concept and avoid mistakes when solving linear equations with one variable.

Generally, the results of this study showed that most students had scored better in the post-test. Moreover, in the learning process of solving linear equation through balancing model most of the students come to actively participate in the classroom activities and developed their skills. This indicates that balancing model teaching approach was an effective tool to develop students' conceptual understanding and it can also create a good learning process. This finding agrees with the findings of (Caglayan and Olive, 2010; Johnson, 1993; Schliemann, 2004; Polly,

2011; Vlassis, 2002; Warren & Cooper, 2005). These findings emphasized the effectiveness of the balancing model teaching approach in enhancing the concept of solving any linear equation problems. But, Surber and Gzesh (1984) findings did not agree with the result of the present finding and the above researchers' findings. The first limitation is emphasized by Surber and Gzesh (1984) the balancing model is not appropriate tools on solving linear equation with subtraction and the balance model seems incapable, and even confuses students, to work with reversible operations.

The limitations mentioned by Surber and Gzesh (1984) findings were the main limitation of the present study finding. But, to minimize those limitations the researcher contribution is used different additional strategies, those are first starting from students past experiences and using different colors representing negative numbers. Generally, in the learning process of solving a linear equation through balancing model, the researcher used this method as an alternative strategy and also used active learning approach but not taught the whole ideas in this way.

5.1.2. The sustainability of using balancing model teaching approach

The second findings of this study were related to establish and sustain the balancing model teaching method on the teaching-learning process of solving any linear equation with one variable. To answer this question, the researcher gathered qualitative data from the researcher's classroom self-observation and an interview with the teacher and the students.

The result of self-observation data shows that while the students learning the concept of linear equation through balancing model teaching method many students had a good attitude for the lesson study and they were happy with the learning process of solving linear equation through balancing model teaching method. This result also corroborates the claim by Hopkins (2012) who suggests that education for sustainable development requires participatory teaching and learning methods that motivate and empower learners to change their behavior and take action for sustainable development. Generally, based on observation data, it was apparent at the beginning of any equation concept during this lesson that the students' attitudes about mathematics were improved.

To answer the second basic research question, the researcher conducted an interview with five students and their mathematics teacher. From the analysis part, table 4.1 and 4.3 showed that the view of the interviewee students changed almost entirely from the pre-intervention interview to the post-intervention interviews. Still with the same participant students during the pre-intervention interview, four students gave a negative response when asked why they were studying mathematics in school and after conducting the teaching experiment through balancing model all the students gave a positive response on the learning process of mathematics as a subject. In the other hand, the teacher interview data showed that the teacher had developed a good attitude towards the balancing model teaching method. The teacher further promised that he will use the balancing model teaching method in his teaching process of solving linear equation concepts for the future. This result is in line with the findings of Carpenter, et al., (1998). They noted that elementary school students were able to use alternative strategies to solve a problem when they learn with balancing model teaching method. They also found that 65% of the learners in their research sample used an alternative strategy before standard algorithms were taught. By the end of their study, 88% of their sample students had used alternative strategies at some point. Supporting this idea, Warschauer, et al., (2014) also suggested that mathematical models or teaching aids make teachers take easier, effective and motivate students' attention to be active learners. Moreover, Johnson (1993) described that using the balancing model to solve linear equations makes the classroom an interesting and exciting place for both teachers and learners as well.

5.2. Implications of the Study

The finding of this study informs empirical information for educational stakeholders particularly mathematics teachers on the effect of using balancing model teaching method in developing the conceptual understanding of students towards linear equation. The finding of this study further revealed that since the implementation of balancing model teaching method in the intervention process had a positive impact on the conceptual understanding and views of grade five section A students and their mathematics teacher. That is, the study indicates that the complete implementation of balancing model teaching method could improve the conceptual understanding of students about the concept of solving linear equation and it also changes their views.

However, this method of teaching did not fully succeed in this study as all students could not improve their conceptual understanding and attitude to the expected level. Therefore, there are many things to be fulfilled in order to implement this method more effectively including giving more time, availing relevant resources as well as students' engagement into the lesson. Even though the preceding limitations, the findings were found very promising in improving the conceptual understanding of students for the abstract concept of mathematics like solving linear equation. Therefore, the implication is that if balancing model teaching method is implemented correctly and completely with enough time and available resources, then all students can improve their conceptual understanding and bring positive attitudinal changes towards learning linear equation.

CHAPTER SIX

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS OF THE STUDY

6.1. Summary

In this part, the researcher tried to summarize the whole process of the present study based on the objective of the study. The purpose of this quasi-experimental single group research was to develop students understanding of solving linear equation with one variable through balancing model teaching method at Ewket Fana Primary School in grade five section A students. Teaching and learning mathematics particularly algebra in many countries including Ethiopia are struggling due to a number of difficulties. So, many students do not understand the fundamental ideas or basic concepts covered in the mathematics class. As a result, the students gave up on mathematics as hopelessly baffling and difficult and develop a negative perception of mathematics. The researcher takes up the challenge to implement an innovation strategy to develop students' conceptual understanding toward linear equation concepts and tried to sustain a balancing model teaching method on the concept of solving linear equation at Ewket Fana Primary School.

Design-based research consisting of three phases, that is, the preparation, teaching experiment and retrospective analysis phases was chosen as an approach to conduct this study. In the first phase of this study, the researcher gathered data on the status of the students' prior background next this the teaching experiment phase, the researcher conducted the lesson study. To gather data, a number of techniques were employed, such as test, interview and observation. These data were analyzed quantitatively and qualitatively in the retrospective analysis phase.

As explained in the analysis part, quantitative data indicated that the performance of the students for the posttest score was improved as compared to students' pre-test result and qualitative data indicated that the students after treatment developed different strategies to solve any linear equation problems and also it makes them to have a good attitude toward mathematics subject. Considering the quantitative and qualitative data confirms the assertion that balancing

model teaching method effectively represented a concrete example of the equality of the two sides of an equation.

6.2. Conclusions

The conclusions focus on the two basic research questions of the study, those are the development of students conceptual understanding of solving linear equation and the sustainability of the balancing model teaching method on the concept of solving any linear equation with one variable.

In order to analyze the students' conceptual understanding, test results and classroom observation data revealed that a significant improvement in students performance after the use of the balancing model in teaching the concept of linear equation and also in the learning process of solving linear equation through balancing model many students could actively participate in the classroom activities and develop more flexible strategies when solving linear equation problems. So, balancing model teaching method was considered important as a tool to developing students' conceptual understanding and performance in focus of linear equations.

To determine whether the alternative teaching strategies was a positive impact on grade five section A students and their mathematics teacher attitude and interests toward equation concept, the researcher asked five representative students and their mathematics teacher related to attitude questions, the result of those data indicated that there was a possibility that alternative teaching strategies (balancing model teaching method as one which) had a positive impact among the students and their mathematics teachers, particularly on solving linear equation concepts.

6.3. Recommendations

Based on the findings of this study and the conclusions made therefore, the researcher forwarded some recommendations that might be useful for the implementation of the learning design in the mathematics classroom and further researchers. These include the following:

- ✓ The education officials should work on capacity building, especially for teachers and on providing facilities for primary schools.

- ✓ Mathematics curriculum designers should design or re-design mathematics textbooks in such a way that the textbook emphasizes are smoothly integrated with the students' real-world situation.
- ✓ Teacher education institution should train a new approach for mathematics teachers.
- ✓ Continues professional development should focus on innovative approaches to develop the teaching of mathematics subjects.
- ✓ Mathematics teachers should apply more innovative teaching methods like balancing model teaching method in teaching abstract concepts like linear equation.
- ✓ Mathematics teachers should assist their students in order to improve their attitude towards mathematics and give awareness to the students on how to use mathematics concepts with real life.
- ✓ Finally, the researcher suggests that future research should be investigated in the area of the current research emphasis on a large population and over a long period of time as well as other grade levels.

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APPENDIX A

Before intervention classroom observation

- How the teachers conduct the lesson, by explaining? By giving a note in the blackboard? Or by giving a chance to the students to solve some guided problems?
- Does the teacher use a real problem to start the learning activity?
- Do the teachers teach the mathematical concept using different teaching strategies?
- Does the teacher mostly use a textbook? What book does use to teach mathematics in fifth grade?
- Does the teacher use other resources materials?
- How do students participate and attitude in the lesson study? Are they are active or passive?
- What are the student's general difficulties in learning mathematics?
- How is their motivation in learning?

APPENDIX B: STUDENTS PRE & POST TEST PROBLEMS

Solving linear equations pre-test

ስም ---------- ክፍል ---------- ቁጥር ---------- ጾታ ----------

የሚከተሉትን ጥያቄዎች በትክክል ከነአሰራሩ ስሩ::

1. ከ አንድ ካልታወቀ ቁጥር እጥፍ ላይ አራት ሲቀነስ የሚለዉን የቃላት ዐ.ነገር በአልገብራዊ መግለጫ ጻፉ?

2. 3 + ሸ x 2 ፤ ሸ = 4፣ ቢሆን የአልገብራዊ መግልጫዉን ዋጋ ፈልጉ

3. $ሀ^2 – 5ሀ$፤ ሀ = 5 ቢሆን የአልገብራዊዉ መግለጫዉን ዋጋ ፈልጉ

4. የሚከተለዉን የእኩልነተ ዐ.ነገር ባዶ ቦታዉን ሙሉ፤ ☐+ 5 = ☐+ 2.

5. ጠ + 4 = 12 ቢሆን የ ጠ ዋጋ ፈልጉ

6. የአንድ ልብስ ዋጋ 93 ብር ነዉ:: ይህ ዋጋ ከሌላዉ የልብስ ዋጋ 3 እጥፍ ነዉ:: የዝቅተኛዉ የልብስ ዋጋ ስንት ነዉ;

Solving linear equation post-test

ስም---------- ክፍል ---------- ቁጥር ----------

የሚከተሉትን ጥያቄዎች በተሰጣችሁ ትእዛዝ መሰረት በትክክል መልሱ፡፡

1. ከአንድ ካለታወቀ ቁጥር ላይ ሰባት ሲቀነስ አስር ይሆናል፡፡ ይህን ዐ.ነገር በእኩልነት ዐ.ነገር ፃፉ?

2. ከ ሶስት ላይ አንድ ያልታወቀ ቁትር ሲደመር አስራ ሁለት ይሆናል፡፡ የሚለዉን ዐ.ነገር በእኩልነት ዐ.ነገር በመፃፍ አሰራሩን በማሳየት የተለዋዋጩን ዋጋ ፈልጉ?

3. ከአንድ ካልታወቀ ቁትር ላይ አምስት ሲቀነስ ሀያ አራት ይሆናል፡፡ የሚለዉን ዐ.ነገር በእኩልነት ዐ.ነገር በመፃፍ አሰራሩን በማሳየት የተለዋዋጩን ዋጋ ፈልጉ?

4. $ሀ + 3 = 8$ የሚለዉን የእኩልነት ዐ.ነገር ሚዛንን በመጠቀም እና የእኩልነት ዐ. ነገር ቀመርን በመጠቀም የተለዋዋጩን ዋጋ ፈልጉ?

5. አንድ ያልታወቀ ቁትር በ ስድስት ሲባዛ ሰላሳ ይሆናል፡፡ የሚለዉን ዐ.ነገር ነእኩልነት ዐ.ነገር በመግለጽ አሰራሩን በማሳየት የተለዋዋጩን ዋጋ ፈልጉ?

6. $4ሸ + 5 = 17$ የሚለዉን የእኩልነት ዐ.ነገር አሰራሩን በማሳየት የተለዋዋጩን ዋጋ አግኙ?

APPENDIX C

During intervention lesson self-observation checklist

- How does a student's participation in the classroom? Are they are active or passive?
- Do all students motivate in the classroom teaching linear equation using the balancing model?
- Do all students' positive attitudes in the learning process of solving linear equation through balancing a model teaching approach?
- How the student conceptual understanding teaching of linear equation using the balancing model?
- What are the student's views understand the concept related to your real life situation?
- Do all students accept the daily lessened?
- What are the student's difficulties in the teaching of linear equation using the balancing model?

APPENDIX D

Classroom teacher interview checklist

- What do see as the value of balancing model for teachers and students?
- What benefits do you think your students obtained by completing this topic using the balancing model?
- What do you see as the perceived benefits of balancing model for helping students understand the meaning of equality symbol?
- What do you see as the perceived benefits of balancing model for helping students understand symbol while solving equations?
- How has the balancing model helped students improve conceptual and procedural understanding?
- Next year, if you are responsible for teaching solving equations, what method will you use?
- Is there anything else you would like to add?

APPENDIX E

Sample of a daily lesson plan

Teacher name: Belaynesh Subject: Mathematics

Topic: solving linear equation Grade: five

Sub-topic: introduction to linear equation Section: A

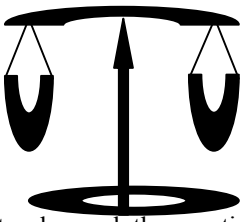
School name: Ewket Fana Date: 11/06/2011 E.C

Rational of the topic: - students often experience the balance of two things in daily life e.g. measuring 1k.g salt, sugar e.t.c. in the shop.

Pre-requisite knowledge: - the students have the basic knowledge of in the previous grade and topic addition and subtraction of whole number and natural number. E. g. 3 + 4 = 7, 5-2 =3.

Specific objective: - at the end of this topic students will be able to:-

- Based on our environment define the meaning of equal relation
- Using a balancing model picture to formulate or construct the equivalent equation correctly.
- Based on the balancing model picture students define the meaning of the equation correctly.

Stage	content	Teacher activity	Time	Student activity	Assessment activity	Remark
Starting activity	Introduction to equation	Ask students an oral question about the meaning of the expression to revise the previous lesson. -ask students what does this picture represents.  -the teachers ask the question to students list the use of this apparatus and the relationship between balancing scale and equation.	5’	-Students will define the meaning of an expression. -students will answer the question raised by a classroom teacher. -The students will list the advantage of balancing model in their real-life situation and define the relationship between balancing scale and equation.	Teachers assess student prior understanding of balancing concept from students’ response.	

Mean activity	Equation formulation	Make students into six groups. -Ask students by the group using a beam balance model to formulate the equation and define the meaning of an equal sign. -ask fast learners more difficult questions. -Based on students' idea present the system of equation formulation and the meaning of an equal sign.	30'	Students by group's measure tiles using a beam balance model and to construct an equation. -Each group using a beam balance model formulate any equation.	Ask students an oral question about what is "=" this symbol and what is the use of this symbol	
Conclusion activity	Introduction of equation & Equation formulation	Summarizing the lesson by asking the students to conclude that the meaning of equation and equation formulation. - Telling these ideas is the basis of solving linear equation that they are going to study in the next class.	5'	-By reflecting the daily lesson such as: define the meaning of equation and the formulation of equation.	Assess students understanding of the meaning of equation and the formulation of an equation based on their reflection	

Teaching-learning material: - (the materials that will be used in your lesson):

- The students' textbook and teachers guide.
- Beam balance model and tiles.

Learner support: - (for slow learner, fast learner…)

- The teachers will assist the slow learners by simplifying the activity providing simpler data and give the fast learner more difficult and complex prepared data to manipulate on.

Prepare by: Belaynesh Kassanew

Signature: - ---------- Date: - 10 / 6 / 2011 E.C

Approved by: - ----------

Signature: - ---------- Date: - ----------

Comment:- ----------

እለታዊ የትምህርት እቅድ

የመምህሩ ስም፡ በላይነሽ ካሳነዉ

የትምህርት ቤቱ ስም፡እዉቀት ፋና

የትምህርት አይነት፡ ሂሳብ

ክፍል፡ 5ኛ

እርእስ፡ የእኩልነት ዐ.ነገርን መፍታት

ሴክሽን፡ ሀ

ንዑስ ርእስ፡ የእኩልነት ዐ.ነገር መግቢያ

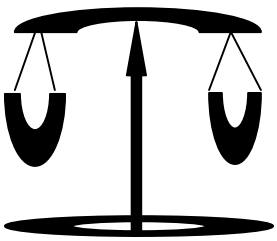
ቀን፡ ዘ / 6 / 2011 ዐ.ም

የትምህርቱ አስፈላጊነት፡ ተማሪዎች በአካባቢያቸዉ ስለ ሁለት ነገሮች እኩልነት መለካት እና መግለጽ እንዲችሉ፡፡ ምሳሌ፡ ነ ኪሎ ጨዉ፣ ስኮር ወዘተ ለመለካት

ቀድሞ በተማሪዎች ሊታወቅ የሚገባ እዉቀት፡ ተማሪዎች ከባለፈዉ ክፍል እና እርእስ ስለ ሙሉ ቁጥሮች እና መቁጠሪያ ቁጥሮች መደመር እና መቀነስ እዉቅና አላቸዉ፡፡

ዝርዝር አላማ፡ ከዚህ ክፍለ ጊዜ ትምህርት በሆላ ተማሪዎች፡-

- ከአካባቢያቸዉ በመንሳት ስለ እኩልነት ትርጉም በትክክል ይገልጻሉ፡፡
- ባላንሲንግ ሞዴልን በመጠቀም የእኩልነት ዐ.ነገርን በትክክል ይመሰርታሉ፡፡
- ባላነሲንግ ሞዴልን በመጠቀም የእኩልነት ዐ.ነገርን ትርጉምን በትክክል ይናገራሉ፡፡

የመማር ማስተማር ደረጃዎች	ጊዜ	የመማር ማስተማር ይዘት	የመምህሩ ተግባር	የተማሪዉ ተግባር	የምዘና ተግባራት	ምርመራ
መግቢያ	5	የእኩለነት ዐ.ነገርን መግቢያ	-ስለ ቁሞች የቃል ጥያቄ በመጠየቅ የባለፈዉን ክፍል ጊዜ ትምህርት ማስታወስ -ተማሪዎችን ይህ ምስል ምን ይባላል ብሎ መጠየቅ  -ተማሪዎችን የዚህን ምስል ጥቅም እንዲዘረዝሩ ማድረግ እና ከእኩልነት ዐ.ነገር ጋር ያልዉን ግንኙነት መጠየቅ	-ተማሪዎች አልጀብራዊ ቁሞችን ትርጉም ማብራሪያ ይሰጣሉ - መምህሩ ሲናገር እዳመጡ ማስታዎሻ ይጽፋሉ -ተማሪዎች በአካባቢያቸዉ ባላነሲንግ ሞዴል ምን ጥቅም እንዳለዉ ይናገራሉ -ተማሪዎች የእኩልነት ዐ.ነገርን እና የባላንሲንግ ሞዴልን ግንኙነት ይናገራሉ	-ተማሪዎቹ ከሚሰጡት ምላሽ መምህሩ የተማሪዎችን ስለ እኩልነት ያላቸዉን ግንዛቤ ይገመግማል	
ዋና ተግባር	30	የእኩልነት ዐ.ነገር	-ስለ እኩልነት ትርጉም እና ባላነሲንግ ሞዴል ጥቅም አጭር ማብራሪያ	-መምህሩ ሲያስተምር ማዳመጥና ማስታወሻ መያዝ	-ተማሪዎችን በቃል “=”	

		ትርጉም	መስጠት -ተማሪዎች በቡድን ሁነዉ ባላነሲነግ ሞዴልን በመጠቀም የእኩልነት ዐ.ነገር እንዲመሰርቱ ማድረግ እና ስለ እኩል ይሆናል ትርጉም እነዲወያዩ ማድረግ -የተወያዩበትን ሀሳብ እንዲገልጹ ማድረግ -ከተማሪዎች ሀሳብ በመነሳት ስለ እኩልነት ዐ.ነገር ትረጉምና የእኩልይሆናል ምልክት ትርጉም ማብራሪያ መስጠት	-በየቡድናቸዉ በመሆን ባላነሲንግ ሞዴልን በመጠቀም የእኩልነት ዐ.ነገር ይመሰርታሉ ከዚህ በመነሳት ስለ እኩል ይሆናል ምለክት ትርጉም ማብራሪያ ይሰጣሉ -የተወያዩበትን ሀሳብ ተራበተራ በየቡድናቸዉ እየወጡ ይገልጻሉ	የዚህን ምልክት ስም መጠየቅ እና ለምን እንደሚጠቅም በመጠየቅ ግንዛቢያቸዉን መመዘን	
የማጠ ቃለያ ተግባር	5	የእኩልነት ዐ.ነገር	-ከሶስቱ የእለቱ ትምህርት አላማዎች በመነሳት ተማሪዎች የእለቱን ትምህርት እንዲያጠቃልሉ መጠየቅ -የእኩልነት ዐ.ነገርን ትርጉምን ማወቅ የእኩልነት ዐ.ነገርን በቀላሉ መደመር እነደሚያስችል በመናገር ቀጣይ እነደሚማሩትና አንብበዉ እንዲመጡ መናገር	-ተማሪዎች ተራ በተራ ስለ ባላነሲንግ ሞዴል ጥቅም፣ የእኩልነት ዐ.ነገር ትርጉም እና የእኩል ይሆናል ምልክት አገልግሎት በቃል ይገልጻሉ	-ተማሪዎች ገለጻ ሲደርጉ የእኩልነት ዐ.ነገር ግንዛቤያቸዉን መገምገም	

መርጃ መሳሪያ፡- የተማሪዎች መማሪያ መጽሀፍ፣ የመምህሩ መምሪያ ፣ባላንሲንግ ሞዴል እና አራት ማእዘን ቅርጽያላቸዉ ቁርጥራጭ ብረቶች፡፡

ለተማሪዎች የሚደረግ ድጋፍ፡- ለዝቅተኛ ተማሪዎች ቀለል ያሉ ምሳሌዎችን በመስጠት ማስረዳት እና ለከፍተኛ ተማሪዎች ከበድ ያሉ ጥያቄዎችን እንዲሰሩ ማድረግ፡፡

አዘጋጅ፡ በላይነሽ ካሳንዉ

ፊርማ፡---------- ቀን፡ 10/ 6/ 2011 ዐ.ም

ያረጋገጠዉ ሰዉ ስም፡----------

ፊርማ፡---------- ቀን፡----------

አስተያየት፡----------
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