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SURFACE AREA OF RANK SPHERE

ANTENEH, TILAHUN

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Wisdom at the source of the Blue Nile

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By

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DEPARTMENT OF MATHEMATICS

COLLEGE OF SCIENCE

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SURFACE AREA OF RANK SPHERE

A Dissertation

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The dissertation titled "**SURFACE AREA OF RANK SPHERE**" by Mr. Anteneh Tilahun is approved for the degree of "Master of Science in Mathematics".

Examiners

	Name	signature
Advisor	____________	____________
~~External~~ Examiner	1 Berhanu Assaye	[signature]
~~Internal~~ Examiner	2 Getachew A	[signature]

Date: ____________

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NOTATIONS

$\|\|x\|\|$	Norm of x
$\bar{0}$	Zero vector given by (0, 0, 0,…,0)
$\langle x_1, x_2, x_3, \ldots, x_n \rangle$	Spanned by $x_1, x_2, x_3, \ldots, x_n$
$\tilde{x}$	Vector x
$\|X\|$	Cardinality of X
$\cup$	Union
$\cap$	Intersection

ABSTRACT

In this report, one of the properties of rank distance code (RD) which is the surface area of rank sphere is discussed .It is showed that the k^{th} *surface area* $S_k(\bar{0})$*,which is the collection of rank norm-k is distance invariant and the weight distributions of the translates*$S_k(x)$ *of* $S_k(\bar{0})$ *are the same for all* $x \in S_t(\bar{0})$*. Ultimately, the weight distribution of* $S_k(x)$ *when* $\|x\|=1$ *is determined.*

CHAPTER -ONE

INTRODUCTIONAND PRIMILLINARY

1.1INTRODUCTION:

In 1948, Claude Elwood Shannon published a paper entitled with "A mathematical theory of communication" that signified the beginning of coding theory[1].It was found convenient to model communication channels as conveyors of symbols from finite sets and represent the effects of channel noise by occasional reception of a symbol other than the transmitted symbol. As the error correcting capability of a code depends mainly on the minimum distance, studying the metric and structural properties of a code becomes a paramount importance. Thus, after the appearance of Shannon's paper, several researchers have made their contributions in designing error correcting codes for different channels and by devising various encoding and decoding techniques.Thus,the hamming metric introduced by R.W.Hamming in 1950, has been considered the most relevant metric for single error correcting codes called Hamming codes[1].The metric hamming distance d_H(x,y) between two vectors x,y is defined to be the number of coordinates in which x and y differ .The major advance came when Bose and Ray-Choudahuri(1960) and Hocquenghem(1959) found a large class of multiple error correcting codes called BCH codes[1].

Codes over finite fields with a rank as the metric, instead of the usual Hamming metric was studied by E.M.Gabidulin in 1985[2] .The same was studied, by Roth (1991) and V. Tarokh et al.(1998)[2]. The metric, rank distance between the vectors x and y is defined as the number of linearly independent coordinates of the resultant vector x-y. Recently, this rank metric codes has great importance for net working, wireless communication and storage equipment [2]. Due to these potentials of applications, studying different structural properties of codes with rank metric have received some attention. Of those different properties, here in this report we concentrated on the surface area of rank sphere in which the rank distance is taken as a metric.

The report is organized as in chapter one; section 1.2, reviews necessary backgrounds (preliminaries).In chapter two; section 2.1, rank sphere and surface area of rank sphere is defined and some results on surfaces areas of rank sphere are analyzed and in section 2.2, weight distribution of a code is defined and it is proved that the translates of surface areas are distance invariant; some results on the weight distribution of the translates of the surface areas are also proved. Ultimately, this report has completed by determining the weight distribution of certain class of translates.

In this report, unless otherwise stated $F_{q^N}^n$ denotes the n-dimensional vector space over the finite fieldF_{q^N}, N is considered to be equal to n and q is taken to be equal to 2.

1.2. PRELIMINARIES:

The aim of this section is to introduce the basic definition, propositions and theorems which are essential for our subsequent discussion of the main concept, Surface Area of Rank Sphere.

Definition1.2.1 A field F is a set of elements with two operations +(addition)and. (Multiplication) satisfying the following properties:

I. F is closed under + and.i.e.,a+b and a.b are in F if a and b are in F .

II. For any a,b ∈ F ; a+b =b+a,a.b=b.a.

III. For any a,b ∈ F ; (a+b)+c=a+(b+c),a.(b.c)=(a.b).c.

IV. For any a,b ∈ F ; a.(b+c)=a.b+a.c.Further identity elements 0 and 1 must exist in satisfying:

V. a+0=a=0+a

VI. a.1=a=1.a

VII. For any a in F, there exists an additive inverse (-a) such that a+(-a)=0=(-a)+a.

VIII. For any a in F, there exists a multiplicative inverse (a^{-1})such thata.a^{-1}=1= a^{-1}.a

The above properties holds true for fields with both finite as well as infinite elements .A field with a finite number of elements (say ,q) is called a **Galois Field** and is denoted by **F(q).**

Definition1.2.2 An element $\alpha \epsilon F(2^r)$ is primitive if$\alpha^m \neq 1, 1 \leq m < 2^r$, $\alpha^{2^r-1}=1$.Equivalently, α is primitive if every non-zero vector in $F(2^r)$ can be expressed as power of α.

Definition 1.2.3 let F be a given field whose elements are called scalars and let V be a non-empty set whose elements are labeled vectors .The set V is called a **vector space** or a **linear space** over the field F if the following axioms are satisfied:

1. For any α, β∈V,α +β∈V.
2. For any α, β∈ V,α +β=β+α.
3. For any α, ***θ***, β∈ V, α+(***θ***+β)=(α+ ***θ***)+ β.
4. There exists a unique vector ***ϕ*** denoted by 0 such that α+0= α=0 +α for all α ∈ V Which is called the zero element.
5. For any vector α∈ V, there exists a unique vector -α∈ V such that α+(-α)=0.
6. For any α∈ F and any vector γ∈V,αγ∈ V.
7. For any element a∈F and any two vectors α, β∈ V,a(α +β)=aα+aβ.
8. For any two scalars a , b∈F and any vector α∈ V,(a+b).α=aα+bα.
9. For any scalars a, b∈F and any vector α∈ V,(ab)α=a(bα).
10. For the unit scalar 1∈F and any vector α∈V,1.α =α.

Definition 1.2.4 A non–empty subset W of a vector space V over a field F is called a vector subspace or simply a subspace of V if W is a vector space over F with respect to addition and scalar multiplication.

Definition 1.2.5 A finite set $\{a_1, a_2, \ldots, a_n\}$, of n-vectors of a vector space V is said to be linearly independent if the relation $c_1a_1 + c_2a_2 + \cdots + c_na_n = 0$ holds only when c_i's are zero.

Definition 1.2.6 A subset S of a vector space V is said to be the basis of V if S is linearly independent and any vector in V other than that of S is a linear combination of the vectors of S.

Definition 1.2.7. The number of elements in a basis of the vector space V is called the dimension of V.

Proposition 1.2.8[5] Let V be an n-dimensional vector space and A be a k-dimensional subspace of it. Then the number of l-dimensional subspaces ofV contains the given k-dimensional subspace A is$\binom{n-k}{l-k}$.

Proposition1.2.9 [5] Let Z be an m-dimensional subspace of a vector space V of dimension n. Then the number of l -dimensional subspaces W of V such that $Z\cap W=\{0\}$ is$\binom{n-m}{l}q^{lm}$.

Definition 1.3.0 Let S be a non- empty subset of a vector space V. Then the set of all linear combinations of elements of S is called the span of S.

Definition 1.3.1 Let X be a vector over a field F.The norm of X denoted by$\|X\|$ is the number satisfying the following conditions:

(1) $0\leq\|X\|$;$0=\|X\|$ iff X=0
(2) $\|cX\|=|c|\|X\|$;a scalar c inF.
(3) $\|X+Y\|\leq\|X\|+\|Y\|$

Definition 1.3.2 For a matrix A its rank is defined as the number of linearly independent rows or columns.

Theorem1.3.3 If A and B are both m x n matrices, then$|\text{rank}(A)-\text{rank}(B)|\leq\text{rank}(A+B)\leq\text{rank}(A)+\text{rank}(B)$.

Definition 1.3.4 Given a matrix A over a field F, then its rank normal form is a matrix having the same rank and order as A.

Theorem1.3.5 Given a matrix A over a field F, there exist non-singular matrices B and C, also over F, such that the product BAC is the rank normal of A.

Definition1.3.6 A codeword (vector) is a sequence of symbols.

Definition1.3.7 code is a set of vectors called code words.

Definition1.3.8 Let **X** be a non-empty set. A mapping d:X x X⟶R is called a metric if the following conditions are satisfied

(1) 0≤d (x,y),for all x , y ϵ X and d(x , y)=0 if and only if x=y.

(2) d (x,y)=d(y,x) for all x,yϵ X.

(3) d (x,z) $\leq$d(x,y)+ d(y,z),for all x,yϵX.

Definition 1.3.9 The rank norm (over F(q)),of the vector a, denoted by rk(a/F(q),or wt(a) or simply$||a||$, is defined to bethe maximum number of linearly independent coordinates of the vector aϵ $F^n_{q^N}$ over$F_q$or the rank of the corresponding matrix A, where A is the m x n matrix obtained by expanding all the coordinates of a by a basis $\beta_0, \beta_1, \beta_2, \beta_3, \dots, \beta_{N-1} of F_{q^N}$over F(q).

Definition 1.4.0 For all a , b $\epsilon F^n_{q^N}$ the rank distance between the vectors a and b is the rank of their difference given as d_r(a,b)=rk(a-b) .

The rank distance (d_r) satisfy definition1.3.8.To see this, since d_r(a,b)=rk(a-b),for a , bϵ $F^n_{q^N}$ and rank is simply counting linearly independent coordinates then rk(a-b) is non negative i.e. condition (1) of definition1.3.8 holds true. Also, for a,b ϵ $F^n_{q^N}$; a-b =a+b =b-a,because q=2 and -1=1 inF_2,then d_r(a,b)=rk(a-b)=rk(b-a)=d_r(b-a) ,i.e.condition (2) of definition 1.3.8holds true. Moreover, from theorem 1.3.3we have that rk(a-b) $\leq$ rk(a-b) +rk(b-c) for all a,b, c ϵ $F^n_{q^N}$.This implies that d_r(a,b) $\leq$ d_r(a,b)+ d_r(b,c),hencecondition (3) of definition 1.3.8 also holds true. Thus, the rank distance denoted by d_r over $F^n_{q^N}$ is a metricwhich referred to as rank metric.

Definition 1.4.1 An$[n, k]$ rank Distance code is a subspace of dimension k in the rank distance space $F^n_{q^N}$.

Example 1.4.2 Consider an RD (rank distance) code C=$\{(0,0), (1,\alpha^2), (\alpha,1), (\alpha^2,\alpha)\}$over F_{2^2}=$\{0,1,\alpha,\alpha^2\}$; where α is a primitive element, where α^2=α+1.

(a) d_r((1,α^2),(α,1))=rk((1-α),(α^2-1)), by definition 1.4.0.

= rk (1+α, α^2+1) =2,since -1=1 in F_2.

Then, the rank distance between the two vectors (code words) (1, α^2) and (α, 1) is2 (i.e. the two vectors are linearly independent).

(b)$d_r((1,\alpha^2),(\alpha^2,\alpha))$= rk(1-$\alpha^2$,$\alpha^2$-α(By definition 1.4.1)

= rk(1+α^2,α^2+α) (since -1=1 inF_2)

=1 (since 1+α^2 and α^2+α are linearly dependent)

Definition1.4.3. For the non-negative integers n, m the guassian polynomial denoted by$\binom{n}{m}$ with respect to the base q is defined as

$$\binom{n}{m} = \begin{cases} \frac{(q^n-1)(q^n-2)...(q^n-q^{m-1})}{(q^m-1)(q^m-2)(q^m-q^{m-1})} & ; \text{ if } 1\leq m \leq n. \\ 1 & ; \quad if m = 0 \\ 0 & ; \ otherwise \end{cases}$$

In which$\binom{n}{m}$ gives the number of m-dimensional subspaces of an n-dimensional vector space over the field F_q. The numbers of vectors of a given rank in a rank space is associated with the Guassian polynomial. Let L_i(n) denote the number of vectors of rank i (0$\leq i \leq n$) in $F^n_{q^N}$,then

L_i(n)=$\begin{bmatrix} n \\ i \end{bmatrix}$ $(2^N-1)(2^N-2)\ldots(2^N-2^{i-1})$, for 1≤i≤ n.

From Definition1.4.3 we have that

(1). L_0(n) =1

(2). L_1(n) =$(2^n-1)^2$and

(3). $\frac{L_K(n)}{L_I(n)} = \frac{((2^n-2)(2^n-4)...(2^n-2^{k-1}))^2}{(2^k-1)(2^k-2)(2^k-4)...(2^k-2^{k-1})}$.

CHAPTER-TWO

SURFACE AREA OF RANK SPHERE

2.1 RANK SPHERE, SURFACE AREA OF RANK SPHERE AND TRANSLATES OF SURFACE AREA OF RANK SPHERE:

This section consists of definition of rank sphere, the i^{th} surface area of rank sphere and translates of surface area of rank sphere. Also. Some results on the translates of the i^{th} surface area of rank sphere included.

Definition2.1.1 [5] For any nonnegative integer t$\leq n$ and x$\epsilon F_{2^N}^n$,define$B_t(x)$={y$\epsilon\epsilon F_{2^N}^n$:$d_R(x,y)\leq t$} to be the rank sphere of radius t with center at x.

Example2.1.2. Construct a rank distance code$F_{2^2}^3$ over the field F_{2^2}={0,1,α, α^2},whereα^2=α+1 and form the rank sphere of radius 1 and center at$\bar{0}$=(0,0,0) which is denoted by $B_1(\bar{0})$.

Now,$F_{2^2}^3$=F_{2^2}xF_{2^2}xF_{2^2}={(0,0,0),(0,0,1),(0,0,α),(0,0,α^2),(0,1,0),(0,1,1),(0,1,α), (0,1α^2,), (0,α,0), (0,α, 1),(0,α, α),(0,α, α^2),(0,α^2, 0),(0,α^2, 1),(0,α^2, α),(0,α^2, α^2),(1,0,0),(1,0,1),(1,0,α), (1,0,α^2), (1,1,0),(1,1,1),(1,1,α), (1,1,α^2),(1,α, 0), (1,α, 1),(1,α, α),(1, α , α^2), (1,α^2, 0), (1,α^2, 1), (1,α^2,α), (1,α^2, α^2), (α, 0,0), (α, 0,1), (α, 0, α), (α, 0, α^2), (α, 1,0), (α, 1,1), (α, 1, α), (α, 1, α^2), (α, α, 0), (α, α, 1), (α, α, α), (α, α, α^2), (α, α^2, 0), (α, α^2, 1), (α, α^2, α), (α, α^2, α^2), (α^2, 0,0), (α^2, 0,1), (α^2, 0, α), (α^2, 0, α^2), (α^2, 1,0), (α^2, 1,1), (α^2, 1, α), (α^2, 1, α^2), (α^2, α, 0), (α^2, α, 1), (α^2, α, α), (α^2, α,α^2), (α^2,α^2,0), (α^2,α^2, 1), (α^2,α^2,α), (α^2, α^2 , α^2)}. Then , from this the rank sphere of radius = 1 and center at $\bar{0}$ = (0, 0, 0) denoted by $B_1(\bar{0})$ will be computed by finding those vectors(i.e. code words) x$\epsilon F_{2^2}^3$ having rank =0 and rank =1.That is $d_r(x, \bar{0}) = rk(x - \bar{0}) = rk(x) \leq 1$. $B_1(\bar{0})$={(0,0,0), (0,0,1), (0,0,α), (0,0,α^2), (0,1,0), (0,1,1), (0,α,0), (0,α, α), (0,α^2, 0), (0,α^2, α^2),(1,0,0),(1,0,1),(1,1,0),(1,1,1),(α, 0,0),(α, 0, α), (α, α, 0),(α, α, α), (α^2, 0,0), (α^2, 0, α^2), (α^2, α^2, 0), (α^2, α^2 , α^2)}.

Definition: 2.1.3[5] For any $x\epsilon F_{2N}^{n}$, defines$_i(x)$= $\{y\epsilon F_{2N}^{n}:d_R(x, y)=i\}$ for any non-negative integer i≤n. $s_i(x)$ is called the ith surface area of the rank sphere with center at x.

Note:(1)$B_t(x)$ $=\cup_{i=0}^{t} S_i(x)$and $S_0(x)=\{x\}$.

(2)$S_t(\bar{0})$ gives all the vectors of rank norm-t in F_{2N}^{n}.Hence $|S_t(\bar{0})|=L_t(n)$.Moreover, $|S_t(x)|=L_t(n)$for any $x\epsilon F_{2N}^{n}$.

Example: 2.1.4 Using the above example2.1.2. we can determine:

(1).$S_1(\bar{0})$(which is the 1stsurface area of the rank sphere with center at $\bar{0}$) will be the set of vectors such that each of them has rank -1.Thus,$S_1(\bar{0})$=$\{(0,0,\alpha),(0,0,\alpha^2),(0,1,0),(0,1,1),(0,\alpha,0),$

$(0,\alpha,\alpha),(0,\alpha^2,0),(0,\alpha^2,\alpha^2),(1,0,0),(1,01),(1,1,0),(\alpha,0,0),(\alpha,0,\alpha),(\alpha,\alpha,0,(\alpha^2,0,0),(\alpha^2,\alpha^2,0)$

$,(\alpha^2,0,\alpha^2)\}$.

(2).$S_2(\bar{0})$ (which is the 2ndsurface area of the rank sphere with center at $\bar{0}$) will be the set of vectors such that each of them has rank 2.

Thus,$S_2(\bar{0})$=$\{(0,1,\alpha),(0,1,\alpha^2),(0,\alpha,1),(0,\alpha,\alpha^2),(0,\alpha^2,1),(0,\alpha^2,\alpha),(1,0,\alpha),(1,0,\alpha^2)1,1,\alpha),(1,1,\alpha),$
$(1,1,\alpha^2),(1,\alpha,0),(1,\alpha,1),(1,\alpha,\alpha),(\alpha,0,1),(1,\alpha^2,0),(1,\alpha^2,\alpha^2),(\alpha,0,\alpha^2),(\alpha,1,0),(\alpha,1,1),(\alpha,1,\alpha),$

$(\alpha,\alpha,1),(\alpha,\alpha,\alpha^2),(\alpha,\alpha^2,0),(\alpha,\alpha^2,\alpha),(\alpha,\alpha^2,\alpha^2),(\alpha^2,0,1),(\alpha^2,0,\alpha),(\alpha^2,1,0),(\alpha^2,1,1),$

$(\alpha^2,1,\alpha^2),(\alpha^2,\alpha,0),(\alpha^2,\alpha,\alpha),(\alpha^2,\alpha,\alpha^2),(\alpha^2,\alpha^2,1),(\alpha^2,\alpha^2,\alpha)\}$

Definition2.1.5 [1] Let C be a code over F(q) and a be any vector of length n. Then, the set a + C =$\{a+x:x\epsilon C\}$ is called a translate(or coset) of C .

Lemma: 2.1.6 [5] S_t(x) =x+$S_t(\bar{0})$ for any $x\epsilon F_{2N}^{n}$ and 0≤t≤n.

Proof: Now $s_t(\bar{0})$ is the set of all norm–t vectors and hence $|s_t(\bar{0})|=L_t(n)$.(By definition2.1.2).Then, for any xϵF_{2N}^{n}, any element in x+$s_t(\bar{0})$ will be of the form x+u where $\|u\|$=t.

Now, d_R (x , x+u) =$\|x-(x+u)\|$=t

i.e.s_t(x) contains all these elements x+u .

Since $|s_t(\mathrm{x})|$=$L_t(n)$, it follows that s_t(x) =x+$s_t(\bar{0})$.

Corollary 2.1.7[5] For any x, y$\epsilon F_{2^N}^n$,$s_t(x)$ =x+$y + s_t(y)$.

Proof: From **lemma 2.1.7**,$s_t(y)$ = y + $s_t(\bar{0})$, then

x+$y + s_t(y)$= x+y + y + $s_t(\bar{0})$.

=x+$s_t(0)$. (y + y =2y = 0 since 2=0 inF_2)

=s_t(x).

Thus,s_t(x) is the translates of the surface area $s_t(\bar{0})$ by x. Moreover, it follows that,B_t(x) = x+$B_t(\bar{0})$ for any x$\epsilon F_{2^N}^n$.

2.2. WEIGHT DISTRIBUTION OF TRANSLATES [7]

In this section weight distribution and distance invariant of a code is defined and also determined whether the kth surface area $s_k(\bar{0})$of rank spherefor 0≤k≤n is distance invariant. In the sequel, weight distribution of some selected class of translates of $s_k(\bar{0})$, that is, of those translates s_k(x) for which ||x||=1 are obtained. Before plunging into it, some essential notations are given .Let $s_{t,k}$ be a set formed by adding a norm-t element with all norm–k elements. Typically$s_{t,k}$ may denote s_k(x) with||x||=t so that$|s_{t,k}| = L_t(n)$ .Let $s_t(\bar{0})\oplus s_k(\bar{0})$ denote the set of all ordered elements of the form u+v where ||u||=t and ||v|| = k so that $|s_t(\bar{0})\oplus s_k(\bar{0})|$=$L_t$(n).$L_K$(n).That is,$s_t(\bar{0})\oplus s_k(\bar{0})$ is formed by adding each norm-t element with every norm-k element, thus$|s_t(\bar{0})\oplus s_k(\bar{0})| = |s_k(\bar{0})\oplus s_t(\bar{0})|$.Let $W_{t,k}^l$ represent the number of norm-l elements in each $s_k(x)$, $||x||$=t where 0≤ $l \leq n$.

To make the above notations and concepts clear let's see the following example.

Example2.2.1 LetC={ (0,0), (0,1), (0, α), (0,α^2), (1,0), (1,1), (1, α), (1,α^2), (α, 0), (α, 1), (α , α),

(α, α^2),(α^2, 0),(α^2, 1),(α^2, α),(α^2, α^2)}be an RD code over F_{2^2}={0,1, α ,α^2} where α is a primitive element and given by the relationα^2 = α+1.Then, let x =(α, 0) ϵ C, where $||x||$

=1.Thus, $S_{1,2}$=S_2(x) ={u + x : uϵC and $\|u\|$=2}.Now, those vectors in C whose rank norm= 2 are $(1, \alpha)$, $(1,\alpha^2)$, $(\alpha,1)$,$(\alpha^2,1)$,(α,α^2),(α^2, α) and x,with$\|x\|$ =1.The following tableshows the sum of these rank=2 norm and x,and also the weight of each resultant vectors.

Table (1)

	+	$X = (\alpha,0)$ (where$\|x\| = 1$)	Weight of Each Vector
Weight =2 Vectors	$(1, \alpha)$	(α^2, α)	2
	$(1,\alpha^2)$	(α^2,α^2)	1
	$(\alpha,1)$	(0,1)	1
	$(\alpha^2,1)$	(1,1)	1
	(α,α^2)	$(0,\alpha^2)$	1
	(α^2,α)	$(1, \alpha)$	2

From table (1)above, we get the following results:

(1) $S_{1,2}$=S_2(x)=$S_2(\alpha, 0)$={(α^2, α),(α^2,α^2),(0,1),(1,1),$(0,\alpha^2)$,$(1, \alpha)$}

(2) $W_{1,2}^1$ =4 (i.e. which is the number of weight 1 vectors in$S_{1,2}$).

(3) $W_{1,2}^2$ =2 (i.e. which is the number of weight 2 vectors in$S_{1,2}$).

Definition2.2.2[7] The weight distribution of a code C of length n is generally defined by{ $W_0(C), W_1(C), \dots W_n(C)$},where$W_i(C)$specifies the number of code words of each possible weight i= 0,1,...,n in C.

Example2.2.3 Consider (RD) code **C**={(1,1,0),(1,1,1),$(1,1,\alpha)$,$(1,1,\alpha^2)$,$(1,\alpha, 0)$,$(1,\alpha, 1)$,$(1,\alpha, \alpha)$, $(1,\alpha,\alpha^2)$,$(1,\alpha^2, 0)$,$(1,\alpha^2, 1)$,$(1,\alpha^2,\alpha)$,$(1,\alpha^2, \alpha^2)$,$(\alpha, 0,0)$,$(\alpha, 0,1)$,$(\alpha, 0, \alpha)$} over a fieldF_{2^2} and letαbe a primitive element of the field F_{2^2} such that$\alpha^2 = \alpha + 1$

(a).Determine $W_1(C), W_2(C), W_3(C)$ and the weight distribution of C ?Based on the definition given above the number of code words having weight 1 is4 and the vectors are

$(1,1,0)$,$(1,1,1)$,$(\alpha,0,0)$,$(\alpha,0,\alpha)$ and the number of code words having weight 2 is 9 and the vectors are$(1,1,\alpha)$,$(1,1,\alpha^2)$,$(1,\alpha,0)$,$(1,\alpha,1)$,$(1,\alpha,\alpha)$,$(1,\alpha^2,0)$,$(1,\alpha^2,1)$,$(1,\alpha^2,\alpha^2)$, $(\alpha,0,1)$ and the number of code words having weight 3 is1 and the vector is$(1,\alpha,\alpha^2)$,thus the weight distribution of the code C is $\{1,9,1\}$.

Definition2.2.4 [7] A code C is called distance invariant if for each i, the cardinality of the set $\{y\epsilon C{:}d_R(x, y)=i\}$ is the same for each xϵC.

From definition2.2.4, it follows that a code C is distance invariant if the weight distributions of all the translates c + C, cϵC is the same.

Example2.2.5 Consider an RD code C=$\{(0,\alpha), (1,\alpha), (\alpha,\alpha^2), (\alpha^2,\alpha^2)\}$ and observe all the translates and their weight distribution to show that C is distant invariant. Now,

C_1=(0,α)+C==$\{(0,0),(1,0),(\alpha,1),(\alpha^2,1)\}$. Thus;

$W_0(C_1)$=1 (i.e. the number of code words in C_1 having weight=0)

$W_1(C_1)$=1 (i.e. the number of code words in C_1 having weight=1)

$W_2(C_1)$=2 (i.e. the number of code words in C_1 having weight=2).

Now, the weight distribution of $C_1$is$\{1,1,2\}$.

C_2=$(1,\alpha)$+C=$\{(1,0),(0,0),(1+\alpha,1),(\alpha,1)\}$.

$W_0(C_2)$=1 (i.e. the number of code words in C_2having weight=0)

$W_1(C_2)$=1 (i.e. the number of code words in C_2 having weight=1)

$W_2(C_2)$=2 (i.e. the number of code words in C_2having weight=2).

Now, the weight distribution of C_2 is$\{1,1,2\}$.

C_3=(α,α^2)+C=$\{(\alpha,1),(\alpha^2,1),(0,0),(1,0)\}$.Thus;

$W_0(C_3)$=1 (i.e. the number of code words in C_3having weight=0)

$W_1(C_3)$=1 (i.e. the number of code words in C_3 having weight=1)

$W_2(C_3)$=2 (i.e. the number of code words in C_3having weight=2).

Now, the weight distribution of C_3 is{1, 1, 2}.

C_4=(α^2, α^2) +C= {$(\alpha^2$,1),$(\alpha, 1)$, (1, 0),(0, 0)}.Thus;

$W_0(C_4)$=1 (i.e. the number of code words in C_4having weight=0)

$W_1(C_4)$=1 (i.e. the number of code words in C_4 having weight=1)

$W_2(C_4)$=2 (i.e. the number of code words in C_4 having weight=2).

Now, the weight distribution of C_4 is {1,1, 2}.

Thus, the weight distribution of all the translates c + C, c ϵ C is the same.

Therefore, the code **C** is **distance invariant.**

Proposition2.2.6 [5] $s_k(\bar{0})$, the collection of all elements of rank norm-k, is distance invariant.

Proof: $s_k(\bar{0})$ is distance invariant if the weight distribution of all its translates, namelys_k(x), for ||x||=k is the same.(By definition 2.2.4).Thus, it is, necessary to proof that weight distribution ofS_k(x) is the same for all x with rank norm k. Any element of s_k(x) will be of the form x+u, where u ϵ $s_k(\bar{0})$ and ||x|| =k.

Let x ϵ $s_k(\bar{0})$ and X be its corresponding n x n associated matrix overF_2.So,||x|| =rank(X)=k. (By definition 1.3.9).Let A be an n x n non-singular matrix over F_2 and B be an n x n non-singular matrix over F_2 such thatAXB is the rank normal of X.(By theorem 1.3.5.)

Then, rank (X) =rank (AXB). (By theorem 1.3.5.)

Call AXB as$\tilde{X}$. Then rank (AXB) =rank ($\tilde{X}$) =k.

Now, there will be an element say$\tilde{x}$ in $S_k(\bar{0})$ whose matrix is $\tilde{X}$

Thus, ||$\tilde{x}$|| =rank ($\tilde{X}$) =k.

Let y$\epsilon s_k(\bar{0})$ and Y be its matrix so that ||y||=rank(Y)=k.

Now, $||x - y||$ =rank(X-Y)

=s, say, where $0 \leq s \leq 2k$.

Moreover, s=rank(X-Y)

=rank (A(X-Y)B).(By theorem 1.3.5.)

=rank (AXB-AYB)

=rank ($\tilde{X}$-$\tilde{Y}$)

(Where $\tilde{Y}$=AYB so that rank ($\tilde{Y}$) =rank (AYB) =rank(Y) =k)

$= ||\tilde{x} - \tilde{y}||$ where $\tilde{y} \in s_k(\bar{0})$ whose matrix is $\tilde{Y}$.

Thus, if $||x - y||$=s there exists $\tilde{y} \in s_k(\bar{0})$ such that $||\tilde{x} - \tilde{y}|| = s$, which implies that x-$s_k(\bar{0})$ and $\tilde{x} - s_k(\bar{0})$ have same the weight distribution. Thus, $S_K(x)$ will have the same weight distribution for any x with $||x||$ =k.

Proposition2.2.7[5] Weight distribution of $S_K(x)$ for x with rank norm t are same for all $x \in s_t(\bar{0})$.

Proof: Let $x \in s_t(\bar{0})$ and X be its corresponding n x n associated matrix over F_2 so that $||x||$ =rank(X) =t.

Let A be an n x n non- singular matrix over F_2 and B be an n x n non-singular matrix over F_2 such that AXB is the rank normal of A. (By theorem 1.3.5.)

Then, rank(X) =rank (AXB). (By theorem 1.3.5.)

Call AXB as $\tilde{X}$.Then rank (AXB) = rank ($\tilde{X}$) = t.

Now there will be an element say $\tilde{x}$ in $s_t(\bar{0})$ whose matrix is $\tilde{X}$

Thus $||\tilde{x}||$ =rank ($\tilde{X}$) =t.

Let $y \in s_k(\bar{0})$ and Y be its associated matrix so that $||y||$=rank(Y) =k.

Now$\|x-y\|$=rank (X-Y)

=s, say, where$|t-k| \leq s \leq t+k$.

Moreover, s=rank(X-Y)

=rank (A(X-Y)B) (By theorem 1.3.5.)

=rank (AXB-AYB)

=rank $(\tilde{X}-\tilde{Y})$

(Where$\tilde{Y}$=AYB so that rank($\tilde{Y}$) =rank (AYB)= rank (Y)=k)

=$\|\tilde{x}-\tilde{y}\|$ where $\tilde{y} \epsilon s_k(\bar{0})$ whose matrix is $\tilde{Y}$.

Thus, if $\|X-Y\|$=s,there exists$\tilde{y} \epsilon s_k(\bar{0})$ such that$\|\tilde{x}-\tilde{y}\|$=s, which implies that x-$s_k(\bar{0})$ and $\tilde{x}-s_k(\bar{0})$ have same weight distribution.Thuss_k(x) and $s_k(\tilde{x})$ have same the weight distribution.

Hence, the proof as x,$\tilde{x}$ being arbitrary in $s_t(\bar{0})$.

Now, having proved that the weight distribution of $s_k(x)$ (with $\|x\|$=t) are the same, the next job at hand is to determine them.

Proposition: 2.2.8 [5] $W_{t,k}^{l}=W_{k,t}^{l} X \frac{L_k(n)}{L_t(n)}$

Proof: Let $s_t(\bar{0})$ =$\{u_1, u_2, \ldots, u_{L_t(n)}\}$, the set of norm-t elements and $s_k(\bar{0})=\{v_1, v_2, \ldots, v_{L_k(n)}\}$,the set of norm-k elements .Each set$s_k(u_i)$contains$W_{t,k}^{l}$ norml-elements and so$s_t(\bar{0}) \oplus s_k(\bar{0})$ will contains $W_{t,k}^{l}$X $L_t(n)$norm-$l$elements by counting the repetition of elements ,as well. Similarly, as eachset $s_t(v_j)$contains$W_{k,t}^{l}$norm$-l$elements,$s_k(\bar{0}) \oplus s_t(\bar{0})$willcontain$W_{k,t}^{l} X L_k(n)$ norm- l elements.Thus, $W_{t,k}^{l} X L_t(n)$=$W_{k,t}^{l} X L_k(n)$ since$|s_k(\bar{0}) \oplus s_t(\bar{0})|=|s_t(\bar{0}) \oplus s_k(\bar{0})|$and hence the result.

Proposition: 2.2.9 [5]$W_{t,k}^{l} = W_{l,k}^{t} X \frac{L_l(n)}{L_t(n)}$.

Proof: Now number of norm- $l$elements in each sets_k(x),||x||=t is$W_{t,k}^{l}$.Hence, in $s_l(\bar{0}) \oplus s_k(\bar{0})$,each norm-t element will be repeated $W_{t,k}^{l}$ times .Therefore, $s_l(\bar{0}) \oplus s_k(\bar{0})$, will contain $W_{t,k}^{l} X L_t(n)$ norm-t elements by counting the repetition of elements as well. This means that, in$s_k(w)$,$||w||= l$,the number of norm-t elements will be $W_{t,k}^{l} X \frac{L_t(n)}{L_l(n)}$.That is, $W_{l,k}^{t}$=$W_{t,k}^{l} X \frac{L_t(n)}{L_l(n)}$from which the proposition follows.

Proposition: 2.3.0 [5]$W_{t,l}^{k} = W_{t,k}^{l}$

Proof: Each set,s_k(x),$||x||$=t, contains $W_{t,k}^{l}$ norm- l elements .When x is added with all norm- l elements, one will get exactly these many number of norm-k elements .Thus $W_{t,l}^{k} = W_{t,k}^{l}$.

Lemma 2.3.1 [5] Let t, k and $l$be such that $|t-k| \leq l \leq t+k$. Then, in$s_t(\bar{0}) \oplus s_k(\bar{0})$,each norm-$l$ element is repeated $W_{l,k}^{t}$ times if and only if $s_{l,k}$ contains $W_{l,k}^{t}$ nom-t elements.

Proof: As $|t-k| \leq l \leq t+k$ it follows that $|t-l| \leq k \leq t+l$and $|l-k| \leq t \leq l+k$. Suppose that,in $s_t(\bar{0}) \oplus s_k(\bar{0})$,each norm- l element is repeated$W_{l,k}^{t}$.Let x be a norm- l element in$s_t(\bar{0}) \oplus s_k(\bar{0})$. Then $x=u_{i_1}+v_{j_1}$

$$x=u_{i_2}+v_{j_2}$$

$$\vdots$$

$$x=u_{i_{W_{l,k}^t}} + v_{j_{W_{l,k}^t}}.$$

Where all u_i's are different norm-t elements and allv_j's are different norm-k element.

Then, it follows that $x+v_{j_1}=u_{i_1}$

$$x+v_{j_2}=u_{i_2}$$

$$\vdots$$

$$x + v_{j_{W_{l,k}^t}} = u_{i_{W_{l,k}^t}}.$$

This implies that, the set $s_k(x)$, $\|x\|= l$ contains $W_{l,k}^{t}$ number of norm-t elements. Thus $s_{l,k}$ contains $W_{l,k}^{t}$norm-t elements.

Conversely, let $s_{l,k}$ contain $W_{l,k}^{t}$ norm-t elements. In particular, for x with $\|x\|= l, s_k(x)$ contains $W_{l,k}^{t}$ norm-t elements. This implies that, in $s_t(\bar{0})\oplus s_k(\bar{0})$,x is repeated $W_{l,k}^{t}$ times .As x is an arbitrary norm- l element ,it follows that ,in$s_t(\bar{0})\oplus s_k(\bar{0})$,each norm-$l$ element is repeated $W_{l,k}^{t}$ times.

Remark2.3.2 As , $s_{l,k}$containing$W_{l,k}^{t}$norm-t elements is a fact that, in $s_t(\bar{0}) \oplus s_k(\bar{0})$,each norm-$l$ element is repeated$W_{l,k}^{t}$ times where $|t-k| \leq l \leq t+k$.

To find weight distribution of certain translates $s_k(x)$ where $\|x\|=1$, denote a k-dimensional subspace of the n – dimensional space F_{2^n} spanned by m elements $x_1, x_2, \dots, x_m \epsilon F_{2^n}$, where k$\leq m$ by$\langle x_1, x_2, \dots, x_m\rangle_k$.

Thereme2.3.3 [5] The weight distribution of $S_{1,k}$is given by $\{W_{1,K}^{K-1}, W_{1,K}^{K}, W_{1,K}^{K+1}\}$, where

$$W_{1,K}^{K-1} = 2^{K-1}(2^K-1)\frac{L_K(n)}{L_1(n)};$$

$$W_{1,K}^{K-1} = (2^{n+1}+2^{k-1}-2^{k+1}-1)(2^k-1)\frac{L_K(n)}{L_1(n)}.\text{and}$$

$$W_{1,K}^{K+1} = (2^n-2^K)^2\frac{L_K(n)}{L_1(n)}$$

Proof: consider the set $S_{1,k}$. In particular, let $\|u\|=1$ and consider the set $s_k(u)$.Each element in $s_k(u)$will be of the form u + x where$\|x\|=k$.Without loss of generality, let$u = (\alpha_1, 0, \dots, 0)$so that$\|u\|=1$and let$x = (x_1, x_2, x_3, \dots, x_n)$ where $\alpha_1, x_1, x_2, x_3, \dots, x_n \epsilon F_{2^n}, \alpha_1 \neq 0$.

When u=$(\alpha_1, 0, \dots, 0)$ is added with x ϵ $s_k(\bar{0})$, the resultant will be of norm-(k-1) or of norm-k or of norm-(k+1).Now u + x=$(\alpha_1+x_1, x_2, x_3, \dots, x_n)$.

Case (a):$\|u+x\|$=k-1.This will happen when $\|(x_2, x_3, \dots, x_n)\|$=k-1,$x_1 \notin \langle x_2, x_3, \dots, x_n\rangle_{k-1}and\alpha_1 + x_1 \epsilon \langle x_2, x_3, \dots, x_n\rangle_{k-1}$,thatis,$x_1 \epsilon\ \alpha_1 + \langle x_2, x_3, \dots, x_n\rangle_{k-1}$.

If $\alpha_1 \in \langle x_2, x_3, \dots, x_n \rangle_{k-1}$,then,$x_1 \in \langle x_2, x_3, \dots, x_n \rangle_{k-1}$.This means the number of choices for x_1 is 0 as $x_1 \notin \langle x_2, x_3, \dots, x_n \rangle_{k-1}$.

So if $\alpha_1 \notin \langle x_2, x_3, \dots, x_n \rangle_{k-1}$ this means the number of choices for x_1 is $2^k - 2^{k-1} = 2^{k-1}$. Hence the number of choices for $(x_2, x_3, \dots, x_n)$ under this case

=Number of (n-1) tuples of norm-(k-1), each of whose space does not contain the

1- dimensional space $\langle \alpha_1 \rangle$(since$\alpha_1 \notin \langle x_2, x_3, \dots, x_n \rangle_{k-1}$)

$=\binom{n-1}{k-1} 2^{(k-1).1} x \{L_{K-1}(n-1) \text{over} F_{2^{K-1}}\}$(by proposition 1.2.9)

$=\binom{n-1}{k-1} 2^{(k-1).1} x \binom{n-1}{k-1} (2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$(by definition 1.4.3)

$=2^{(k-1)} \binom{n-1}{k-1}\binom{n-1}{k-1} (2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$.................... (*)

Thus, the possible choices for x= $(x_1, x_2, x_3, \dots, x_n)$ will be

$W_{1,K}^{K-1} = 2^{(k-1)} x 2^{(k-1)} \binom{n-1}{k-1}\binom{n-1}{k-1} (2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$. (Since the

Possible choices ofx_1 is$2^{k-1}$and that of $(x_2, x_3, \dots, x_n)$ is given in $*$)

$=(2^{(k-1)})^2 x \binom{n-1}{k-1}\binom{n-1}{k-1} (2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$.

$=(2^{(k-1)})^2 x \frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{((2^k-2)(2^k-4)\dots(2^k-2^{k-1}))^2} (2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$.

(by definition 1.4.3)

$= (2^{(k-1)})^2 x \frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{((2^k-2)(2^k-4)\dots(2^k-2^{k-1}))^2} (2^k-2)(2^k-4)\dots(2^k-2^{k-1})(\frac{1}{2^{k-1}})$

$=(2^{(k-1)}) \frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{(2^k-2)(2^k-4)\dots(2^k-2^{k-1})}$(By simplification)

$=2^{(k-1)}(2^k-1)\frac{L_K(n)}{L_I(n)}$. (Since$\frac{L_K(n)}{L_I(n)} = \frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{(2^k-1)(2^k-2)(2^k-4)\dots(2^k-2^{k-1})}$by definition 1.4.3)

Case (b):$\|u+x\| = k$ .Here arises two possibilities:

(b)(1): When$\|(x_2, x_3, \dots, x_n)\|$ =k and$x_1 \in \langle x_2, x_3, \dots, x_n \rangle_k$ and so $\alpha_1 + x_1 \in \langle x_2, x_3, \dots, x_n \rangle_k$ or

(b)(2): When$\|(x_2, x_3, \dots, x_n)\|$=k-1 and$x_1 \notin \langle x_2, x_3, \dots, x_n\rangle_k$ andso$\alpha_1 + x_1 \notin \langle x_2, x_3, \dots, x_n\rangle_{k-1}$.

Possibility (b) (1):

Now$\|(x_2, x_3, \dots, x_n)\|$ =k ,$x_1 \in \langle x_2, x_3, \dots, x_n\rangle_k$ and$x_1 \in \alpha_1 + \langle x_2, x_3, \dots, x_n\rangle_k$.

If$\alpha_1 \in \langle x_2, x_3, \dots, x_n\rangle_k$ then $x_1 \in \langle x_2, x_3, \dots, x_n\rangle_k$ and number of choices for x_1 is2^k.

And number of choices for$(x_2, x_3, \dots, x_n)$

=Number of (n-1) –tuples of norm-k, each of whose space contains $\langle \alpha_I \rangle$ (since$\alpha_1 \in \langle x_2, x_3, \dots, x_n\rangle_{k-1}$)

=$\binom{n-1}{k-1}$x$\{L_k(n-1) \text{over} F_{2^k}\}$. (By proposition 1.2.8)

=$\binom{n-1}{k-1}\binom{n-1}{k}(2^k-1)(2^k-2)\dots(2^k-2^{(k-1)})$.(By definition 1.4.3)......................... (1 *)

If $\alpha_1 \notin \langle x_2, x_3, \dots, x_n\rangle_k$, then $x_1 \in \alpha_1 + \langle x_2, x_3, \dots, x_n\rangle_k$ .But $x_1 \in \langle x_2, x_3, \dots, x_n\rangle_k$ and so number of choices for x_1 here is 0.

Possibility (b) (2):

Now, $\|(x_2, x_3, \dots, x_n)\|$ =k-1 ,$x_1 \notin \langle x_2, x_3, \dots, x_n\rangle_{k-1}andx_1 \notin \alpha_1 + \langle x_2, x_3, \dots, x_n\rangle_{k-1}$.

If $\alpha_1 \in \langle x_2, x_3, \dots, x_n\rangle_{k-1}$ then the number of choices for x_1 is $2^n - 2^{K-1}$.

And number of choices for $(x_2, x_3, \dots, x_n)$will be

=Number of (n-1)-tuples of norm–(k-1), each of whose space contains $\langle \alpha_I \rangle$ (since$\alpha_1 \in \langle x_2, x_3, \dots, x_n\rangle_{k-1}$)

=$\binom{n-1}{K-1-1}$X$\{L_{K-1}(n-1) \text{over} F_{2^{K-1}}\}$(by proposition 1.2.8)

=$\binom{n-1}{k-2}\binom{n-1}{k-1}(2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$.(By definition 1.4.3)............... (2 *)

If $\alpha_1 \notin \langle x_2, x_3, \dots, x_n\rangle_{k-1}$ then the number of choices for x_I is$2^n - 2^K$.
And number of choices for$(x_2, x_3, \dots, x_n)$

= Number of (n-1)-tuples of norm–(k-1),each of whose space that does not contain $\langle\alpha_1\rangle$. (Since $\alpha_1 \notin \langle x_2, x_3, \dots, x_n\rangle_{k-1}$)

$=\binom{n-1}{k-1}2^{(k-1).1}\{L_{K-1}(n-1)\text{over}F_{2^{K-1}}\}$.(By proposition 1.2.9)

$=\binom{n-1}{k-1}2^{(k-1)}\binom{n-1}{k-1}(2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})$.(By definition 1.4.3).............. (3∗)

Thus, under this case(b), using $(1*), (2*), (3*)$ the number of possible choices of $(x_1, x_2, x_3, \dots, x_n)$ will be

$$W_{1,K}^{K} = 2^K\binom{n-1}{k-1}\binom{n-1}{k}(2^K-1)(2^{(k)}-2)\dots(2^K-2^{(k-1)}) + (2^n-2^{K-1})\binom{n-1}{k-2}\binom{n-1}{k-1}$$
$$(2^{(k-1)}-1)(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})+(2^n-2^K)\binom{n-1}{k-1}2^{(k-1)}\binom{n-1}{k-1}(2^{(k-1)}-1)$$
$$(2^{(k-1)}-2)\dots(2^{(k-1)}-2^{(k-2)})\dots\dots\dots\dots\dots(4)$$

$$=\frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{(2^k-2)(2^k-4)\dots(2^k-2^{k-1})}(2^n-2^k)+\frac{(2^n-2^{k-1})((2^n-2)\dots(2^n-2^{k-2}))^2(2^n-2^{k-1})}{2(2^k-4)\dots(2^k-2^{k-1})}$$
$$+\frac{\left((2^n-2)(2^n-4)\dots(2^n-2^{k-1})\right)^2}{(2^k-2)(2^k-4)\dots(2^k-2^{k-1})}(2^n-2^k) \text{ (After simplification of 4)}$$

$$=(2^n-2^k)(2^k-1)\frac{L_K(n)}{L_1(n)}+\frac{(2^n-2^{k-1})}{2}\frac{(2^k-2)}{(2^n-2^{k-1})}(2^k-1)\frac{L_K(n)}{L_1(n)}+(2^n-2^k)(2^k-1)\frac{L_K(n)}{L_1(n)}\text{(since}$$
$$\frac{L_K(n)}{L_1(n)}=\frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{(2^k-1)(2^k-2)(2^k-4)\dots(2^k-2^{k-1})}\text{ by definition 1.4.3)}$$

$$=(2^n-2^k)(2^k-1)\frac{L_K(n)}{L_1(n)}+(2^{k-1}-1)(2^k-1)\frac{L_K(n)}{L_1(n)}+(2^n-2^k)(2^k-1)\frac{L_K(n)}{L_1(n)}$$

$$=(2^n+2^n+2^{k-1}-2^{k+1}-1)(2^k-1)\frac{L_K(n)}{L_1(n)}.$$

$$=(2^{n+1}+2^{k-1}-2^{k+1}-1)(2^k-1)\frac{L_K(n)}{L_1(n)}.$$

Case (c):$\|u+x\|$=K+1.This will happen when $\|(x_2, x_3, \dots, x_n)\|$ =k,and $x_1 \in \langle x_2, x_3, \dots, x_n\rangle_k$ so that $\alpha_1 + x_1 \notin \langle x_2, x_3, \dots, x_n\rangle_k$,that is,$x_1 \notin \langle x_2, x_3, \dots, x_n\rangle_k$.

If $\alpha_1 \in \langle x_2, x_3, \dots, x_n\rangle_k$then number of choices for x_1 is 0.

So,if $\alpha_1 \notin \langle x_2, x_3, \dots, x_n\rangle_k$ then the number of choices for $x_1$is 2^k.

Number of choices for $(x_2, x_3, \dots, x_n)$

=Number of (n-1)-tuples of norm-k, each of whose space does not contain$\langle \alpha_l \rangle$.(Since$\alpha_1 \notin \langle x_2, x_3, \dots, x_n \rangle_{k-1}$)

=$\binom{n-1}{k}(2^k)\{L_K(n-1) \text{over} F_{2^K}\}$(By proposition 1.2.9)

=$\binom{n-1}{k}(2^k)\binom{n-1}{k}(2^K\text{-}1)(2^{(k)}\text{-}2)\dots(2^K\text{-}2^{(k-1)}$. (By definition 1.4.3)............ (4*)

Thus, using (4*) and since the possible choices of x_1 is 2^k,then number of possiblechoices of $x=(x_1, x_2, x_3, \dots, x_n)$will be

$W_{1,K}^{K+1}=(2^k)^2\binom{n-1}{k}\binom{n-1}{k}(2^K\text{-}1)(2^{(k)}\text{-}2)\dots(2^K\text{-}2^{(k-1)})$

$=(2^k)^2(\frac{(2^{n-1}-1)((2^{n-1}-2)(2^{n-1}-4)\dots(2^{n-1}-2^{k-1}))^2}{((2^k-1)(2^k-2)\dots(2^k-2^{k-1}))^2}(2^K\text{-}1)(2^{(k)}\text{-}2)\dots(2^K\text{-}2^{(k-1)})$

$= (\frac{(2^n-2)(2^n-4)\dots(2^n-2^k))^2}{(2^k-1)(2^k-2)\dots(2^k-2^{k-1})}$

$=(\frac{(2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2(2^n-2^k)^2)}{(2^k-1)(2^k-2)\dots(2^k-2^{k-1})})$

$=(2^n-2^K)(2^n-2^K)\frac{L_K(n)}{L_1(n)}$(since $\frac{L_K(n)}{L_1(n)} = \frac{((2^n-2)(2^n-4)\dots(2^n-2^{k-1}))^2}{(2^k-1)(2^k-2)(2^k-4)\dots(2^k-2^{k-1})}$).

$=(2^n-2^K)^2\frac{L_K(n)}{L_1(n)}$

Hence the results follow.

SUMMARY

In this report, one of the properties of rank metric codes, which is called surface area of rank sphere and its translates and weight distribution of those translates are discussed. That is, for any non-negative integer $t \leq n$ and $x \epsilon F_{2^N}^n$, the rank sphere of radius t with center at x is defined by $B_t(x)$={$y\epsilon F_{2^N}^n: d_R(x,y) \leq t$}.Now, for any $X\epsilon F_{2^N}^n$ and non-negative integer i≤n S_i(x) ={$y\epsilon F_{2^N}^n: d_R(x,y) = i$} is called the i^{th}surface area of rank sphere with center at x. A code $C \subset F_{2^N}^n$ is called distance invariant code if for each i the cardinality of the set{$y\epsilon F_{2^N}^n: d_R(x,y) = i$}is the same for each .The translates $S_{t,k}$ of the k^{th} surface area of rank sphere which is denoted by $S_k\ (x)$ with $||x|| = t$ and$|S_{t,k}|$ =$L_t(n)$ of surface area of $s_k(\bar{0})$, which is the collection of all elements of rank norm-k, is distance invariant and the weight distributions of the translates $S_k\ (x)$ of $s_k(\bar{0})$ are the same for all $x \in s_t(\bar{0}$. Ultimately, the weight distribution of $S_k\ (x)$ when $||x|| = 1$ is determined.

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