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Designing a Professional Development Program for Elementary Mathematics Teachers to Support Productive þÿStruggle using Guskey s Mode

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COLLEGE OF EDUCATION

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DEPARTMENT OF MATHEMATICS AND SCIENCE EDUCATION

DESIGNING A PROFESSIONAL DEVELOPMENT PROGRAM FOR ELEMENTARY MATHEMATICS TEACHERS TO SUPPORT PRODUCTIVE STRUGGLE USING GUSKEY'S MODEL

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Designing a Professional Development Program for Elementary Mathematics Teachers to Support Productive Struggle using Guskey's Model

By:

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Education in Teaching Mathematics

Advisor: Berie Getie (PhD)

June 2026

Bahir Dar, Ethiopia

DECLARATION

I hereby declare that this thesis, entitled "Designing a Professional Development Program for Elementary Mathematics Teachers to Support Productive Struggle using Guskey's Model," is my original work. This thesis has not been submitted to any other university or institution for the award of any academic degree, diploma, or certificate.

Name of candidate: **Kassanew Gietu Getie** Signature: _______________ Date: June 2026

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Approval of Thesis for Defense

I hereby certify that I have supervised, read, and evaluated this thesis entitled "Designing Professional Development Program for Elementary Mathematics Teachers to Support Productive Struggle using Guskey's Model" by **Kassanew Gietu**, prepared under my guidance. I recommended the thesis to be submitted for oral defense.

Berie Getie (PhD)	______________________	June, 2026
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Approval of Thesis for Defense Result

As members of the board of examiners, we examined this thesis entitled "Designing Professional Development Program for Elementary Mathematics Teachers to Support Productive Struggle using Guskey's Model" by **Kassanew Gietu.** We hereby certify that the thesis is accepted for fulfilling the requirements for the award of the degree of Master of Education in Teaching Mathematics.

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ABSTRACT

Despite the recognized importance of productive struggle (PS) in fostering conceptual understanding in mathematics, a persistent gap exists between its theoretical endorsement and classroom implementation. In Ethiopia, teacher centered instruction dominates, leading to low proficiency. Teachers often view struggle as harmful and revert to directive practices, such as telling and over-scaffolding that remove cognitive demand from students. Traditional Professional Development (PD) fails to bridge this gap by attempting to change beliefs before changing practice. This study designed and evaluated a contextually relevant PD intervention using Guskey's model of teacher change, which deals belief change follow, rather than precede, observable improvements in student outcomes.

Employing a qualitative Design Based Research (DBR) methodology, the study involved two early-careers Grade 3 mathematics teachers and 30 students in Bahir Dar. The intervention included a baseline followed by two iterative cycles. The PD equipped teachers with the strategies to support PS. Data from classroom audio and video recordings, student artifacts, and semi-structured interviews were collected and then analyzed thematically.

Findings showed a substantial change in teachers' practices. In the baseline, teachers acted as struggle removers through procedural questioning and immediate intervention. By Cycle 2, they transitioned to struggle facilitators, effective questioning, purposeful wait time, encouraging persistence, and acknowledging the normalcy of mistakes. Consequently, student engagement evolved from passive reception to multimodal representation, while perseverance increased dramatically, evidenced by self-correction of their thinking and multiple attempts. Crucially, the study validated Guskey's sequence; only after observing tangible student engagement and perseverance did teachers' beliefs shift from viewing struggle as instructional failure to viewing it as a necessary catalyst for learning. The study concludes that experiential, video reflection based PD grounded in Guskey's model can effectively transform mathematics instruction by shifting the focus from preventing struggle to facilitating it.

Keywords*:* Productive Struggle, Professional Development, Guskey's Model of Teacher Change, Design Based Research, High Cognitive Demand Tasks.

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LIST OF ABBREVIATION

AL: Active Learning

DBR: Design Based Research

FDRE: Federal Democratic Republic of Ethiopia

HCD: High Cognitive Demand

NCTM: National Council of Teachers of Mathematics

PBL: Problem Based Learning

PD: Professional Development

PF: Productive Failure

PS: Productive Struggle

RQ: Research Question

SCM: Student centered Methods

TFE: Teach For Ethiopia

ZPD: Zone of Proximal Development

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CHAPTER ONE: INTRODUCTION

1.1. Background of the Study

Mathematics education globally has shifted from traditional rote memorization and procedural fluency toward deep conceptual understanding, resilient problem solving, and the development of growth mindsets. At the core of this transformation is Productive Struggle (PS), which first appeared notably in the Second Handbook of Research on Mathematics Teaching and Learning and has since gained substantial scholarly attention, especially after being popularized by Warshauer in 2011 (Young et al., 2023). Warshauer (2014) characterizes PS as: an intellectual effort students expend to make sense of mathematical concepts; providing challenges that fall within a student's reasonable capability; advancing thinking and deepening understanding if supported carefully toward resolution. The National Council of Teachers of Mathematics NCTM (2014) defines PS as a core practice in effective mathematics teaching, where students delve more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions.

PS contrasts with unproductive frustration by emphasizing progress through errors, perseverance, and sense-making, ultimately strengthening neural pathways and promoting long term retention (Kapur, 2010, 2014). It develops grit, creative problem solving, and strong habits of mind such as perseverance and flexible thinking (Mind Research Institute, 2021; Renaissance, 2020, as cited in Young et al., 2024). PS helps learners move beyond surface-level memorization and encourages them to grapple with complex concepts, thus leading to more meaningful and retained knowledge(Young et al., 2023).

Grounded in cognitive and constructivist learning theories, PS requires students to apply prior knowledge to novel, challenging tasks and learn from their errors, fostering meaningful knowledge construction (Hiebert & Grouws, 2007; Casler-Failing, 2024). Research on productive failure (PF) further shows that initial struggles, even without immediate success, enable students to better apply their learning to new problem situations, enhancing transferability and promoting growth mindsets over fixed abilities (Kapur, 2008, 2010, 2014).

In classroom settings, PS is facilitated through High Cognitive Demand (HCD) tasks that encourage multiple solution pathways, connections to underlying mathematical concepts, and inclusivity via enabling and extending prompts (Livy et al., 2018; Stein et al., 2008). These tasks, categorized in the mathematical tasks framework as procedures with connections or doing mathematics, promote deeper understanding compared to lower-level memorization or procedural tasks (Smith et al., 1998). Studies such as the QUASAR project have demonstrated significant learning gains for students engaging in HCD tasks(Silver & Kay, 1996; Stein & Lane, 1996). A clear relationship exists between the consistent selection and implementation of HCD tasks and students' conceptual understanding of mathematics (Parrish & Bryd, 2022). Effective implementation requires teachers to anticipate misconceptions, maintain challenge without discouragement, and establish norms such as "If you are not struggling, you are not learning" (NCTM, 2014). Supporting PS also involves recognizing the four primary types of student struggle identified by Warshauer (2014); getting started, carrying out processes, uncertainty in explaining and sense making, and expressing misconceptions or errors, and responding productively through probing questions, encouragement, adequate wait time, and acknowledgment (Warshauer, 2011, 2015).

Despite these global insights, there is no empirical research of PS in Ethiopia. In Ethiopia, educational sector has been bringing in more active and student centered learning since the introduction of the new education policy of 1994. However, the process has been very difficult for the teachers (Keski-Mäenpää, 2018) , rote memorization and teacher centered instruction still dominate classrooms. Keski-Mäenpää (2018) found existing practical arrangements at the schools in Ethiopia do not support the use of a student centered teaching method. For example, a detailed curriculum, annual tests, a high student-teacher ratio and a lack of teaching and learning materials support teacher-led teaching and make it difficult to implement student centered teaching methods. In order to launch active learning methods successfully in Ethiopian school context, major changes are needed in these arrangements and in teacher training as well. In urban public primary schools in Bahir Dar, Amhara Region, where Teach For Ethiopia (TFE) numeracy fellows teach, the challenges are pronounced: overcrowded classrooms (often 60-70+ students), severely limited resources (insufficient textbooks, manipulative, or teaching aids), and heavy reliance on direct instruction to cover the syllabus quickly.

Although supporting PS in learning is complex (Bolyard et al., 2023), Professional Development (PD) is essential for equipping teachers with the skills to support PS effectively. Effective PD is ongoing, collaborative, and situated in classroom contexts, impacting beliefs and behaviors through enhanced skills and improved teaching (Tesfamicael & Ayalew, 2021; Darling-Hammond et al., 2017; Suk Yoon et al., 2007; Çetin & Bayrakcı, 2019). As Çetin & Bayrakcı (2019) note, "PD has an impact on teachers' beliefs and behavior. The relationship between teachers' beliefs and their practice is not straightforward or simple; on the contrary, it is dialectical, 'moving back and forth between change in belief and change in classroom practice'" (p. 33). Guskey's model of teacher change (2002) provides a robust theoretical framework for this research, reversing traditional assumptions by asserting that changes in attitudes and beliefs follow evidence of improved student outcomes, rather than preceding them.

1.2. Statement of the Problem

Despite its recognized importance, the translation of PS from theory into classroom practice remains a persistent challenge in mathematics education. Although many teachers are exposed to and express commitment toward implementing PS, the lack of clear guidance on how to effectively enact it often leads to inconsistent practices, reduced instructional effectiveness, and frustration for both teachers and students (Young et al., 2023).

Research consistently indicates that teachers tend to support PS at a conceptual level but struggle to implement it in practice. Instead of facilitating sustained student thinking, teachers often revert to directive, procedural instruction, prematurely resolving student difficulties and limiting opportunities for meaningful engagement (Russo et al., 2021; Warshauer, 2015; Parrish & Bryd, 2022). Furthermore, empirical research examining how teachers can effectively support PS in real classroom contexts remains limited (Warshauer, 2011), indicating a significant gap in both research and practice.

Several factors contribute to this implementation challenge. Culturally, struggle is often perceived as a negative experience rather than a valuable learning opportunity. Teachers' prior experiences with rote and teacher centered learning, combined with limited professional preparation and pressure to produce quick answers, further discourage the use of PS (Stigler & Hiebert, 1998; Tasos et al., 2005). Results of research have demonstrated that teachers' past experiences as learners can influence how they interpret, make sense of, and implement reform practices in their own teaching. Teachers, and often parents, tend to cling to dominant, traditional teaching approaches, similar to what they likely experienced as learners, out of fear that moving away from these practices would hurt student learning (NCTM, 2014). In addition, the notion of PS is relatively new in the K-12 teaching vernacular. Therefore, it is not widely known or understood. Without a clear understanding of what it means, it would be difficult for teachers to implement a practice of supporting it in learning. Even when teachers attempt to use HCD tasks these tasks often decline in complexity during implementation as teachers intervene quickly when students experience difficulty. For example, teachers may provide step-by-step solutions, reduce the task to procedural steps, or limit student participation, thereby removing the productive aspect of struggle and undermining conceptual learning.

This challenge is further compounded by teachers' limited ability to recognize and respond productively to student struggle. Instead of using strategies such as probing questions, providing wait time, encouraging persistence, and acknowledging effort, teachers frequently rely on telling, over-scaffolding, or correcting errors directly (Freeburn & Arbaugh, 2017; Warshauer, 2014). This suggests that teachers often lack a clear understanding of what PS looks like in practice and how to support it effectively in classroom settings.

The problem is particularly pronounced in the Ethiopian context, where traditional, teacher centered instruction and rote memorization dominate classroom practice. Although national education policies have emphasized active and student centered learning since the 1994 education reform, implementation has remained difficult due to school arrangement and lack of teacher training (Keski-Mäenpää, 2018). According to Telore and Dametew (2023) studies teachers have positive perception towards active learning strategies but practice of implementation was not at a satisfactory level. Teachers implemented activities such as brain storming, asking and answering questions, discussion and group works. The researchers recommended trainings for the teachers and active involvement of all concerned bodies to facilitate active learning in the schools. Additionally, studies indicate that teachers face challenges in implementing student centered approaches, such as problem based learning(PBL), due to a lack of training, limited experience, and inadequate instructional support (Muluye et al., 2025).

In the local classroom context, these challenges are clearly evident. Based on the researcher's teaching experience, many teachers rely heavily on lecture based instruction and procedural teaching methods, even when active learning is encouraged. Students are rarely given opportunities to engage with challenging tasks independently, and teachers often intervene immediately when students encounter difficulty. As a result, students become dependent on teacher guidance, show limited perseverance, and expect direct answers rather than engaging in problem solving processes. Errors are often discouraged or negatively perceived, further reducing students' willingness to take risks and struggle productively.

Although PD programs are intended to address such instructional gaps, they often fail to provide sustained, practice oriented support. Many PD initiatives focus on short term training without continuous follow up, limiting their impact on classroom practice and student learning outcomes (Darling-Hammond et al., 2017; Ischinger, 2009). Consequently, teachers may gain theoretical knowledge but struggle to translate it into effective instructional practices.

Therefore, there is a critical need for research that not only examines the gap between teachers' beliefs and practices regarding PS but also designs and refines contextually relevant PD interventions that support teachers in implementing PS effectively. This study addresses this need by employing a Design Based Research (DBR) approach to develop a PD program grounded in Guskey's model of teacher change, which emphasizes that changes in teachers' beliefs occur after they observe improvements in student learning outcomes. Through iterative cycles of implementation and refinement, this study seeks to support teachers in enacting PS and improving students' engagement and perseverance in mathematics learning.

1.3. Objectives of the Study

The main purpose of this study is to design, implement, and evaluate a contextually relevant PD intervention that equips mathematics teachers with the knowledge and skills to effectively support PS in their classrooms.

Specifically, the study aims to:

1. Assess the influence of the PD intervention on teachers' classroom practices, particularly their use of questioning, encouragement, wait time, and acknowledgement when supporting PS.

2. Determine the changes observed in students' engagement and perseverance when teachers implement strategies to support PS.
3. Understand how teachers' beliefs and attitudes toward PS evolve as they engage in the intervention, implement new practices, and witness improvements in their students' engagement and perseverance.

1.4. Research Questions

The study is guided by the following research questions:

1. How does the PD intervention influence mathematics teachers' classroom practices in supporting PS?
2. What changes in students' engagement and perseverance are observed when teachers implement strategies to support PS in mathematics classrooms?
3. How do mathematics teachers' beliefs and attitudes about PS change after implementing the intervention and observing students' engagement and perseverance?

1.5. Significance of the Study

This study holds considerable theoretical, practical, and contextual significance. It addresses a critical gap in mathematics education research by examining how PD can enable teachers to effectively support PS.

Theoretically, the study contributes to the existing literature by providing an empirical examination of Guskey (2002) model of teacher change within the domain of mathematics education and PS. While Guskey's model has been widely cited, few studies have tested its application in facilitating specific pedagogical shifts such as supporting PS. By investigating how teachers' beliefs and practices evolve through iterative cycles of implementation, reflection, and observation of student engagement and perseverance, this research offers insights into the process of teacher change. Furthermore, the study enriches the literature on PS (Warshauer, 2015; Hiebert & Grouws, 2007) by documenting how the four types of student struggle can be identified and addressed through targeted teacher strategies in authentic classroom settings. The findings also provide empirical support for constructivist and cognitive theories that emphasize the construction of deep, transferable mathematical understanding through active intellectual engagement.

Practically, the study offers valuable contributions for improving mathematics instruction. It will produce a contextually grounded PD model focused on experiential learning, implementation of HCD tasks, identification of student struggle, and the use of specific strategies (probing questions, wait time, encouragement, and acknowledgement) to support PS. This framework can serve as a practical template for teacher educators, instructional coaches, and school leaders working in similar resource-constrained settings. At the policy level, the findings offer empirical justification for investing in sustained, classroom-based PD rather than traditional short term workshops.

Contextually and socially, the study is particularly significant for Ethiopia's education system. By promoting PS, the research directly addresses the persistent challenges of low mathematics achievement, rote memorization, and limited conceptual understanding in Ethiopian primary schools. Shifting classroom practices toward PS can help build stronger conceptual foundations, enhance student perseverance, and foster growth mindsets (Dweck, 2006). In an examination oriented system with large class sizes and limited resources, this study demonstrates how western developed pedagogical approaches can be meaningfully adapted to local realities. Ultimately, the research contributes to the development of resilient, problem solving learners who are better equipped to meet the demands of the 21st century.

1.6. Delimitations of the Study

This study is delimited to examining the effectiveness of a PD intervention in supporting PS in Grade 3 mathematics classrooms. Geographically, the study is limited to two purposively selected urban public primary schools in Bahir Dar City, Amhara National Regional State, Ethiopia. These schools are typical of many government primary schools in the region, characterized by large class sizes and limited instructional resources.

The study involves Grade 3 mathematics teachers with their students. The teachers were selected from each school to capture implementation across varied levels of teaching experience within a manageable context. Although this small sample enables in-depth analysis, the findings are context-specific and not intended for broad generalization.

The intervention focuses specifically on fraction-related tasks and the implementation of four key teacher strategies for supporting PS: questioning, encouragement, giving time (wait time), and acknowledgement (Warshauer, 2015). While other instructional practices may influence PS, they are not included in this study.

1.7. Operational Definition of Terms

Acknowledgement: A teacher strategy that explicitly communicates to students that confusion, struggle, and errors are a normal and necessary part of learning mathematics.

Design Based Research (DBR): An iterative research methodology where the researcher and practitioners collaboratively design, implement, analyze, and refine educational interventions through successive cycles to improve practice.

Encouragement: A teacher strategy that encourage students to explain, affirms a student's effort, persistence, and progress toward a solution without lowering the cognitive demand of the task.

Guskey model of teacher change: A theoretical framework positing that lasting changes in teachers' beliefs and attitudes occur after they implement new practices and observe positive changes in student learning outcomes.

Productive Failure: The process where in students initially fail to solve a complex problem, but the cognitive effort expended during that struggle prepares them to deeply understand the correct concept upon reflection or re-instruction.

Productive Struggle (PS): The intellectual effort students invest in solving challenging but accessible mathematical tasks, characterized by progress through effort, sense making, and perseverance rather than unproductive frustration.

Professional Development (PD): A sustained, experiential training program designed to enhance educators' knowledge and skills, in this study, specifically the facilitation of PS.

Questioning: The use of probing prompts that require students to focus on their thinking, identify their struggle, and make sense of tasks.

Student Engagement: The observable cognitive and affective investment of students in a mathematical task, evidenced by the use of multiple representations, active participation, and verbal articulation of reasoning.

Student Perseverance: The sustained effort of students to work through mathematical challenges without giving up, evidenced by self-correction, multiple solution attempts, and revised work.

Teacher Belief and Attitude: A teacher's subjective evaluation and perception regarding the value of student struggle.

Unproductive Struggle: Struggle that results in frustration, task abandonment, or failure to make mathematical progress due to a lack of appropriate scaffolding or the task being beyond the student's current capacity.

Wait Time: The deliberate duration of silence a teacher maintains after posing a question or after a student stops talking, allowing students time to process and formulate a response without premature and over teacher intervention.

CHAPTER TWO: LITERATURE REVIEW

2.1. Introduction

This chapter reviews the theoretical and empirical literature related to productive struggle in mathematics education and the professional development of teachers. It examines the value of struggle in mathematical learning, the nature and types of student struggle, instructional strategies for supporting productive struggle, teacher responses to student difficulties, and the role of high-cognitive-demand tasks in fostering meaningful mathematical engagement. The chapter further explores professional development and Guskey's model of teacher change as theoretical foundations for understanding how teachers adopt and sustain new instructional practices. Finally, international and Ethiopian empirical studies are reviewed to identify existing knowledge, contextual challenges, and research gaps that justify the present study and inform the development of its conceptual framework.

2.2. The Value of Struggle in Mathematics

Mathematics learning is inherently a process of sense-making rather than the mere acquisition of procedures. Deep mathematical understanding develops when learners engage with challenging problems, explore alternative strategies, justify their reasoning, and persist through uncertainty (Kilpatrick, Swafford, & Findell, 2001) . From this perspective, struggle is not a sign of failure but a fundamental feature of learning mathematics. The National Research Council identified productive disposition as a key strand of mathematical proficiency, defining it as the habitual inclination to see mathematics as sensible, useful, and worthwhile while believing in one's own capability to learn it (Kilpatrick, Swafford, & Findell, 2001) .Productive disposition serves as both the motivation and the confidence that sustain students when they encounter difficult mathematical tasks. Students who believe that effort leads to understanding are more likely to persevere, engage in reasoning, and view challenges as opportunities for learning rather than evidence of inability.

However, productive disposition can only lead to meaningful learning when students are given opportunities to grapple with challenging mathematical ideas. Research consistently shows that cognitively demanding tasks promote deeper understanding because they require students to think, reason, and make connections rather than simply follow procedures (Hiebert & Grouws (2007). Within such tasks, errors and difficulties become valuable learning resources, helping students refine their thinking and construct stronger conceptual understanding (Borasi, 1994) . Just as productive disposition is the fuel that motivates and guide learners, productive struggle is the vehicle that carries them toward understanding. Through sustained engagement with mathematical challenges, students develop not only conceptual knowledge but also the resilience, persistence, and problem-solving habits that characterize successful mathematical learners.

2.3. Struggle in Learning

The notion of struggle in learning is not novel. For years, cognitive learning theorists (e.g., Piaget, 1960; Skemp, 1971) have suggested that experiencing struggle, disequilibrium, or impasse during learning motivates learners to find a resolution, leading to new learning (Bolyard et al., 2023). PS can also be considered intimately related to Lev Vygotsky's familiar Zone of Proximal Development (ZPD) theory. Vygotsky describes ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers." PS thus can be understood as the space a student inhabits whilst engaging in an appropriately challenging task that sits within their ZPD (Dixon & Mahoney, 2024). Carol Dweck's mindset theory also supports it. Dweck (2006) distinguishes between a fixed mindset (the belief that intelligence is static) and a growth mindset (the belief that intelligence can be developed through effort and perseverance). PS inherently requires a growth mindset (Dweck, 2006). When students believe that their intellectual capacity can grow, they view struggle not as an indicator of innate inability, but as a necessary process for brain development and learning (Kapur, 2014). Kapur (2008) described productive failure (PF), the notion that undertaking problem solving efforts that do not lead to a solution can positively impact learning. More recently, Baker et al. (2020) characterized PS as utilizing existing knowledge, perseverance, and sense-making to solve new and novel problems. A key idea across theses constructs is that struggle occurs as a result of efforts to engage in sense-making, create new connections, and deepen understanding (as cited in Bolyard et al., 2023).

Research has supported the positive role struggle plays in learning. Hiebert & Grouws (2007) found providing students opportunities to "struggle with important mathematics" (p. 387) to be a main feature that supported meaningful learning. Researchers examining the effects of engaging students in problem solving tasks before instruction found the approach benefited students' learning, even when those initial efforts were not successful (Kapur & Rummel, 2012; Kapur, 2010). Scaffolding plays an important role. Several studies have found that providing scaffolding support after learners have attempted to solve a problem on their own and reached a point of stalled progress to be effective (Kapur & Rummel, 2012). Other studies have highlighted the importance of opportunities to collaborate and reflect as students engage in PS. Results of these studies support the practice of providing space for students to share, discuss, and evaluate a variety of solutions and approaches as a key process in supporting deeper learning (Kapur & Rummel, 2012).

Supporting PS in learning is complex. An examination of the literature revealed a variety of instructional features that have been explicated as supporting PS in learning (Bolyard et al., 2023). Some include establishing a learning environment that values perseverance and normalizes failure; engaging students in high-level tasks; valuing students' ideas and contributions; scaffolding students' sense-making; focusing on justification and explanation; and positioning students as contributors to the learning process.

Most literature discussing productive struggle in learning is grounded in mathematics, likely because it is a specific practice listed in NCTM's recommendation of effective teaching practices. NCTM (2014) articulated eight teaching practices for effective mathematics instruction: establishing goals to focus learning, implementing tasks that promote reasoning and problem solving, using and connecting representations, facilitating mathematical discourse, building procedural fluency from conceptual understanding, supporting productive struggle, and eliciting and using student thinking. According to NCTM (2014), "Effective mathematics teaching consistently provides students, individually and collectively, with opportunities and supports to engage in PS as they grapple with mathematical ideas and relationships" (p. 48). NCTM (2014) emphasizes that PS involves "delving more deeply into understanding the mathematical structure of problems and relationships among mathematical ideas, instead of simply seeking correct solutions" (p. 48). NCTM (2014) notes that through this process, "Students come to realize that they are capable of doing well in mathematics with effort and perseverance in reasoning, sense making and problem solving" (p. 10). This ties directly to the concept of perseverance, which Young et al (2023) identify as a key component of PS. Young et al (2023) define PS as the process of engaging with challenging tasks that require effort, critical thinking, and persistence to solve, involving obstacles, mistakes, and cognitive discomfort while actively working towards a solution.

NCTM (2014) provides clear guidelines for both teachers and students, emphasizing the shared responsibility in the mathematics classroom.

Table 1: Support PS in Learning Mathematics, adapted from NCTM (2014)

Teachers doing	Students doing
• Anticipating what students might struggle with during the lesson, being prepared to support them productively. • Giving time to struggle with the task, asking question that support their thinking without stepping in to do the work. • Acknowledge students that confusion and errors are the natural part of learning by facilitating discussion on mistake, misconception, and struggles. • Praising students for their effort in making sense of mathematical idea and perseverance on reasoning the tasks.	• Struggling at mathematical tasks and, but knowing that breakthroughs often emerge from confusion and struggle. • Asking question related to the source of their struggle. • Persevering in solving problems, don't give up. • Helping one another without telling their classmate what the answer is.

NCTM (2014) advises that "As teachers plan lessons, key components for them to consider are the student struggles and misconceptions that might arise. Thinking about these in advance allows teachers to plan ways to support students productively, Without removing the opportunities for students to develop deeper understanding of the mathematics" (pp. 48-49). This proactive approach is often reinforced by classroom norms such as, “If you are not struggling, you are not learning,” which reframes cognitive dissonance as a positive and expected component of the learning process (NCTM, 2014).

While research has supported the importance of struggle in learning, implementing this practice can be difficult for teachers. US cultural norms view struggle in learning negatively, particularly in science and mathematics, and view the teacher’s role as removing struggle(Stigler & Hiebrt, 2004). Results of research have demonstrated that teachers’ past experiences as learners can influence how they interpret, make sense of, and implement reform practices in their own teaching. Teachers, and often parents, tend to cling to dominant, traditional teaching approaches, similar to what they likely experienced as learners, out of fear that moving away from these practices would hurt student learning (NCTM, 2014)). In addition, the notion of PS is relatively new in the K-12 teaching vernacular. Therefore, it is not widely known or understood. Without a clear understanding of what it means, it would be difficult for teachers to implement a practice of supporting it in learning (Bolyard et al., 2023).

2.4. Types of Student Struggle

For teachers to effectively support PS, they must recognize the various forms of intellectual difficulty students face (Warshauer, 2014). Warshauer (2014) identified four main types of student struggle in mathematics.

Getting Started

These struggles most often occur when students initially engage with a task, either individually or in small groups. They represent a fundamental confusion about the task requirements or a lack of an initial approach. Students experiencing getting started struggles may exhibit a range of behaviors, including:

- Confusion about what the task is asking, vocalized through statements like, “I don’t know what to do” or “I’m very confused”.
- Claiming to forget a prior concept, such as, “I have absolutely no idea….I don’t remember that far”.
- Gestures of uncertainty and resignation, such as looking away, thinking, sitting back.

- No work on paper, indicating an inability to begin the task at all.

Carrying Out a Process

This type of struggle occurs when students have successfully initiated a plan but encounter an impasse while attempting to execute a procedure. The difficulty lies in the implementation of the strategy. Indicators of this struggle include:

- Encountering an impasse despite having formulated a plan.
- Being unable to implement a process from a representation they have formulated, for example, being unable to translate a drawn diagram into a mathematical equation.
- Being unable to carry out an algorithm or process effectively.
- Forgetting facts or formulas necessary for the next step.
- Making computational mistakes that hinder the continuation of the process.

Experiencing Uncertainty in Explaining and Sense-Making

This struggle tends to occur in the latter part of a task, particularly as students are asked to share, justify, or reflect on their work in small groups or with the whole class. The core issue is difficulty articulating their thinking and making sense of their own or others' solutions. This is characterized by:

- Difficulty in explaining their work, expressed through statements like, "I know what I'm thinking, I just can't show the exact way".
- Expressing uncertainty about their solution or the steps they took.
- Being unclear about the reasons for their choice of strategy.
- Being unable to make sense of their own work or the solutions proposed by their peers.

Expressing Misconceptions and Errors

This struggle involves specific mathematical mistakes or misunderstandings that affect the problem solving process. Errors in mathematics are generally categorized into two distinct types:

- Conceptual errors: These arise when students misunderstand underlying relationships and principles within a problem.

- Careless errors: These result from unintended and avoidable lapses in procedure, computation, or transcribing information.

2.5. Instructional Strategies for Supporting Productive Struggle

Recognizing the types of student struggles is only the first step; the critical factor in making struggle productive lies in the teacher's instructional response (Warshauer, 2015). Research suggests that a range of teacher support and responses are possible during episodes of student struggle to advance students productively towards a resolution of understanding (Warshauer, 2014). However, teachers must use strategies that keep tasks at a high level of cognitive demand, such as asking probing questions, encouraging explanation, and avoiding premature scaffolding that lowers rigor (Dominguez & Svihla, 2023; Livy et al., 2018). Livy et al. (2018) stress that teachers should use questions that extend thinking, such as "Why does this work?", "What have you tried?", or "Can you justify your decision?", rather than providing procedural hints that do the thinking for the student. Hiebert & Grouws (2007) emphasize strategies like providing adequate wait time, letting students struggle with uncertainty, and creating opportunities for them to examine different solution paths. Warshauer (2015) specifically identified four essential strategies for teachers when students are in the midst of struggle:

Question: The teacher asks questions that help students focus on their thinking and identify the source of their struggle, then encourages students to build on their thinking or look at other ways to approach the problem.

Encourage: Teachers encourage students to reflect on their work and support student struggle in their effort and perseverance, not just in getting the correct answer. Teachers are also encouraged to foster persistence through using affirming statements such as: "You did not give up even though you were stuck … you tried something different … you tried more than one answer …"(Livy et al., 2018).

Give Time: Teachers provide adequate time and support for students to manage their struggles through adversity and failure by not stepping in too soon or too much, thereby taking the intellectual work away from the students.

Acknowledge: Teachers explicitly acknowledge that struggle is an important and necessary part of learning and doing mathematics.

In addition to these four core strategies, other classroom practices help support student struggle and make it productive (Pasquale, 2016). These include setting explicit goals at the beginning of the lesson, setting problems in a familiar setting whenever possible to leverage student experience, supporting students by providing appropriate tools and representations, grouping students heterogeneously, establishing high mathematical expectations, providing time for group reflection, and comparing student outcomes to original goals (Pasquale, 2016).

Crucially, teachers must establish class norms that explicitly support PS, such as these examples(Pasquale, 2016):

- Being wrong is an opportunity to learn.
- Being correct is an impetus to help others.
- Everyone is responsible for each other’s learning.

2.6. Teacher Responses to Student Struggle

Recent studies have illustrated many possible ways teachers can use to respond to the students’ PS in mathematics (Freeburn & Arbaugh, 2017; Warshauer, 2015). A primary goal of teacher response is to maintain the task’s HCD (Smith et al., 1998). According to Warshauer(2015) teachers respond to the student’s PS in the following four ways.

Table 2: Teacher response summary adapted from (Warshauer, 2014).

Teacher response	characterizations
Telling	Supply information. Suggest strategy, method, or technique. Correct error. Relate to simpler problem. Model method or technique.
Direct guidance	Redirect student thinking. Breakdown problem into smaller parts. Alter problem to analogy.
Probing guidance	Ask for reason and justification. Offer idea based on students thinking. Seek explanation that could get at an error. Ask for written work of students.
Affordance	Ask for detailed explanation. Build on student thinking Press for justification and sense making individually or with group. Afford time for students to work.

2.7. The Role of High Cognitive Demand Tasks in Promoting Productive Struggle

PS cannot occur in a vacuum; it requires a context that necessitates intellectual effort (Hiebert & Grouws, 2007). Curriculum developers, text writers, subject specialists, and researchers should provide supplemental materials organized by tasks that promote critical thinking, reasoning, problem solving and modeling are preferable than those meaningless repetitive tasks (Tesfamicael & Ayalew, 2021). PS happens when students work through challenging tasks with just enough support to persevere. Therefore, the selection and implementation of tasks are a central part of a teacher's instructional toolkit, and what students learn is often defined by the tasks they are given(Christiansen & Walther, 1986). In order to move students toward developing a deep conceptual understanding of mathematics, classroom teaching must incorporate opportunities for students to grapple with meaningful tasks (Lampert, 2001; NCTM, 2014).

The well-established mathematical tasks framework proposed by Smith et al (1998) categorizes tasks into four levels of cognitive demand: two lower level (memorization and procedures without connections tasks) and two higher level (procedures with connections and doing mathematics tasks). Memorization tasks are generally algorithmic and in most cases require reproducing a learned fact, rule, or definition that was memorized(Smith et al., 1998). Procedures without connections tasks are generally procedural, with little ambiguity as to the steps required to complete the work; they focus on correct answers rather than understanding(Smith et al., 1998). Because these lower level tasks do not require students to engage in sense making or relational thinking, they do not elicit PS (Hiebert & Grouws, 2007).

Conversely, tasks for PS must be HCD tasks(Livy et al., 2018). Procedures with connections tasks require procedures but with the purpose of developing connections to underlying mathematical concepts; multiple representations or solution paths are possible (Smith et al., 1998). Doing mathematics tasks require students to analyze the task, explore ideas contained within, connect them to other concepts, and make explanations(Smith et al., 1998). These HCD tasks are the fertile ground for PS because they require students to exert the intellectual effort necessary to make sense of mathematics (Hiebert & Grouws, 2007; Warshauer, 2015). Students engaged in HCD tasks showed the highest student learning gains in studies such as the QUASAR Project(Silver, 1996; Stein & Lane, 1996).

However, the cognitive demand of a task can easily be diminished during implementation. Teachers promote PS by choosing challenging mathematical tasks, implementing them effectively, and requiring students to justify their thinking(Parrish & Bryd, 2022). Providing problems that students cannot immediately solve, encouraging independent exploration, using purposeful questioning, and letting students observe peer's strategies all support deeper engagement and understanding (Parrish & Bryd, 2022). If a teacher steps in too soon to model a method or simplify the problem, the task declines into a procedural exercise, diminishing students of the opportunity to struggle productively(Smith et al., 1998; NCTM, 2014). However, exposing students to complex problem solving tasks which are beyond their cognitive levels, skills, and abilities can result in PFs on the part of students (Kapur, 2010). Therefore, a delicate balance must be struck where the task is challenging yet achievable within the student's zone of proximal development (McLeod, 2025; Warshauer, 2015).

Challenging tasks are defined by Russo (2016) as having four key features: They involve absorbing mathematical problems; There exist multiple solution pathways; There is a concern for a whole class focus on the same problem; Tasks enable inclusivity with enabling and extending prompts(Russo, 2016). Crucially, a well-designed mathematics lesson should include consolidation tasks, enabling prompts (for students experiencing difficulty), and extending prompts (for students who completed the task quickly)(Livy et al., 2018). This approach, known as differentiated task delivery, ensures that the cognitive demand of the core problem is maintained for all learners(Livy et al., 2018).

Research suggests that students' out of school experiences influence their motivation and perseverance with difficult mathematical tasks(Pasquale, 2016). Therefore, tasks that have a familiar or real life context are more meaningful (Clarke & Roche, 2018; Taylor, 2015, as cited in Pasquale, 2016). Because students are able to draw on their everyday experience to solve these kinds of tasks, they are more motivated to stick with the task(Pasquale, 2016). To demonstrate how a task can be adapted through enabling and extending prompts without compromising cognitive demand, consider the following multiplicative task (suitable for Year 5 students) presented by Livy et al. (2018):

Task: I did a multiplication question correctly for homework, but my printer ran out of ink. I remember it looked like 2_ × 3_ = _ _ 0. What might be the digits that did not print? (Give as many answers as you can).

Enabling prompt: What might be the missing numbers? 1 _ × _ = _ 0.

Extending prompt: Convince me that you have all the possible solutions.

Using this approach, all students have access to a challenging task where the cognitive demand is not compromised by over simplifying the task.

2.8. Professional Development for Teacher Learning

Given the nature of teacher beliefs and the cultural norms surrounding mathematical struggle, shifting classroom practice toward supporting PS requires intentional and sustained intervention (Darling-Hammond et al., 2017). A teacher may have good intentions in mind but may cause damage to students or the system due to a lack of training and skill. To keep teachers on par with the changing dynamics of the educational landscape and to help them perform better, not only teacher education programs are fundamental, but continuous teacher development programs are also essential (Nauman, 2017, as cited in Çetin & Bayrakcı (2019)). PD is fundamentally about teachers learning, learning how to learn, and transforming their knowledge into practice for the benefit of their students' growth (Avalos, 2010). Guskey (2002) formally defines PD as a continuous, job embedded process designed to enhance educators' knowledge, skills, and professional judgment to ultimately improve instruction and student outcomes.

Historically, PD in education has been characterized by short workshops that fail to give lasting change (Guskey, 2002). However, the literature confirms that effective PD can support deep changes in teaching if it is situated in classroom practice, is ongoing, and is collaborative with other teachers (Çetin & Bayrakcı, 2019). High quality programs move beyond one-off workshops, emphasizing sustained duration, active learning, and collective participation (Darling-Hammond et al., 2017). PD has a significant influence on the beliefs and behavior of the teacher and their classroom practice (Mitskova, 2010, as cited in Çetin & Bayrakcı, 2019)

To effectively train teachers to support PS, PD must follow principles of effective practice. Darling-Hammond et al (2017) identified seven features of effective PD: it is content focused, it incorporates active learning, it supports collaboration, it uses models of effective practice, it provides coaching and expert support, it offers feedback and reflection, and it is of sustained duration. Specifically, effective PD must be content-focused, deepening teachers' knowledge in specific mathematical subjects, particularly regarding HCDs tasks and the nature of PS. It must also be collaborative, allowing teachers to work in professional learning communities to analyze student struggles and rehearse supportive strategies. Finally, it must be practical, involving modeling, coaching, and expert feedback so teachers can safely practice the uncomfortable act of withholding answers and probing student thinking(Suk Yoon et al., 2007).

The mechanism by which PD impacts student outcomes is sequential. Suk Yoon et al (2007) posit that PD affects student achievement through a three step mechanism. First, PD enhances teacher knowledge and skills. Second, better knowledge and skills improve classroom teaching. Third, improved teaching raises student outcome. Crucially, if one link in this chain is weak or missing for example, if a teacher fails to apply new ideas from PD to classroom instruction better student learning cannot be expected (Suk Yoon et al., 2007). This highlights the necessity of bridging the gap between PD sessions and actual classroom implementation, ensuring that teachers are supported to translate new knowledge about PS into instructional practices.

2.9. Guskey Model of Teacher Change

Despite the general acceptance of PD as essential to improvement in education, reviews of PD research consistently point out the ineffectiveness of most programs (Guskey, 2002). A variety of factors undoubtedly contribute to this ineffectiveness. However, it has been suggested that the majority of programs fail because they do not take into account two crucial factors: 1) what motivates teachers to engage in PD, and 2) the process by which change in teachers typically occurs (Guskey, 1986, as cited in Guskey, 2002). To design a PD program that successfully encourages elementary mathematics teachers to support PS, it is vital to ground the intervention in a robust theory of teacher change, that is, Guskey's model of teacher change (Guskey, 2002).

Guskey's model challenges the conventional wisdom regarding how and why teachers adopt new practices from PD. Historically, the prevailing assumption was that PD would first change a teacher's attitudes and beliefs, which would then lead to new classroom practices, and finally result in improved student outcomes (Guskey, 2002). This linear, belief first approach suggests that teachers must be convinced of the value of a new strategy, such as PS, before they will attempt to implement it. However, as evidenced by the persistent gap between teachers' espoused beliefs and their enacted practices regarding PS (Philipp, 2007), changing beliefs through exposure to theory alone is difficult.

Guskey (2002) proposed a fundamental reversal of this sequence, asserting that for change to be meaningful and lasting, positive changes in student learning must precede changes in a teacher's attitudes and beliefs. According to the model, significant change in teachers' attitudes and beliefs occurs primarily after they gain evidence of improvements in student learning outcomes. The crucial point is that it is not the PD per se, but the experience of successful implementation that changes teachers' attitudes and beliefs (Guskey, 2002). Teachers believe a new practice works because they have seen it work in their own classrooms, and that experience solidifies their belief and commitment.

Guskey's model is predicated on the idea that change is primarily an experientially based learning process for teachers. Practices that are found to work, that is, those that teachers find useful in helping students attain desired learning outcomes, are retained and repeated. Those that do not work or yield no tangible evidence of success are generally abandoned(Guskey, 2002). Therefore, demonstrable results in terms of student learning outcomes are the key to the endurance of any change in instructional practice. The core insight of the model is that efficacy follows evidence: teachers are not persuaded by theory only, but by positive results in their own classrooms, which solidifies their belief in the value of the new practice(Guskey, 2002).

The process unfolds sequentially as follows:-

- ✓ PD experience: Teachers first participate in a PD program or training.
- ✓ Implementation of the new practice: Teachers then implement the new strategy in their classroom practice.
- ✓ Changes in student learning outcomes: Teachers subsequently observe positive changes in student learning outcomes.
- ✓ Change in attitudes and beliefs: only then do teacher internalize a change in attitudes and beliefs toward the new practice, committing to it because they have seen tangible evidence of its success.

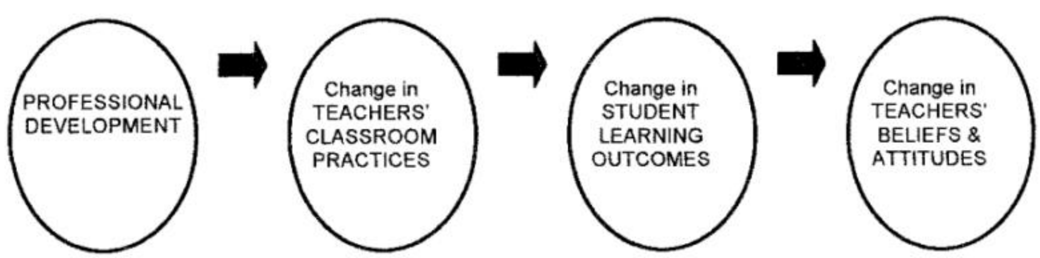

Figure 1: Guskey model of teacher change (Guskey, 2002)

2.10. Empirical Review

To contextualize the proposed study within the broader global and regional landscape, it is essential to examine the empirical research conducted on PS, teacher support strategies, and PD across different educational contexts. This review categorizes the empirical evidence into international and Ethiopian studies to highlight both the universal challenges of implementing PS and the context-specific barriers that necessitate the intervention.

2.10.1. International Studies on Productive Struggle

Internationally, empirical research has extensively documented the benefits of PS and the challenges teachers face in facilitating it. The landmark QUASAR Project demonstrated that when students were exposed to HCDs and engaged in PS, they showed the highest student learning gains compared to students in traditional classrooms(Silver, 1996; Stein & Lane, 1996). Warshauer (2014) conducted seminal empirical work identifying the types of student struggles and teacher responses, finding that teachers often default to telling or direct guidance rather than probing guidance or affordance, thereby inadvertently lowering the cognitive demand of tasks.

(Kapur, 2008, 2010, 2012, 2014) extensive empirical work on PF across international contexts (Singapore) has consistently shown that students who struggle with generating solutions before instruction outperform their peers who receive direct instruction on delayed post-tests measuring conceptual understanding and transfer. However, there is a persistent disparity between teachers' espoused beliefs and enacted practices; even when international teachers value PS, they frequently revert to directive methods in the moment of student difficulty due to cultural norms that view struggle negatively (Stigler & Hiebert, 2004). Furthermore, Darling-Hammond et al (2017) and Suk Yoon et al (2007) have empirically validated that international PD programs only succeed in changing classroom practice when they are sustained, content-focused, and actively bridge the gap between teacher knowledge and student outcomes.

2.10.2. Ethiopian Studies

In the Ethiopian context, the empirical landscape mirrors the broader African challenges but with distinct systemic barriers. To date, however, there is a lack of empirical research explicitly investigating PS as defined by Hiebert & Grouws (2007) or Warshauer (2015) in Ethiopian elementary mathematics classrooms. Because PS is fundamentally a student centered, constructivist process, its operationalization in Ethiopia should be understood through the empirical research on Active Learning (AL) and Student centered Methods (SCM) in the Ethiopian education system.

The Ethiopian educational sector has been bringing in more active and student centered learning since the introduction of the new education policy of 1994. However, the process has been very difficult for the teachers (Keski-Mäenpää, 2018) . As Keski-Mäenpää (2018) notes, active learning in the Ethiopian school context remains widely phrased, poorly practiced.

Empirical evidence attributes the persistence of traditional instruction to severe practical, cultural, and professional constraints. Implementing student centered teaching faces many challenges. According to Ayalewu et al.,(2025) the implementation of problem based learning (PBL) models faces significant challenges as perceived by participating teachers; from these, a lack of adequate knowledge and training on PBL, and insufficient teacher experience with PBL, are listed as the primary problems. This aligns with Keski-Mäenpää (2018) findings that existing practical arrangements at schools, such as a detailed curriculum, a high student teacher ratio, and a lack of teaching materials make it extremely difficult to implement student centered methods.

The PD provided to Ethiopian mathematics teachers has been widely criticized as ineffective. Keski-Mäenpää (2018) concluded that in order to launch active learning methods successfully in the Ethiopian school context, major changes are needed in practical arrangements and in teacher training. Current teacher training often consists of short-term, top-down workshops focused on theory rather than practical, sustained, and experiential learning. Consequently, there is no evidence of sustained pedagogical change. There is no known study in Ethiopia that utilizes Guskey's model of teacher change, which posits that lasting belief change follows observable student success, to design a PD intervention specifically aimed at supporting PS.

2.11. Conceptual Framework of the Study

This study is grounded in an adapted version of Guskey (2002) model of teacher change, which provides a sequential process for understanding how PD can lead to sustained improvements in teaching practices, particularly in supporting PS in mathematics education.

The conceptual framework integrates key elements from the literature within an experiential PD approach. The framework operates sequentially as follows:

Phase 1: PD intervention:- The process begins with targeted PD that moves beyond theoretical exposition (Darling-Hammond et al., 2017). Grounded in the literature, the PD immerses teachers in HCD tasks(Livy et al., 2018) and trains them to identify the four types of student struggles (Warshauer, 2014). Crucially, the PD explicitly trains teachers on the four essential strategies to support PS (Warshauer, 2015; Hiebert & Grouws, 2007).

Phase 2: Implementation in to classroom:- Following the PD, teachers implement the four PS support strategies (Warshauer, 2015) in their mathematics classrooms.

Phase 3: Changes in student engagement and perseverance: - As teachers implement these support strategies, they observe changes in student engagement and perseverance primarily evidenced by increased student engagement (use of multiple representations, active participation) and perseverance (willingness to revise thinking, self-correction, and sustained effort through difficulty).

Phase 4: Change in Teacher Attitudes and Beliefs:- According to Guskey's model, it is only after teachers observe these positive changes in student engagement and perseverance that their core attitudes and beliefs shift (Guskey, 2002). This shift solidifies their commitment to the new instructional practices.

The Reflective Redesign Cycle (DBR)

Crucially, because this study employs DBR as its research design, the process does not end at Phase 4. A Reflective Redesign Cycle connects the change in teacher beliefs and attitudes back to the PD intervention. The reflections from Cycle 1 inform the refinement of the PD and classroom tasks for Cycle 2. The visual representation of this adapted conceptual framework is presented in Figure 2.1:

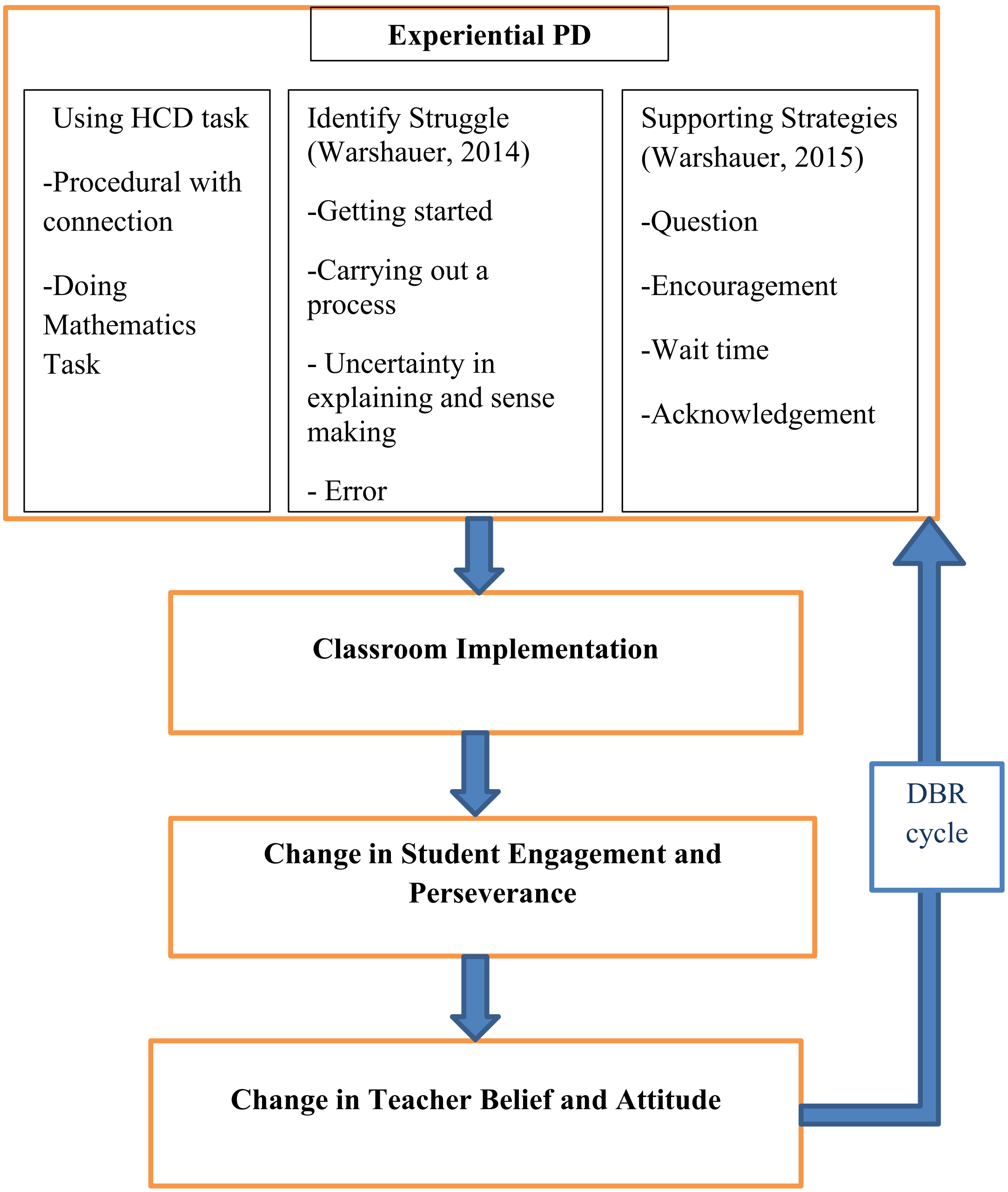

Figure 2: Conceptual framework adapted from Guskey (2002) model of teacher change

CHAPTER THREE: RESEARCH METHODOLOGY

3.1. Introduction

This chapter presents the detailed methodology employed in this study. It outlines the philosophical assumptions guiding the research, the research approach, the research design, the study area and context, the population and sampling procedures, the structure of the intervention cycles, the data collection methods and instruments, the data analysis procedures, measures of trustworthiness, ethical considerations. The chapter provides a clear rationale for each methodological choice.

3.2. Research Paradigm

The philosophical foundation of this study is grounded in the interpretivist paradigm, also referred to as the constructivist paradigm. Interpretivism posits that reality is socially constructed, multiple, and subjective. It asserts that knowledge is best understood through the meanings individuals assign to their lived experiences, rather than through the discovery of singular, objective truths that exist independent of human cognition(Creswell, 2013). This paradigm stands in contrast to positivism, which assumes a single, objective reality that can be measured and quantified. For interpretivists, the world is not discovered; it is interpreted.

Ontologically, interpretivism assumes that multiple realities exist, each shaped by the distinct context, culture, and personal histories of the individuals involved (Erciyes, 2020). In the context of this study, the reality of a mathematics classroom, what constitutes a struggle, whether it is viewed as productive or harmful, how a teacher responds to confusion, is not an objective fact but a subjective experience co-created by the teacher and the students. A teacher who views struggle as a failure of instruction experiences a vastly different classroom reality than a teacher who views struggle as a necessary catalyst for learning.

Epistemologically, interpretivism holds that knowledge is co-constructed between the researcher and the participants; the researcher is not a detached, objective observer, but rather an active participant in the meaning making process(Creswell, 2013). In DBR, this epistemological stance is vital. The researcher and the teachers collaborated to design the tasks, interpret the classroom video data, and reflect on the outcomes. The knowledge generated from this study is therefore not an objective truth imposed upon the teachers, but a shared understanding developed.

Methodologically, interpretivism favors' qualitative approaches that allow for the deep exploration of participants' subjective experiences(Cohen et al., 2009). The interpretivist paradigm is highly appropriate for this study because the phenomena under investigation, teachers' instructional choices during moments of student struggle, students' affective and cognitive engagement, and teachers' beliefs about the value of struggle, are deeply subjective and context-dependent. By adopting an interpretivist stance, this study does not seek to measure the amount of struggle or generalize a universal law of instruction. Instead, it seeks to understand how teachers interpret and make sense of a PD intervention within their specific classroom realities, and how those interpretations translate into practice.

3.3. Research Approach

This study employed a purely qualitative research approach. Qualitative research is a systematic, subjective approach used to describe life experiences and give them meaning, emphasizing the importance of the participants' perspectives (Creswell, 2013). Unlike quantitative or mixed-methods approaches that rely on statistical analysis, numerical frequencies, or hypothesis testing, a purely qualitative approach prioritizes thick description, contextual detail, and the complex realities of human interaction.

This study is purely qualitative because the primary research questions deals about how practices evolve, what changes are observed, and how beliefs shift. Answering these questions requires analyzing the precise phrasing of classroom dialogues, observing of teacher and student body language, interpreting of student artifacts, and understanding teachers' reflection.

3.4. Research Design

The study adopted DBR as its primary research design. "DBR evolved near the beginning of the 21st century and was heralded as a practical research methodology that could effectively bridge the chasm between research and practice in formal education"(Anderson & Shattuck, 2012, P. 16). DBR is a methodology designed by and for educators that seeks to increase the translation of education research into improved practice. It stresses the need for theory building and the development of design principles that guide, inform, and improve both practice and research in educational contexts (Anderson & Shattuck, 2012).

Brown (1992) and Collins (1992) are widely acknowledged as early contributors to the definition and activation of DBR. They described it as a methodology that requires: addressing complex problems in real contexts in collaboration with practitioners; integrating known and hypothetical design principles with technological affordances to render plausible solutions to these complex problems; and conducting rigorous and reflective inquiry to test and refine innovative learning environments as well as to define new design principles (as Herrington et al., 2007 cited). Unlike traditional experimental designs that seek to control variables to test isolated hypotheses, DBR embraces the complexity of educational settings and views the context as an integral part of the phenomenon being studied (Barab Sasha, 2004).

Key characteristics of DBR according to Anderson & Shattuck (2012) are: being situated in authentic, real classrooms rather than laboratories; emphasizing collaboration between researchers, teachers, and students; involving multiple iterations; using existing theories to inform design while aiming to generate new design principles; and seeking to improve educational practice directly while also advancing knowledge. Expanding on these characteristics, the Design-Based Research Collective (2003) emphasizes that DBR is fundamentally interventionist and iterative. It involves continuous cycles of design, enactment, analysis, and redesign. Through these iterative cycles, the intervention is progressively refined, and the evolving designs provide real-time feedback that informs both the next iteration of the intervention and the broader theoretical understanding (Mckenney & Reeves, 2012).

Furthermore, a hallmark of DBR is its dual yield. As noted by Cobb et al (2003), the methodology is uniquely positioned to produce two distinct but interrelated outcomes: a functional, locally effective educational intervention, and empirically grounded "humble theories" or design principles. These principles offer domain-specific guidance that extends beyond the immediate study, allowing other educators and researchers to adapt and apply the findings to similar, yet distinct, educational contexts. Consequently, DBR was deemed the most appropriate design for this study, as it allows for the systematic development and refinement of a contextually relevant professional development intervention for supporting productive struggle while simultaneously generating theoretical insights relevant to the process of teacher change and the facilitation of productive struggle in resource-constrained mathematics classrooms.

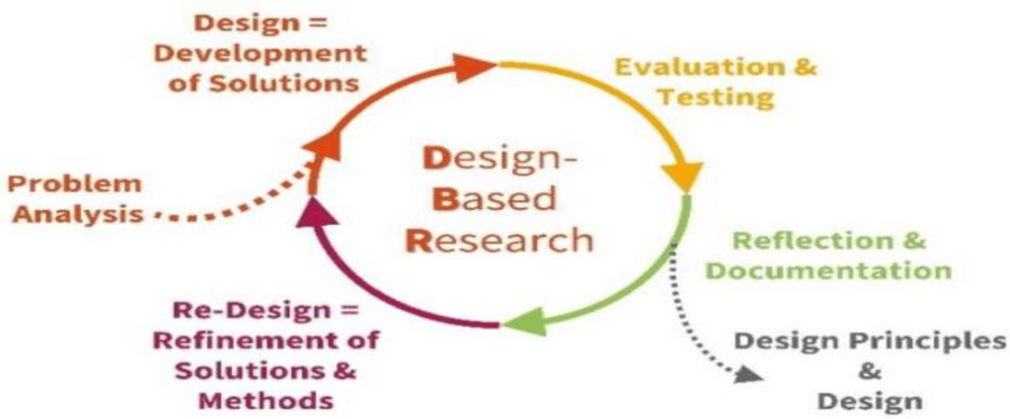

Figure 3: DBR cycle based on Herrington et al. (2007)

3.5. Study Area and Context

The study was conducted in Bahir Dar City, the capital of the Amhara National Regional State, Ethiopia. Geographically and institutionally, the research was delimited to urban government primary schools within the Bahir Dar City administration where Teach for Ethiopia (TFE) social enterprise employs numeracy fellows.

Understanding the study area is critical to interpreting the findings. The Ethiopian educational context is characterized by systemic challenges that heavily influence instructional practices. The 1994 Education and Training Policy mandated active, student centered learning, yet traditional, teacher centered instruction and rote memorization still dominate classrooms (Kati Keski-Mäenpää, 2018).

TFE recruits recent university graduates, provides them with structured pre-service training (General pedagogy;- active learning, lesson planning, classroom management and organization, assessment for/of learning, teachers ethics, special need education, structured pedagogy, action research and numeracy boost as well as leadership, teaching as a collective leadership, print rich classroom, joy full learning, and teacher lead assessment) and places them in Bahir Dar, Debre markos, and Centeral Ethiopia as literacy (language teacher) and numeracy fellow (mathematics teacher) in public schools. At the time of this study, TFE currently employs 13 numeracy fellows (including the researcher) teaching in selected public schools in Bahir Dar. The researcher is also employed by TFE as a numeracy fellow in Bahir Dar district, which enables contextual familiarity and professional access while maintaining ethical research boundaries. The selected schools are characterized by large class sizes (often 60-70+ students), limited instructional resources (insufficient textbooks, manipulative, or teaching aids), and a heavy reliance on direct instruction to cover a detailed curriculum quickly.

3.6. Population, Sample, and Sampling Technique

3.1.1. Population

The target populations for this study are early-career elementary mathematics teachers employed by TFE who were teaching in Bahir Dar. These teachers typically have between five months and three years of teaching experience, operate within a common pre-service training provided by TFE, offering baseline for examining how a targeted DBR intervention focused on PS influences instructional practices and beliefs. Early-career teachers were selected as the population because their instructional habits are still forming; they are generally less entrenched in traditional methods than government teachers and are more receptive to reform based pedagogies, making them ideal participants for this study.

3.1.2. Sample and Sampling Technique

The study utilized purposive sampling to select participants (Creswell, 2013). Two Grade 3 mathematics teachers from two different public primary schools were purposively selected. Teacher A is male with two years of teaching experience and has degree in applied physics, while Teacher B is female with five months of experience has degree in computer science. They were selected because they were accessible and willing to participate in the study.

Additionally, a total of 30 Grade 3 students (15 from each teacher's class) were purposefully selected as participants for artifact collection and observation. The selection of these 30 students was conducted collaboratively by the researcher and the teachers using stratified purposive sampling based on the students' first semester academic scores. From each class, the selection was stratified to include: 5 high achieving students, 5 middle achieving students, and 5 low achieving students.

This stratification was essential for the study. PS manifests differently across achievement levels, and teachers must learn to support it universally for all levels. A high achieving student might quickly encounter struggle during doing mathematics tasks, requiring the teacher to use extending prompts, whereas a low achieving student might struggle primarily with getting started, requiring enabling prompts (Warshauer, 2014; Livy et al., 2018).

3.7. Design of the Professional Development Intervention

The intervention was structured into three iterative cycles, guided by Guskey (2002) model of teacher change and the principles of DBR. Each cycle built upon the reflections of the previous, embodying the cyclical nature of design, implementation, analysis, and redesign.

Baseline

The baseline established the empirical starting point prior to any PD intervention. During the baseline, the two teachers implemented textbook tasks focused on division with remainders using their usual instructional approaches. Data were collected to document existing classroom practices, specifically the use of the four PS strategies, as well as baseline levels of student engagement and perseverance, and teacher initial beliefs and attitudes about struggle. Based on the baseline data, thereby, confirming the necessity of the intervention.

Cycle 1: Initial PD and Implementation

Cycle 1 focused on building the teachers' foundational understanding of PS and introducing the four support strategies proposed by Warshauer (2015). The cycle began with introducing the concept of PS, the four types of student struggle; the four strategies of supporting PS, HCD tasks were discussed. The researcher give sample task for the teachers modeled the four strategies using, allowing the teachers to experience struggle as learner's first-hand. Then again the researcher prepared a model conversation between student and teacher in the task they experience and give to them as handout.

Subsequently, the researcher and teachers collaboratively planned a lesson. A fraction task involving the sharing of bread was selected and collaboratively designed. The teachers then implemented this task in their own classrooms. Post lesson data were collected and analyzed to assess the initial changes. These findings directly informed the redesign of Cycle 2.

Cycle 2: Refined PD and Implementation

Cycle 2 was designed based on the analysis and reflections of Cycle 1. It incorporated a structured video reflection of cycle 1 lesson. Each teacher reviewed the video recording and transcript of their Cycle 1 lesson with the researcher. Using Warshauer's four strategies, they identified missed opportunities for wait time, encouragement, instances of premature telling, and moments where acknowledgement could have normalized student confusion.

Following this reflection, the teachers collaboratively redesigned a new fraction focused task. The teachers implemented the refined lesson with a conscious focus on the four strategies of supporting PS. The data from Cycle 2 collected and analyzed.

3.8. Sources of Data and Data Collection Instruments

To ensure comprehensive understanding and triangulation, multiple qualitative data sources were collected across the three cycles.

Classroom Observations and Audio-Video Recordings

All mathematics lessons during the PS tasks were video recorded with the participants' consent. Video recording was essential as it captured the moment to moment dynamics of the classroom. Audio recorders were also used to record the student and teacher conversation. The audio and video captured the exact duration of wait time, the phrasing of probing questions, the tone of encouragement, acknowledgement and the immediate behavioral responses of students. These recordings were later transcribed verbatim, to serve as the primary data source for thematic analysis.

Student Artifacts

The written work of the 30 purposively selected students was collected immediately after each lesson. Artifacts included drawings, fraction notations, number lines, erased attempts, and written verbal explanations. In qualitative research, artifacts are not only pieces of paper; they are physical manifestations of cognitive engagement and perseverance. Indicators such as multiple erased answers, trial and error partitioning of shapes, the use of multiple representations, and written verbal reasoning demonstrated how students were wrestling with the mathematical concepts. These artifacts were triangulated with the video data.

Semi-Structured Teacher Interviews

Semi-structured interviews were conducted with the two teachers at the end of the baseline, after Cycle 1, and after Cycle 2. The interview questions were designed to map onto Guskey's model of teacher change. Baseline questions explored espoused beliefs about struggle and usual teaching practices. Post Cycle 1 and 2 questions focused on observing student outcomes and belief shifts. The semi-structured interview allowed the researcher to follow the teachers' natural thought processes while ensuring alignment with the research questions.

Collaborative Reflection Meetings

Structured reflection meetings were held between the researcher and the teachers between cycles. These meetings were a core mechanism of the DBR process. During these sessions, the group discussed findings from the current cycle, celebrated instructional successes, identified persistent challenges, and collaboratively redesigned the tasks and strategies for the next iteration. Researcher field notes were maintained during these meetings.

3.9. Data Analysis Procedures

Data analysis followed Ahmed et al (2025) thematic analysis within the interpretivist tradition. Thematic analysis is a method for identifying, analyzing, and reporting patterns (themes) within data. It organizes and describes the data set in rich detail, and interprets various aspects of the research topic. The analysis was ongoing and iterative, data from the Baseline were analyzed to inform the design of Cycle 1 and data of cycle 1 were also analyzed immediately to inform the re-design of Cycle 2. The six-phase was employed as follows:

Phase 1: Familiarization with the data: The researcher immersed himself in the data by repeatedly watching the audio and video recordings, reading and re-reading the interview and audio and video recordings transcript, and examining the student artifacts.

Phase 2: Generating initial codes: Systematic codes were generated across the entire dataset. The coding utilized a hybrid approach. A priori codes were derived directly from the theoretical framework, including Warshauer (2015) four strategies, the four types of student struggle, and Guskey's sequence of change. Inductive codes were also generated from the data itself, such as, contextual barriers of implementing PS.

Phase 3: Searching for themes: The initial codes were joined into broader themes that aligned with the research questions. For example, codes like asking how, asking why, and probing guidance were grouped under the theme of questioning. Codes like erased drawings, multiple trying, and self-correction were grouped under the theme of student perseverance.

Phase 4: Reviewing themes: The generated themes were reviewed against the coded data extracts and the entire dataset to ensure internal coherence and external distinctiveness.

Phase 5: Defining and naming themes: Each theme was sharply defined, and its essence was captured in a concise name. The analysis produced themes directly corresponding to the research questions: the change of instructional practices, the manifestation of student engagement and perseverance, and the change of teachers' belief.

Phase 6: Producing the report: The final narrative was produced, weaving together the thematic findings with transcript excerpts and artifact descriptions to tell evidence-based story of the intervention's impact across the baseline, Cycle 1, and Cycle 2.

3.10. Trustworthiness of the Study

To ensure the trustworthiness of the findings, the study rigorously adhered to Lincoln and Guba's four criteria of measuring trustworthiness (Alexander, 2019):

Credibility: Achieved through extensive engagement in the field over multiple cycles and persistent observation. Data triangulation was employed; every claim about a teacher's practice was supported by video evidence, the teacher's own interview reflections, and student artifacts. Furthermore, member checking was utilized during the reflection sessions between cycles, where teachers verified the accuracy of their transcribed interactions and the researcher's interpretations.

Transferability: Facilitated by providing detail and rich descriptions of the context, the participants, and the classroom interactions. By detailing the constraints of the schools, the specific cultural perceptions of struggle, and the exact iterative changes in the teachers' practices, student engagement and perseverance, and teacher belief, the study provides enough contexts for readers to check the applicability of the findings to other settings.

Dependability: Maintained through a detailed audit trail. The researcher documented all methodological decisions, the evolution of the coding scheme, and the specific theoretical logic used to move from raw data to final themes. The iterative nature of the DBR process itself, with its built in cycles of testing and revision inherently enhances dependability.

Confirmability: Ensured by connecting interpretations directly in the data and utilizing established theoretical frameworks Warshauer (2015), and Guskey (2002) to guide the analysis, thereby minimizing researcher subjectivity or bias. The triangulation of data further ensured that the findings reflected the participants' realities rather than the researcher's preconceptions.

3.11. Ethical Considerations

Ethical considerations were rigorously upheld throughout the study, adhering to international guidelines (World Medical Association, 2013; FDRE, 2014). Prior to data collection, official permission letter was given from Bahir Dar University School of teacher education and the school principals.

Written informed consent was obtained from the two participating teachers, ensuring they understood the study's purpose, procedures, and their right to withdraw without penalty. Because the study involved children (Grades 3), guardian consent was obtained via consent forms distributed through the schools. Additionally, child assent was collected through age-appropriate verbal explanations before each observed lesson.

Confidentiality was strictly maintained. All participants and schools were assigned pseudonyms (Teacher A, Teacher B, School 1, and School 2) in all transcripts, reports, and the final thesis. Audio-video recordings and transcripts were stored on password protected devices accessible only to the researcher.

CHAPTER FOUR: DATA ANALYSIS AND FINDINGS

4.1. Introduction

This chapter presents the analysis and findings of the data collected during each cycle of the study, organized according to the three cycles of the DBR intervention: the baseline, cycle 1, and cycle 2. The analysis is structured around the research questions guiding this study.

The chapter is organized in four sections: the first section presents the baseline analysis, establishing the empirical initial point of the teachers' practices, student engagement and perseverance, teacher's belief prior to the intervention. The second presents the cycle 1 analysis, examining the initial impact of the PD and the emerging shifts in teacher implementation of PS strategies and student engagement and perseverance, and teacher belief. The third, presents the cycle 2 analyses, illustrating the refined teachers implementation of PS strategies and the resulting student engagement and perseverance, and teacher belief shifts. The final section provides a comprehensive summary of the findings, directly synthesizing the results in relation to the three research questions and the theoretical framework.

4.2. Baseline Analysis: Existing Classroom Practice and Challenges

4.2.1. Description of Baseline

The baseline was conducted to identify the existing teaching practices of the two participating teachers, the students' engagement and perseverance, and initial belief of the two teachers, prior to the PD intervention. During baseline, the teachers utilized normal textbook tasks and usual instructional methods. No training on PS had yet been provided. The purpose of baseline was to establish an empirical initial point and to identify the specific classroom practices that required improvement. Both teachers utilized varied mathematical task involving packing books into cartons, requiring students to compute quotients and interpret remainders.

4.2.2. Baseline Findings

4.2.2.1. Implementation of PS Strategies

The findings from the baseline data indicate that the four strategies for supporting PS; questioning, wait time, encouragement, and acknowledgement, were minimally and inconsistently implemented in both teacher's classrooms. Instruction was highly teacher-centered, focusing on procedural guidance rather than conceptual sense-making.

Questioning

Classroom questioning was largely procedural and funneling, aimed at getting correct answers rather than promoting reasoning. Teachers frequently asked closed, rapid fire questions that guided students toward a single and teacher directed method. This aligns with Warshauer (2014) identified teacher responses specifically direct guidance and telling, which lowers the cognitive demand of the tasks.

Teacher A's interaction with the student showed a heavy reliance on procedural questioning that removed the challenge of the task. After introducing the task, Teacher A immediately began funneling their thinking toward a single method, repeated subtraction:

Teacher A: there are 342 books here, then I packed 4, how many left?

Student: 338

Teacher A: ok 338, again I take 4 books and packed, how many carton packed?

Student: silent

Teacher A: 2, how many books left?

Student: 334

Rather than asking students how they might approach the problem or allowing them to devise another and their own strategy, Teacher A directed them through the steps sequentially. When students struggled to grasp the repeated subtraction, he quickly directed them to division, eliminating any struggle of the students: "Divide it, 342 divided by 4…. you should use the method that gives you the answer in short period of time." This explicitly prioritized procedural speed and correct answer rather than conceptual understanding, limited students the opportunity to grapple with the mathematical structure of the problem (Hiebert & Grouws, 2007).

Similarly, Teacher B utilized highly directive questioning that led students step-by-step through the long division procedure. When a student struggled with long division of 134 to 4, Teacher B told the step-by-step procedure:

Student: How do you divide 13 by 4?

Teacher B: How many times does it get?

Student: silent

Teacher B: It can't be equal, but give it 3 times.

Student: 3 times 4 is 12.

Teacher B: 12, so how much is 13-12?

Student: 1

Teacher B: There are 4 here. So you say it is 14. What is 14 divided by 4?

Student: 4 times

Teacher B: 4 times will be 16.

Student: Can't.

Teacher B: It's not that it can't, but it can. How many times will it get and how many left over?

Student: 4

Teacher B: it gets 3 times what is left over?

Student: 2

Teacher B: 2 will get left over, good boy. When you first divided, you gave 3, 1 remained, 14, then you gave 3, what is left over?

Student: 2

Teacher B: 2 ok, so how many boxes are needed?

Student: 2

Teacher B: 2 alone can't pack all this. 33 boxes are needed. How many books are left?

Student: (silent)

Teacher B: It is 2. So, 33 cartons are needed to pack 134 books. 2 books will be left over

In this dialogue, Teacher B did not ask the students to reason, justify, or do it by their own. Instead, told the steps ("give it 3 times," "So you say it is 14") and answer immediately. This kind of questioning reduces the task to a set of memorized procedures, categorizing the teacher's response as telling (Warshauer, 2014).

Furthermore, when a student made a conceptual error by stating "…14 is not divided by 4." Teacher B immediately corrected it rather than questioning the student to remove the misconception by their own:

Teacher B: ok tell me

*Studen*t: 134 divided by 4; 13 divided by 4 is 3; 13-12 is 1; 14 is not divided by 4.

Teacher B: It is not possible.

Student: Yes

Teacher B: No, 3 times but there is left over….it is possible. It may not divide equally, don't say it is not possible.

Wait Time

In the baseline observations, both Teacher A and Teacher B demonstrated severely limited wait time. Consistent with the literature on unproductive struggle, students were not given sufficient opportunity to think, reason, respond before the teacher explaining or providing answers (Warshauer, 2015).

Teacher A explicitly noted his role as a rescuer, stating, "I am here to help you if you have question, ask me." When students struggled to start the task, he did not give time for cognitive processing, reasoning. Instead, he immediately re-explained the question, drew a carton on the board, and began demonstrating it. Specially, Teacher A physically took over the students' work. When he got student that didn't begin or was stuck he took the student's pen and wrote on their paper and showed how they should do by showing the steps. He repeatedly does it for more than five students.

When a student was silent after Teacher A's questioning, he immediately told the answer.

Teacher A: we say no remainder at the end if there is…? What?

Student: (silent)

Teacher A: 0, yes. After we divide, subtracts and at the end if we get 0, there is no remainder. If not, there is remainder.

This extreme lack of giving thinking time completely removes the intellectual effort required for learning (Hiebert & Grouws, 2007).

Similarly, Teacher B filled students silence immediately. For example, when a student remained silent after being asked how many times 13 divides by 4, Teacher B instantly supplied the answer ("It can't be equal, but give it 3 times"). This absence of adequate wait time prevented students from engaging in the neural strengthening associated with solving with mathematical concepts (Hiebert & Grouws, 2007).

Encouragement

In the baseline, encouragement was infrequent and generic. Both teacher's never encouraged student to explain their work rather they asked procedural question. In Teacher A's class, encouragement was largely absent, as the focus remained on procedural explanation. Teacher A also did not encourage the students' effort and persistence. Teacher B occasionally used phrases such as "good boy" and "cheer up, if there is confusion you can ask?" but these were disconnected from encouraging persistence or cognitive effort.

Teacher B: for example let's take 4 chalk: one … two … three … four. Now this is packed. One packed carton. So to pack the entire textbook how many cartons (like this) you required?

Student: we pack 4, 4, 4, 4… Then,

Teacher B: yes, good.

Teacher B: cheer up, if there is confusion you can ask?

This does not encourage students to persist through the confusion by themselves, it encouraged students to ask question when they were confused.

Acknowledgement

Acknowledgement was entirely absent in both classrooms. Both teachers did not validate students' confusion or normalize mistakes as normal and stepping stones to learning.

When students expressed confusion, the teachers treated it as a deficit to be immediately remedied. For example, when students in both Teacher A's and Teacher B's class were silent when confused, they immediately filled it, showing being silent is not good, it also mean student should not be silent when asked and should be fast responder.

The implicit classroom norm communicated that struggle was a failure to be fixed by the teacher rather than a productive step in the learning process. Interview data indicate that Teacher A possessed a superficial awareness of positive reinforcement "When i give acknowledgement after trying a little or giving up, they try to do more", but practices of acknowledging the value of struggle were completely absent in classroom observations.

4.2.2.2. Student Engagement and Perseverance

Student Engagement

Student engagement during the baseline was low, as evidenced by both teacher's classroom observation and student artifacts. A significant number of students did not attempt the task, displaying passive participation and heavy reliance on teacher explanations. In Teacher A's classroom, 5 students out of 15 students did not attempt anything in the given paper, while in Teacher B's classroom, 6 students out of 15 students wrote nothing on their papers until the teacher solved it.

Some students demonstrated only superficial/surface engagement. Artifacts from Teacher A's class showed students writing sequences of 4, 4, 4, 4, 4, 4, 4, 4 ... Or listing the multiples of 4 indicating engagement at a procedural level without deeper understanding. Similarly, one student drew a number line but could not attempt anything. In Teacher B's class, artifacts revealed similar superficial attempts. Classroom interactions were dominated by teacher talk, with repeated instances of student silence, indicating that many students were not cognitively involved in the learning process.

Student Perseverance

The baseline data shows that students demonstrated limited perseverance. A few students showed some persistence; for example, 4 students in Teacher A's class started division of 342 to 4 and used division many times to get the quotient and remainder. Similarly, few artifacts from Teacher B's class contained erased answers, and revision. However, majority of students were unable to sustain their attempts and work. Many started adding 4s but could not continue, started division but stopped (one student in Teacher B's class starting division five times but failing to reach the quotient and remainder), many students attempted to draw rectangles (as carton) but stopped. This early abandonment of the task indicates unproductive struggle. Consistent with NCTM (2014), these findings suggest that without the necessary instructional support, specifically questioning, wait time, encouragement, and acknowledgement, students lacked the resources to persist through difficulty.

4.2.2.3. Teachers' Beliefs and Practices

The baseline interview data indicate a clear mismatch between teachers' stated beliefs about PS and their actual classroom practices. Both teachers expressed positive theoretical beliefs about the importance of allowing students to struggle. Teacher A stated that "a challenging question is very important and useful for children," and Teacher B emphasized that "bringing the answer by themselves will make the lesson more understandable." Teacher B even claimed, "I don't give answers when students are struggling... I give them a hint so that they can struggle with it by their own."

However, classroom observations revealed totally different reality. Teacher A described his approach as "bottom up... I first explain to them simple questions... I go from easy to difficult; I give difficult questions to the fast students... I show for them." Similarly, Teacher B stated, "I show them how to solve a difficult question in a small way so that they don't have to go straightly to the difficult question." This was showed in classroom interactions, where both teachers frequently intervened to explain procedures or directly demonstrate the procedures. Such practices limit opportunities for students to engage in sustained cognitive effort (Hiebert & Grouws, 2007) and indicate that teachers viewed struggle as something to be removed rather than facilitated (Stigler & Hiebert, 2004).

Both teachers attributed this discrepancy to systemic challenges, such as overcrowded classrooms (60-70 students) and broad curricular content. Teacher B noted, “If the number of student is small, it is possible to reach everyone... but it will be a barrier to manage 70 students while teaching them.” These contextual challenges reinforced directive teaching methods, allowing to familiar, comfortable approaches rather than the uncomfortable practice of struggle.

4.2.3. Reflection from Baseline

The baseline findings showed that both classrooms were dominated by teacher centered instruction, procedural questioning, immediate teacher intervention, and limited opportunities for independent reasoning, low encouragement and absence of acknowledgement. Students exhibited low engagement and weak perseverance. Although the teachers expressed positive beliefs about struggle in interviews, these beliefs were not reflected in practice. This baseline finding clearly displays unproductive struggle. The high reliance on procedural questioning, rapid intervention, and minimal use of supportive strategies prevented students from engaging in meaningful mathematical cognitive effort. The findings highlight a critical gap between teachers’ espoused beliefs and their instructional practices, underscoring the urgent need for a targeted PD intervention to help teachers to support PS.

4.3. Cycle 1 Analysis

4.3.1. Design of Cycle 1

Cycle 1 was designed to develop teacher’s basic understanding of PS and introduce practical strategies for supporting students during challenging tasks. Based on the baseline findings, which revealed teacher centered instruction, procedural questioning, and immediate intervention, the first cycle focused on helping teachers understand PS and apply the four support strategies (Warshauer, 2015).

The PD included conceptual training on the meaning of PS, common types of student struggle, HCD tasks, and the four strategies of supporting PS. The second component involved experiential PD, where the researcher modeled the four strategies by giving sample tasks for them as a student. Finally, the researcher and teachers collaboratively prepared a task, and plan.

4.3.2. Cycle 1 Findings

Cycle 1 data from classroom audio and video transcriptions, student artifacts, and teacher interviews show a clear but still developing change toward PS in both teachers' classroom. The PD intervention influenced the teachers' use of questioning, wait time, encouragement, and acknowledgement, which in turn impacted student engagement, perseverance, and teacher beliefs shift.

4.3.2.1. Implementation of PS Strategies

Questioning

Questioning is highly visible strategy in Cycle 1. Rather than giving direct answers, both teachers frequently used questions to guide thinking and probe reasoning, attempting to shift from telling to probing guidance (Warshauer, 2014).

Teacher B demonstrated highly effective probing questioning to help students overcome the common misconception of counting pieces instead of comparing area. When a student incorrectly claimed Beza ate more because she ate two pieces, Teacher B used a sequence of questions to redirect the student's attention to the size of the shaded parts:

Teacher B: who ate more, Ali or Beza?

Student: Beza.

Teacher B: Why did you say Beza?

Student: Beza ate two pieces.

Teacher B: Look at the shaded parts?

Student: Ali's is one, but it is wide; Beza's are two small ones.

Teacher B: From the shaded parts, which is wider?

Student: They are equal.

Teacher B: What does that mean?

Student: When two quarters are together, it becomes equal to the other half one.

Teacher B: So, who ate more, Ali or Beza?

Student: They ate equal.

This dialogue is productive questioning that maintains the cognitive demand of the task(Smith et al., 1998).

Teacher B used the following dialogue to justify the students reasoning.

Teacher B: Did you divide it?

Student: Yes (showing it).

Teacher B: Out of the three, who ate more?

Student: Leul, because Beza has two left over, but Leul has only one left.

Teacher B: Compare what they ate.

Student: The shaded part?

Teacher B: Yes.

Student: Two pieces are shaded for both of them.

Teacher B: Which one is larger in size?

Student: I think it's Leul's.

Teacher B: Look at it carefully?

Student: Leul's is larger; therefore, what Leul ate is more.

Teacher B also used questioning to address careless errors and reading misconceptions, as seen when a student confused dividing by four with eating four:

Teacher B: Why did you say Beza?

Student: Beza ate four.

Teacher B: Did she eat four?

Student: Yes.

Teacher B: Read the question, does it say she ate four?

Student: She divided it into four.

Teacher B: Does she divide it into four mean she ate four?

Student: (He started to draw the bread and began dividing it into four).

Teacher A utilized questioning to elicit justification. When a student correctly identified that Ali and Beza ate equal amounts, Teacher A pressed for the underlying reasoning:

Student: Ali and Beza ate is equal.

Teacher A: How did you make it equal?

Student: (showing the two loaf of bread she drew) this is Ali's; it is divided into two. This is Beza's; it is divided into four. What Ali ate is a half (1/2). Since Beza ate two, it is two quarters (¼), when put together, it becomes a half. Therefore, the portions they ate are equal.

Teacher A used questioning to help students reconsider unsure conclusions. When one student claimed that all three children ate equal amounts, the teacher redirected attention to the representation:

Student: all three of them ate equal.

Teacher A: why?

Student: because….Leul cut it into three and ate two. Ali ate one, and Beza ate two.

Teacher A: and what about the amount they ate? Among Ali, Beza and Leul, who ate more?

Student: they ate equal.

Teacher A: look again, compare what they ate?

Student: ate more …uh…..Leul.

Teacher A also used questions when doing leftovers.

Teacher A: if the bread three of them left over were put together, would it be more than or less than one whole bread?

Student: it would be more.

Teacher A: how do you know?

Student: because both Ali's and Beza's are half. Half plus half gives us a whole. Even if Leul's leftover isn't counted as a half, when it's added, it goes over the whole.

However, both teachers still challenged with over supporting through directive and step-by-step questioning when students hesitated, reverting to direct guidance (Warshauer, 2014). When a student in Teacher A's class struggled to compare the fractions, Teacher A took over the challenge through funneling:

Teacher A: How much did Ali eat?

Student: A half.

Teacher A: And Beza?

Student: A half.

Teacher A: And Leul?

Student: This one.

Teacher A: Into how many did Leul divide it?

Student: Into three.

Teacher A: Show me how he divided it.

Student: (silent)

Teacher A: What do we do to find out that ate the most?

Student: Compare.

Teacher A: How do you compare them?

Although intended to be supportive, this rapid sequence led the student step-by-step, moving the reasoning work from the student to the teacher.

Similarly, Teacher B sometimes overwhelmed students with consecutive prompts.

Teacher B: Okay, who ate more?

Student: Beza and Leul.

Teacher B: who ate more, Ali or Beza?

Student: Beza.

Teacher B: Why?

Student: Ali ate one, but Beza ate two.

Teacher B: Show me which one Ali ate?

Student: (Pointing to the half) this one.

Teacher B: Identify it; which one is it?

Student: (Shading) He ate this one.

Teacher B: And what Beza ate?

Student: (She shaded the two quarters).

Teacher B: Which is more?

Student: (Pointing to what Beza ate) this.

Teacher B: How?

Student: (Silent)

Teacher B: Look at the shaded parts.

Student: I mean it's equal.

This rapid succession reduced the learner's opportunity to pause and reason independently.

Wait Time

Wait time was influenced by the intervention, representing a significant change from the baseline where silence was immediately filled by the teacher. Teacher B explicitly utilized extended wait time at the beginning of the lesson. The teacher becomes silent and revolves around for 4 minutes without support. When a student wanted direct help, Teacher B redirected responsibility back to them:

Student: Teacher, does that one has an answer? (Pointing his finger to the board)

Teacher B: It seems it doesn't have an answer? Let's see, look at it.

Student: How do I work it out?

Teacher B: Use any method you want and show me your work.

This deliberate removal of immediate support allowed students time to struggle in order to start the task.

Teacher B gives time when a student struggled to divide Leul's bread into three equal parts.

Teacher B: Show me from the drawing.

Student: (He remained silent, unable to divide Leul's bread into three equal pieces).

Teacher B: try it, I will see you when you finish.

After a while

Student: I have finished.

Teacher B: How did you divide it?

Student: (He divided the circular bread into two, and then divided one of those halves into two).

Teacher B: Are the pieces equal?

Student: One is larger.

Teacher B: The question says Leul divided the bread given to him into three equal parts.

Student: Okay, wait.

Teacher B: Keep going.

Teacher B also gives time when another student struggled to divide Leul bread into three equal parts:

Teacher B: Okay, when Leul divides it into three, all three must be equal, right?

Student: Yes.

Teacher B: And is this equal?

Student: I don’t know.

Teacher B: Which of the parts is larger?

Student: This one.

Teacher B: Try to divide it equally?

Student: But I don’t know how.

Teacher B: If I gave you one bread and told you to divide it equally for three children, how would you give it to them?

Student: Okay (drawing the bread).

After a while, the teacher goes to the girl and sees the drawing; she had divided it correctly.

Student: (Showed it to Teacher B).

Teacher A also began in giving explicit wait time. When a student struggled to do the third question regarding the leftovers, Teacher A stated, “Let me give you some time; work on it.” Later, when a student drew unequal bread, Teacher A did not fix it but said, “If you don’t draw them equally, you cannot know the answer. So, draw them exactly equal. I will come back when you finish.” This intentional giving of time provided the student the necessary space to reason independently.

Teacher A also gave time when the students compare leftover.

Teacher A: What about the leftovers? If the leftover were put together, would they be more than or less than one full bread?

Student: (become silent and thinking).

The teacher moved to other student and after a while come back

Teacher A: Have you finished?

Student: Yes, it is more.

Teacher A: How do you know?

Student: Because Ali cut the bread into two and ate the half...

Teacher A: Show me how it is more?

Student: Ali’s half and Beza’s two quarters make one whole. When Leul’s is added, it is more than one whole.

This episode shows that the given time enabled students to formulate reasoning.

The interview data reflect teachers’ awareness of wait time. Teacher B noted, “I allowed approximately 10 minutes of wait time for students to think and work independently before intervening.” Teacher A also stated, “I provided sufficient wait time for students to think and work independently.” While both teachers believed they were implementing wait time effectively, the classroom evidence shows that the challenge lies not only in providing time but in ensuring that the time leads to meaningful engagement, persistence/sustained struggle.

Encouragement

Encouragement was another strategy used by both teachers during Cycle 1 to sustain confidence, motivate persistence, and validate effort (Warshauer, 2015).

Teacher A set a strong tone of encouragement at the very beginning of the lesson, framing mathematics as a discipline of effort rather than innate ability: “Mathematical knowledge doesn’t just come to you; it comes through trying. You keep trying, and if there is anything you don’t understand, you can ask.”

He also encouraged perseverance when students were partially correct, validating their progress while maintaining the challenge: “Your method of sharing is correct, but try to work on it again to find out exactly how much they received.” Furthermore, both teachers frequently utilized general praise to affirm correct answer, reasoning and revisions. Teacher B frequently used phrases such as “Good boy!”, “Very good boy!”, and “Good girl!” For example, when a student deduced that Leul ate more because his leftover was the smallest from the three, Teacher B responded, “Very good boy!” Teacher A similarly validated and extended reasoning: “Good girl,” and “Good boy. Look at the drawings, the numbers, and the fractions to understand. Try it.”

Teacher B also encouraged multiple representations, pushing students to extend their thinking beyond their initial approach:

Teacher B: What happens when they are put together?

Student: Ali’s and Beza’s make one whole; then Leul’s third is left over. Therefore, since it is more than one whole by a third, the answer is more.

Teacher B: Good girl! Now show me by working it out in a different way?

Student: In which way?

Teacher B: You used circular bread; try it with four-sided (rectangular) bread?

When a student struggled to divide into thirds and said, “But I don’t know how,” Teacher B encouraged persistence by simply stating, “If I gave you one bread and told you to divide it equally for three children, how would you give it to them?

Teacher B: Try to divide it equally?

Student: But I don’t know how.

Teacher B: If I gave you one bread and told you to divide it equally for three children, how would you give it to them?

Student: Okay (started drawing of bread).

Interview data confirm these classroom practices. Teacher A stated, “I encouraged students during their activities.” Teacher B similarly explained, “I encouraged them with phrases such as good, very good, and well done.” These indicate that encouragement was intentionally used strategy promoted through the intervention, but it is generic.

Acknowledgement

In Cycle 1, Acknowledgement became visible. It began to emerge as a distinct strategy, primarily through the normalization of mistakes/error at the launching of the task. Teacher A acknowledges mistakes as normal during the independent work time: “Don’t worry if you don’t get it right away; work in any way you feel is best. Don’t worry about making mistakes, okay? Just do what you think is right.” This statement explicitly acknowledges struggle, normalizing errors and reframing them as a natural part of the process rather than failures to be removed. Teacher B couldn’t acknowledge struggle/mistake.

However, in the moment acknowledgement of specific student struggle/confusion was less developed. When students were stuck during the task, teachers often quickly shifted to questioning or encouragement rather than pausing to validate the specific struggle/confusion occurred as necessary and important. For example, when a student in Teacher B’s class could not divide Leul’s bread into three equal parts, Teacher B said, “The question says Leul divided the bread given to him into three equal parts,” followed by “keep going.” While supportive, the teacher did not acknowledge the difficulty of the task itself (for example, the teacher can acknowledge by telling “It is really hard to divide bread into three equal parts, isn’t it?”). This shows that while teachers conceptually know the importance of normalizing mistakes/errors globally, their ability to acknowledge struggle in the moment was still developing.

4.3.2.2. Student engagement and perseverance

Student Engagement

Implementing PS strategies had a positive effect on student engagement. Students became more active, attempting tasks independently, and utilizing multiple representations rather than following teacher-directed algorithm.

Teacher B reported in her interview: “Almost all students became actively involved in the lesson. Students were more participative, understood their roles, and were encouraged to express their ideas and work independently.” She noted a dramatic shift from the baseline, “Only a few students wait directive procedures while the majority involved in the task by their own.” Teacher A confirmed this: “More students were actively involved in solving the task and were willing to try different approaches.”

Student artifacts provided evidence for their deep cognitive engagement. In Teacher A’s class, artifacts showed a clear change from the baseline’s passive acceptance to active representation by their own. Many students drew both circular and rectangular breads, labeled the drawings with the names of the children (Ali, Beza, Leul), and expressed the parts using fractions (1/2, 1/4, 1/3). One student’s artifact contained 18 circular drawings that are divided and shaded differently, with attempts by rectangular bread and a number line. Another student used a crisscross technique to compare fractions, moving beyond visual drawings into symbolic mathematics.

In Teacher B’s class, artifacts showed deep cognitive engagement through different representations. Students used circular drawings, rectangular breads, and even hexagonal shapes to represent the bread. One student’s paper contained 15 rectangular breads, 8 divided, and 3 shaded, with open figures and signs indicating active trying/exploration. Another paper had 12 differently divided rectangular breads and 6 divided circular drawings. The papers full of drawings, irregular shapes, indicated that students were cognitively engaged with how to represent the fractions, rather than simply telling the answer.

Student Perseverance

The Cycle 1 data indicate stronger perseverance, as many students continued working through mistakes and revised their answer and thinking instead of giving up, directly contrasting what is seen in the baseline.

The evidence of perseverance was visible in the student artifacts through the presence of erased answers and multiple attempts. In Teacher A's class, one artifact showed an erased answer for the first question "it is Ali", which was corrected to "it is equal," and an erased answer for the second question "Ali", corrected to "Leul" by themselves. Another student erased answers three times and made multiple trials to divide rectangular bread into three equal parts before finally dividing it correctly. One remarkable artifact contained 20 circular breads drawn through trial and error, 17 divided differently, and two number lines (one erased). Another student drew 9 rectangular drawings, with the first 6 trials showing incorrect division into three equal parts, but succeeding on the next three drawn breads. Teacher A confirmed this in his interview: "Evidence of their engagement and perseverance included the use of different strategies, drawings, multiple attempts, erased answers, and self-corrections."

In Teacher B's class, artifacts demonstrated sustained effort. Papers contained multiple times erased answer and the correct response written in front, erased circular drawings, and different kind erased drawings. One student's paper showed 15 circular drawings with many irregular shapes and trial drawings, indicating sustained effort to partition the shapes correctly. Another student wrote the answer three times, with erased and corrected drawings.

Perseverance was also observed in classroom interactions. When one student in Teacher B's class struggled to divide Leul's bread into three equal parts, and the teacher questioned him, the student responded, "Okay, wait," and continued trying. In Teacher A's class, when a student understood his drawing was unequal, he noted, "I made a mistake when I was drawing the lines," and independently chose to redraw it. When another student in Teacher A's class was confused about comparing fractions, she continued to reason aloud: "Is a third plus a third a quarter? ... Does it become a half? ... How much is a third plus a third?" These examples show that students were developing the persistence necessary to work through the carrying out a process and expressing misconception types of struggle (Warshauer, 2014). However, as Teacher A noted, "a few students still wanted to give up when they faced challenges and expected answers from the teacher," indicating perseverance of the student was still developing.

4.3.2.3. Teachers' Beliefs and Attitudes toward PS

Cycle 1 data provide early empirical support for Guskey (2002) model of teacher change, which posits that significant change in teachers' attitudes and beliefs occurs primarily after they observe positive changes in student learning outcomes.

Teacher A explicitly described this sequence. He noted that his belief changed "after observing my students' performance before and after applying this approach." He stated: "Previously, I believed that students should receive correct answers directly from the teacher. However, after implementing these strategies, I now believe that students should generate responses themselves, either independently or with peer support." This change towards a constructivist belief was directly linked to his observation of student engagement and perseverance: "The increased participation and perseverance of students influenced my decision to allow them more time to struggle... Seeing students actively engaged and attempting different strategies helped me understand that giving them time to think... supports their learning."

Teacher B demonstrated a similar shift regarding students' mathematical capability: "Before this lesson, I believed that it was easier to directly explain solutions to students and guide them quickly. However, I realized that students are capable of exploring and solving problems by themselves if given time." She further acknowledged the nature of PS: "I understood that students can investigate and think deeply when they are allowed to struggle productively."

Both teachers developed more positive attitudes toward patience and student autonomy. However, both also noted that contextual barriers, specifically large class sizes and time constraints, made it difficult to support all students equally, highlighting the real world challenges of implementing PS in the Ethiopian context.

4.3.3. Reflection and Implication for Cycle 2

Cycle 1 demonstrated meaningful progress compared to the baseline. Teachers intentionally implemented the four PS strategies, and instruction became significantly less teacher centered. Students showed remarkable improvements in engagement and perseverance, as evidenced by artifacts full of trial, error, and revision. Crucially, observing these student outcomes initiated a shift in teacher beliefs, validating the Guskey sequence.

However, challenges remained. Questioning was sometimes overly directive and funneling, wait time was cut short sometimes when students hesitated, and not purposeful, the teachers encouragement was focused on correct answer sometimes and acknowledgement was largely limited to general class statements rather than specific validations of struggle. These findings informed the iterative design of Cycle 2, which focused on refining the PD intervention.

4.4. Cycle 2 Analysis

4.4.1. Re-Design of Cycle 2

Cycle 2 was designed to strengthen teachers' implementation of PS strategies through structured video reflection and collaborative redesign. Based on Cycle 1 video data and the transcription we discussed on moments where questioning is directive, teacher should acknowledge, encouraging is essential, and time should be given. Then, the researcher and teachers jointly revised the lesson design. Greater emphasis was placed on designing an open ended task with HCD, providing extended wait time, utilizing probing questions that prompt justification rather than funneling, encouraging student explanation, persistence, and effort and explicitly acknowledging the normalcy of struggle.

While both teachers participated in the collaborative planning and implemented the Cycle 2 task, data collection for this cycle was delimited to Teacher A. Due to logistical constraints, including limited participant availability and the intensive time required for verbatim transcription of classroom audio and video, interview within the research timeframe, audio-video and interview data could not be collected and analyzed for Teacher B.

4.4.2. Cycle 2 Findings

Cycle 2 provides strong evidence that the PD intervention continued to strengthen Teacher A's classroom practice. Compared with Cycle 1, Teacher A's support was more deliberate, patient, and focused on eliciting students' own reasoning. The analysis below focuses deeply on Teacher A's implementation, student engagement and perseverance, and belief shifts.

4.4.2.1. Implementation of PS Strategies

Cycle 2 shows that Teacher A implemented the four strategies of questioning, wait time, encouragement, and acknowledgement with significantly greater intentionality and consistency than in previous cycles.

Questioning

Questioning was the most dominant and refined strategy in Cycle 2. Teacher A moved away from the step-by-step funneling observed in Cycle 1 and instead utilized probing guidance and affordance (Warshauer, 2014), asking questions that forced students to re-examine their own logic, justify their strategies, and extend their thinking.

When students presented mathematical misconceptions, Teacher A used probing questions to create cognitive dissonance rather than simply correcting the error. For example, when a student incorrectly added fractions, the interaction was as follows:

Student: We divide the two injera into three parts, and they each get two thirds. A third plus a third equals a half.

Teacher A: How can a third plus a third be a half?

Student 1: (Silent)

Teacher A: Look again and see whether a third and a third actually make a half.

Rather than telling the student the correct sum, Teacher A's question placed the responsibility of verification back on the student, maintaining the cognitive demand of the task. Similarly, when another Student claimed that "a third and a third is a quarter," Teacher A simply asked, "How did a third and a third become a quarter?" allowing the student to self-correct.

Teacher A also used questioning to push students toward mathematical precision and generalization. When a student could only provide a vague estimate ("More than 1 whole"), Teacher A pressed for exactness: "It could be more than 1 whole, but how much exactly?"

Teacher A: Okay. In total, how much injera does one child get?

Student: More than 1 whole.

Teacher A: It could be more than 1 whole, but how much exactly?

Student: (Silent)

Teacher A: The way you explained it now is correct. How much does one child get? Think about it and work it out.

Student: (Starts thinking)

Teacher A also questioned the student to explain the way he share the 5 injera to 3 child's as follows:

Teacher A: Okay, how did you do it?

Student: By dividing each injera into 3, they take one piece from each of the 5 *injera*.

Teacher A: So, how much did each get?

Student: A third, a third, a third, a third, and a third.

Teacher A: So, when you share the 5 injera among the children, how much does each get?

Student: A third.

Teacher A: (Pointing to one of the drawings) is it only this part that one child gets?

Student: No.

Teacher A: Then how is it?

Student: They get a third from each of the 5 injera.

Furthermore, Teacher A extended the task for students who did the answer quickly, using questioning to promote deeper conceptual transfer:

Student: When we share 5 injera for 3, each gets 5/3 in fractions.

Teacher A: Okay. What if one more friend is added?

Student: Since there are 5 injera and 4 of them, they get one each from 4 injera... Then one remains. We divide the remaining into 4... Each gets 1 full and a quarter... 5/4.

Teacher A: Correct, good job.

By asking, “What if one more friend is added?” Teacher A challenged the student to generalize the relationship between the numerator and denominator, transforming a simple sharing task into a deep exploration.

The interview data support this pattern. Teacher A stated: “I used questioning by asking students how they shared the five injera, how they got their answers, and how they knew their method was correct. Instead of telling them the answer directly, I asked them to explain their thinking.”

This suggests that questioning had become a conscious instructional strategy.

Wait Time

Wait time was purposeful and extended in Cycle 2. Teacher A frequently gave time after posing a challenge, explicitly signaling to students that they had the time to reason.

The most powerful example of productive wait time occurred during interactions are as follows. When a student struggled to add five individual thirds into a total amount, Teacher A validated the method but explicitly delayed intervention:

Student: They get a third from each of the 5 injera.

Teacher A: That is a good way of sharing. Now, try to work out the total amount one child gets.

(The teacher returns after a while)

Teacher A: How did you do it?

Student: One child gets 1 injera and two thirds.

Teacher A: How do you know?

Student: I added them up.

Teacher A: Okay, how?

Student: Three thirds make 1 whole injera, and the other two are two thirds. So, it becomes 1 injera and two thirds.

Similarly, when another student claimed a third and a third made a quarter, Teacher A did not correct him but stated, “Okay, look at it carefully and I will see you.” After an extended wait time, the student independently repaired the misconception:

Teacher A: How did a third and a third become a quarter?

Student: (Silent)

Teacher A: Okay, look at it carefully and I will see you.

(After a while)

Student: I have fixed it now.

Teacher A: Okay.

Student: They get one each. We divide this injera into three and give one full injera and a third to each child. When we divide this one into three and give it to them, then one child gets one injera and two thirds. This one third and this one third make two thirds.

Teacher A: Good job, that’s a good approach. Now, express what each received as a fraction.

Student: Okay.

(After a while)

Student: Teacher, I have finished.

Teacher A: Okay.

Student: What one child received is one injera and two thirds, so it is 1/3.

Teacher A: What mean 1/3?

Student: When one whole thing is divided into 3, it is called 1/3.

Teacher A: Didn't you say they received 1 whole and two thirds? How is this expressed as a fraction?

Student: (Silent)

Teacher A: Try again.

(After a while)

Student: The injera to be shared is 5, and the children sharing are 3. Therefore, each gets 5/3.

These conversations allow students to work through the carrying out a process and uncertainty in sense making struggles (Warshauer, 2014). So, they experience the breakthrough of self-correction. Teacher A also explicitly give time to manage frustration, saying, "I will give you time; keep trying," validating that the cognitive effort required time.
Teacher A later confirmed this in interview data: "I used wait time by giving students enough time to draw, calculate, and revise their work before I intervened."

Encouragement

Encouragement was utilized strategically to validate progress, persistence, build confidence, and push students toward multiple representations. Teacher A consistently reduced fear of mistakes and motivated persistence.

When students were partially correct, Teacher A encouraged them to continue. For example, when a student struggled to add the parts that one child should get, Teacher A encouraged and validated the student effort while maintaining the challenge:

Teacher A: Then how do you give it to the three children?

Student: One injera... uh... (Silent).
Teacher A: You are in a correct beginning, but in total, how much does one child get?

Student: They get 1/6.

Teacher A: Your method of sharing is correct, but try to work on it again to find out exactly how much they received.

When a student struggled to share the 5 injera to 3 children and did mistake the teacher encourages the student effort by saying “keep trying”:

Student: There are 5 loaf of bread. When we give one each to the three of them, 2 remain. Then we divide them into 4, and they get a quarter each.

Teacher A: Why did you divide it into 4?

Student: To share it for the three children, so each gets 1 and a quarter.

Teacher A: Show me what one child gets.

Student: Here, 1 and a quarter; for the second one, 1 and a quarter.

Teacher A: How can you give to the third one?

Student: Oh... it became more than 1 and a quarter... (Starts trying again).

Teacher A: Okay, keep trying.

Teacher A also encouraged students not to stop at the first correct answer, but to seek alternative strategies: for example,

Teacher A: How do you know?

Student: I added them up.

Teacher A: Okay, how?

Student: Three thirds make 1 whole injera, and the other two are two thirds. So, it becomes 1 injera and two thirds.

Teacher A: Yes, you can do that. That is a good method. Now, try another way of sharing.

Student: Okay.

Teacher A: How do they get one whole?

Student: Since there are 5, when divided by 3, they get one whole and 2 are left over.

Teacher A: Okay, go on, smart girl.

Student: We then divide the 2 into 3 and give them a third each meaning from one injera.

Teacher A: Okay.

Student: Then there is another whole injera left. We divide that into 3 as well, and when we give them the thirds, they each get 1 whole and two thirds.

Teacher A: Excellent! That is a very good method. Now, show me by working it out in another sharing method.

This shows encouragement beyond praise. Was used to sustain effort and promote multiple strategies. This kind of encouragement aligns with Dweck (2006) growth mindset, reinforcing that mathematical proficiency comes from sustained effort and exploration.
Interview data confirm this practice. Teacher A stated: "I also encouraged them by telling them not to fear mistakes and to use any method they preferred."

Acknowledgement

In Cycle 2, Teacher A's use of acknowledgement became fully aligned with Warshauer's (2015) definition, explicitly communicating that struggle, confusion, and errors are a natural and necessary part of learning mathematics.

At the outset of the lesson, Teacher A established a norm of acknowledgement: "Do not worry about making mistakes, just put down whatever is in your head." This acknowledged that mistakes are an expected and normal part of the process, not a failure. In the moment acknowledgement of specific struggle was also observed. When a student explicitly asked, “Teacher, I don’t understand this part. How much is a third and a third?” Teacher A did not rush to provide the algorithm. Instead, he acknowledged the validity of the confusion and directed the student to use their existing representations: “It is difficult for many students. Try it however you think, and I will come back.”

Student: Teacher, I don’t understand this part.

Teacher A: Okay.

Student: How much is a third and a third?

Teacher A: How is a third expressed as a fraction?

Student: 1/3.

Teacher A: How are two thirds expressed as a fraction?

Student: Okay, two thirds... uh...

Teacher A: It is difficult for many students. Try it however you think, and I will come back.

Furthermore, when another student self-corrected the misconception about thirds and quarters, Teacher A responded, “Good job, that’s a good approach.”

Student: I have fixed it now.

Teacher A: Okay.

Student: They get one each. We divide this injera into 3 and give 1 injera and a third to one child. When we divide this one into 3 and give it to them, then one child gets one injera and two thirds. This one third and this one third make two thirds.

Teacher A: Good job, that’s a good approach.

The above conversation was not only praise for a right answer; it was an acknowledgement of the revision process, validating the recovering of the student from an error, which is, normalizing error. In his interview, Teacher A confirmed this change by stating, “I no longer see mistakes as failure; I see them as part of learning,” showing a deep understanding of acknowledgement.

4.4.2.2. Student engagement and perseverance

Student Engagement

Implementing PS strategies had a positive effect on student engagement in Cycle 2. Students moved to active, different mathematical representation. Teacher A confirmed this in his interview: "More students were active during the lesson and wanted to share their ideas... I noticed greater confidence because students were less afraid of mistakes."

Student artifacts provided evidence of deep cognitive engagement. Students took ownership of the task, utilizing multiple representations to make sense of the sharing process of the injera. Artifacts showed that students used circular drawings, rectangular drawings, child-like emojis to represent the three children, curved lines connecting the share of injera to the corresponding children, verbal reasoning (one full injera and two-thirds), improper fractions (5/3, 5/4, 5/2), and mixed numbers.

One student drew five circular injera, gave one to each of three emojis, and then used curved lines to connect the remaining two thirds to the children. Another student explicitly wrote out the additive process: "third + third + third + third + third," and then synthesized it as "one full injera and 2 thirds." Another artifact showed a student who not only solved the three child scenario (5/3) but extended it to four children (5/4) and then to two children (5/2), concluding that "the children receive the most injera when shared between only two people." This generalization shows that students were not only following steps, they were actively engaged in mathematical sense making of the task (Hiebert & Grouws, 2007).

Student Perseverance

The Cycle 2 data show strong student perseverance. Instead of giving up when their initial attempts failed, students demonstrated a willingness to revise their thinking, redraw their models, and try alternative strategies.

Most of the evidence of perseverance comes from the student artifacts, which were full of multiple attempts and erased work. One artifact noted, the student tried seven times and had difficulty dividing the circular injera into three equal thirds, resulting in several wrongly divided drawings, yet the student continued until they successfully partitioned the circles. Another student drew five circular drawings and initially divided each into four unequal parts, then erased it. Again drew five circles, there are more erased drawings before the final correct attempt. One artifact contained many erased writings and circular drawings with corrected versions. The student made many attempts to divide the 5 circles into three child; they also tried drawing a rectangle divided into three. Eventually, the student drew five circles, divided two circles into three each, and wrote the share of each child.

Perseverance was also shown in classroom interactions. When one student incorrectly divided the remaining injera into quarters to share for three children, the teacher pointed out the inconsistency, then the student recognized it (Oh... it became more than 1 and a quarter...) and began trying again, while the teacher simply encouraged, “Okay, keep trying.” When another student confused about thirds and quarters, and struggled to represent the share in fraction, he did not stop; the teacher gave wait time three times, the student fixed the drawing, and successfully explained the correction and represent the share by the fraction. The real dialogue is attached in the above wait time section of cycle 2.

Teacher A summarized this change in his interview time: “Most students continued working when they faced difficulty... Many students kept trying until they understood.”

4.4.2.3. Teacher Beliefs and Attitudes toward PS

The interview data from Cycle 2 show substantial change in Teacher A’s beliefs, providing robust empirical support for Guskey (2002) model of teacher change. Teacher A explicitly detailed this sequence. By implementing PS strategies (practice), he observed the change in student engagement and perseverance (student outcomes). Seeing this evidence leads him to change pedagogical philosophy (beliefs): “My belief has changed positively. Before, I thought students needed more direct explanation before they could solve challenging tasks. Now I believe students learn mathematics better when they are given time to think, struggle, and discover ideas with guidance.”

He explicitly connected his shifted beliefs to the observable perseverance of his students: “i also trust students more because i saw that they can solve difficult problems when given the opportunity.” Furthermore, his attitude also transformed: “my attitude has also changed because i am now more patient when students are confused. I no longer see mistakes as failure; I see them as part of learning." This realizes that mistakes are stepping stones rather than failure, represents the culmination of the Guskey sequence, moving Teacher A from a transmission model of teaching to a constructivist facilitation of PS.

4.4.3. Reflection from Cycle 2

The findings from Cycle 2 show substantial improvement in the implementation of PS strategies, student engagement and perseverance, and teachers' beliefs and attitudes compared with both the baseline and Cycle 1. Teacher A demonstrated intentional, patient, and refined use of questioning, wait time, encouragement, and acknowledgement. Students actively participated, used multiple representations, explained their reasoning, and showed high initiation to persist through challenges and revise their thinking.

The change observed in Cycle 2 demonstrates a meaningful transition toward PS as conceptualized in the literature. Teacher A's strategic use of probing questions, extended wait time, encouraging effort and explicit acknowledgement of struggle helped keep the cognitive demand of the task while scaffolding students through difficulty/confusion (Warshauer, 2015). The increase in student engagement and perseverance and the teacher's changed belief and attitudes suggest that the PD intervention successfully fostered conditions for meaningful learning, validating the conceptual framework and Guskey's model of teacher change.

4.5. Summary of Findings

The findings drawn from classroom observations, audio and video recordings, student artifacts, and teacher interviews in all cycles are summarized below in relation to the three research questions:

RQ1: How does the PD intervention influence mathematics teachers' classroom practices in supporting PS?

The findings showed a substantial change in teachers' classroom practices across the three cycles. During the baseline, instruction was highly teacher centered, characterized by telling and direct guidance (Warshauer, 2014). Questioning was largely directive, aimed at reaching students to last correct answers rather than promoting reasoning. Wait time was severely limited, with teachers frequently rescuing students from struggle, most notably by taking their pens, immediately telling the way and doing the work for them. Encouragement was general not on effort and persistence, and acknowledgement of struggle was absent.

Following the PD intervention, teachers increasingly adopted practices that kept the cognitive demand of mathematical tasks. By Cycle 1, questioning changed towards probing guidance, prompting students to justify their reasoning and visualize their strategies. Wait time was extended, allowing students to struggle with initial confusion/mistake rather than being immediately rescued. Teachers encouraged multiple representations, persistence, validate progress, extend thinking, and also praise correct answer. But acknowledgement was a developing strategy.

By Cycle 2, Teacher A's practices became highly refined: questioning was used to create cognitive dissonance and promote precision and generalization. The teacher also extended the questioning and utilized probing guidance and affordance. Wait time was purposefully utilized to afford students the space to self-correct and synthesize their reasoning. Teacher encouraged and validate the effort of students beyond praising correct answer, the persistence not to stop at a correct answer rather extend by another strategy. Teachers moved to explicitly normalizing mistakes and validating the revision process as necessary (Warshauer, 2015; NCTM, 2014).

RQ2: What changes in students' engagement and perseverance are observed when teachers implement strategies to support PS in mathematics classrooms?

The implementation of PS strategies had a positive effect on both student engagement and perseverance. In the baseline, student engagement was poor and superficial; many students did not attempt the task at all, depending entirely on the teacher to demonstrate the step-by-step procedure. Perseverance was minimal, with most students giving up at the first sign of difficulty, indicating Unproductive struggle.

As teachers shifted their practices in Cycles 1 and 2, students transitioned to active mathematical agents. Engagement evolved from simply computing algorithms to utilizing multiple representations, drawing circular and rectangular models, using emojis, drawing connecting lines, fractional notation, and verbal reasoning to make sense of the problems. Student perseverance also increased dramatically. Artifacts from Cycles 1 and 2 provided evidence of sustained cognitive effort, characterized by multiple erased attempts, trial and error partitioning, and students independently revising their misconceptions (self-correcting the division of a circle into equal thirds after multiple failed attempts). Rather than giving up when stuck, students showed a willingness to keep trying, indicating persistence central to PS (Warshauer, 2014; Young et al., 2024)

RQ3: How do mathematics teachers' beliefs and attitudes about PS change after implementing the intervention and observing students' engagement and perseverance?

The findings provide robust empirical support for Guskey's (2002) model of teacher change. At baseline, a mismatch existed between teachers' espoused beliefs (valuing struggle conceptually) and their enacted practices (removing struggle through direct instruction), the teacher attributed this lack of practice to contextual barriers like overcrowded classrooms, time constraint, and also broad curricula.

The belief change did not occur only from attending PD sessions. Instead, it unfolded sequentially. As teachers implemented the new PS strategies (practice), they observed a dramatic increase in student engagement, the use of diverse representations, and, most importantly, students' ability to independently correct their own errors (persistence). Witnessing these tangible student successes catalyzed a fundamental shift in teachers' beliefs. Teacher A explicitly transitioned from a transmission model of teaching ("Previously, I believed that students should receive correct answers directly from the teacher") to a constructivist facilitation ("Now I believe students learn mathematics better when they are given time to think, struggle, and discover ideas with guidance"). Furthermore, their attitudes toward student struggle transformed; mistakes were no longer viewed as failures requiring immediate teacher rescue, but as essential stepping stones for learning. This increased trust in students' during confusion represents the final stage of the Guskey sequence, increasing the teachers' commitment to supporting PS.

CHAPTER FIVE: DISCUSSION, RECOMMENDATIONS, AND CONCLUSION

5.1. Introduction

This chapter presents the discussion, conclusions, and recommendations of the study based on the data analysis and findings detailed in chapter four. The chapter is organized as follows: the first section discusses the major findings in relation to the research questions and existing literature. The second section outlines the theoretical and practical implications of the study. The third section acknowledges the limitations, and next section provides recommendations. The final section presents the conclusion.

5.2. Discussion of Major Findings

The discussion is structured according to the three research questions guiding the study, synthesizing the baseline, Cycle 1, and Cycle 2 findings with the broader literature on PS, instructional practices, and teacher change.

5.2.1. The Influence of the PD Intervention on Teachers' Classroom Practices

The first research question investigated how the PD intervention influenced mathematics teachers' classroom practices in supporting PS. The findings revealed a profound change from teacher centered, directive instruction to student centered facilitation, mediated by the explicit introduction and refinement of Warshauer (2015) four strategies: questioning, wait time, encouragement, and acknowledgement.

Consistent with the literature, both teachers initially operated as struggle removers (Parrish & Bryd, 2022; Stigler & Hiebert, 2004). Teacher A's extreme lack of wait time, literally taking the pen from the student to do the work, exemplified the telling response that lowers cognitive demand (Warshauer, 2014). However, the experiential PD intervention facilitated a significant change. By Cycle 2, both teachers demonstrated a marked change toward probing guidance and affordance (Warshauer, 2014).

Regarding questioning, the intervention process highlighted a critical developmental stage. In the baseline the questioning is directive leading students to the correct answer only, and In Cycle 1, teachers attempted to use questioning but sometimes reverted to funneling and step-by-step prompts that ultimately removed the cognitive load from the students. This aligns with research suggesting that teachers struggle to maintain cognitive demand during moments of student frustration (NCTM, 2014; Russo et al., 2021). However, the video-reflection component in Cycle 2 allowed teachers to see this tendency in their own practice. Consequently, in Cycle 2, Teacher A's questioning shifted to creating cognitive dissonance (e.g., "How can a third plus a third be a half?"), forcing students to re-examine their own logic and reasoning, a practice essential for conceptual understanding (Hiebert & Grouws, 2007).

The implementation of wait time also shifts. In the baseline, silence was immediately filled by the teacher. By Cycle 2, wait time became an intentional pedagogical tool, affording students the space to self-correct and synthesize reasoning, as seen when Teacher A explicitly stated, "I'll give you time; keep trying." This validated Hiebert & Grouws (2007) assertion that students must be allowed to grapple with uncertainty. While encouragement (praising effort) was present in Cycle 1, acknowledgement (normalizing mistakes/confusion as a necessary part of learning) was absent in the baseline and underdeveloped in Cycle 1. By Cycle 2, Teacher A explicitly acknowledged the difficulty of tasks and the value of the revision process, fulfilling Warshauer (2015) four strategy, that is often missed in traditional instruction.

5.2.2. Changes in Students' Engagement and Perseverance

The second research question explored the changes in students' engagement and perseverance when teachers implemented PS strategies. The findings showed a clear transformation from passive, superficial engagement to active, multi-modal cognitive engagement and sustained persistence.

In the baseline, student engagement was characterized by task abandonment, silence, and superficial procedural attempts indicative of Unproductive struggle (Warshauer, 2014). However, as teachers implemented HCD tasks and supported them with the four strategies, student artifacts and classroom observations revealed a dramatic increase in cognitive engagement. Students utilized multiple representations, circular drawings, rectangular models, emojis, and fractional notations, demonstrating that they were actively trying to make sense of the mathematics rather than merely following steps (NCTM, 2014).

Perseverance also shifted to a classroom norm. Artifacts from Cycle 1 and 2 show evidence of erased answers, trial and error partitioning, and multiple attempts to correct unequal divisions. The fact that a student attempted to divide a circle into equal thirds seven times before succeeding aligns with Kapur (2014) theory of PF; students were willing to persist through initial failure because the teacher's use of wait time, encouragement, and acknowledgement signaled that the struggle was both normal and valuable (Young et al., 2023). The shift in the Ethiopian context is particularly noteworthy; students who were conditioned to expect spoon feeding and view errors as incompetence began to view struggle as the mechanism for learning (Dweck, 2006).

5.2.3. The Evolution of Teachers' Beliefs and Attitudes (Guskey's Model)

The third research question examined how mathematics teachers' beliefs and attitudes about PS changed after implementing the intervention and observing student engagement and perseverance. The findings supports Guskey (2002) model of teacher change.

At baseline, a mismatch existed between the teachers' espoused beliefs and their enacted practices. This discrepancy is well documented in the literature, often attributed to cultural norms and lack of training (Stigler & Hiebert, 2004). The PD intervention did not attempt to change beliefs through argumentation only. Instead, following Guskey's sequence, it equipped teachers with practical strategies, which they implemented in classroom, leading to observable student engagement and perseverance.

It was only after teachers witnessed tangible evidence of student success, observing students successfully self-correct, and revise misconceptions, and watching students use of multiple representations that their core beliefs shifted. Teacher A's reflection, "I no longer see mistakes as failure; I see them as part of learning," encapsulates the culmination of Guskey's sequence. The intervention confirmed that efficacy follows evidence; once teachers trusted that students could handle the struggle, their attitudes transformed to patience and trust, increasing their commitment to PS practices (Guskey, 2002).

5.3. Design Principles for Effective Professional Development

This iterative DBR study yielded 3 principles:

The first design principle dictates that PD for supporting PS must immerse teachers as learners in HCD tasks before explicitly teaching pedagogical theories. The theoretical rationale for this principle is anchored in constructivism; just as students must experience cognitive disequilibrium to construct new mathematical knowledge, teachers must personally experience the discomfort and subsequent sense-making of PS to understand their students. Without this experiential foundation, teachers cannot genuinely appreciate the necessity of struggle, nor can they accurately recognize student success when it occurs. Empirically, this was validated during the transition from the baseline to Cycle 1. In the baseline, both teachers removed struggle practically, by removing cognitive demand of task by taking pens from students and providing immediate step-by-step solutions. By requiring teachers to grapple with challenging fraction tasks during the initial PD session, they developed the essential empathy needed to tolerate their students' confusion, which served as a prerequisite for their willingness to implement Warshauer's (2015) support strategies in the classroom.

The second design principle asserts that PD must utilize structured video reflection to explicitly bridge the persistent gap between teachers' espoused beliefs and their enacted classroom practices. Video reflection makes invisible instructional habits visible by holding a mirror to the teacher's actual practice. The empirical justification for this principle emerged clearly through the re-design process. During Cycle 1, both teachers believed they were successfully implementing extended wait time and probing questions, encouraging effort and persistence; however, audio and video transcription revealed they were still relying on funneling and over-scaffolding, encouraging correct answer. Consequently, the Cycle 2 redesign incorporated structured video reflection, where teachers analyzed their own Cycle 1 transcripts against Warshauer's framework. This intervention resulted in a measurable pedagogical shift, most notably demonstrated by Teacher A's transition from directive funneling in Cycle 1 to purposeful wait-time, probing guidance, and encouraging effort and persistence in Cycle 2.

The third design principle emphasizes the necessity of contextualizing HCD tasks using local cultural artifacts to lower the barrier to entry for students without reducing the cognitive demand of the task. Empirical evidence for this principle was occurred when comparing the baseline and

Cycle 2 outcomes. During the baseline, textbook division tasks resulted in high levels of student passivity, with multiple students submitting entirely blank papers. Conversely, the Cycle 2 task, which required sharing culturally familiar injera among children, prompted engagement. The familiar context allowed students to enter the task through drawing and partitioning, which sustained their perseverance through multiple erased attempts and self-corrections.

5.4. Implications of the Study

5.4.1. Theoretical Implications

This study provides empirical test of Guskey's model of teacher change within the specific domain of PS in mathematics education. While the model is widely cited, its application in facilitating specific pedagogical shifts like PS in low resource contexts remains under-researched. This study validates the sequence of the model, proving that constructivist reform can take root even in rigid, teacher centered cultural contexts, provided the PD follows the experiential sequence of practice leading to student outcomes. Furthermore, the study enriches Warshauer's (2014) framework by demonstrating that the four strategies are not naturally intuitive; they require repeated reflective practices to master them.

5.4.2. Practical Implications

For teacher educators, curriculum developers, and organizations like TFE, this study offers a validated, practical PD model. It demonstrates that shifting from passive training to active, addresses authentic classroom challenges. The findings provide concrete evidence that PD must prioritize the four strategies of supporting PS. At the policy level, the study provides empirical justification for shifting towards sustained, classroom based, experiential PD rather than traditional short term workshops, which have historically failed to change practice in Ethiopia (Keski-Mäenpää, 2018; Muluye et al., 2025).

5.4.3. Contextual Implications

By promoting PS, the research directly addresses the persistent challenges of low mathematics achievement and rote memorization in Ethiopian primary schools. By fostering growth mindsets (Dweck, 2006) and perseverance, this pedagogical shift equips students with the resilient problem solving skills necessary for solving complex real challenges, moving Ethiopian education away from rote absorption toward active intellectual empowerment.

5.5. Limitations of the Study

While the study yielded significant insights, it is not without limitations. First, the sample size was limited to two early-career teachers in Bahir Dar. While appropriate for qualitative DBR, the findings are context specific and not intended for broad statistical generalization to all Ethiopian teachers. Second, the study was conducted over two DBR cycles. While enough to observe practice and belief changes, the long term sustainability of these changes beyond the intervention period remains unknown. Finally, the study relied heavily on video observation and interviews; incorporating quantitative measures of student achievement (conceptual understanding) could provide a more holistic view of PS outcomes in future studies. Additionally, due to logistical constraints, Cycle 2 data was only collected and analyzed for Teacher A.

5.6. Recommendations

Based on the findings and implications of this study, the following recommendations are proposed for teacher educators, PD designers, school leaders, and future researcher.

5.6.1. Recommendations for PD Design and Teacher Training

- ✓ Prioritize experiential learning (teachers as learners first): PD programs must deny purely theory. Before teachers can be expected to facilitate PS in their classrooms, they must first experience struggle as learners themselves. Trainers should immerse teachers in HCD tasks, allowing them to feel the cognitive discomfort and subsequent breakthrough of sense making, which builds a deep understanding of the strategy.
- ✓ Use Guskey's model in training: Teacher trainers and PD facilitators must structure their programs according to Guskey's model of teacher change. Instead of attempting to change teachers' beliefs through lectures or theoretical arguments first, training should equip teachers with practical strategies, encourage immediate classroom implementation, and then guide them to observe the resulting improvements in student. Belief change will naturally follow this tangible evidence of student success.
- ✓ Integrate structured video reflection: Schools and PD providers must incorporate video reflection as a core component of training. Recording and analyzing their own classroom instruction is a powerful catalyst for teachers to improve practices. It allows teachers to see their missed opportunities for the strategy.

5.6.2. Recommendations for Classroom Practice and Curriculum

- ✓ Contextualize mathematical tasks: Teachers and curriculum designers should adapt mathematical tasks using culturally relevant, local materials (such as sharing injera, bread, or local scenarios). This practice lowers the barrier to entry for struggling students and makes the problem accessible without reducing the cognitive demand of the task.
- ✓ Shift from telling to probing: Teachers must consciously remove the culture to immediately rescue students by providing step-by-step solutions (telling). Instead, they should use probing guidance and affordance.

5.7. Conclusion

This study addresses the critical gap between the theoretical value of PS and its actual implementation in the Ethiopian mathematics classroom. By employing a DBR approach grounded in Guskey’s model of teacher change, the research demonstrated that sustainable pedagogical reform is possible even in traditional educational environments. The intervention successfully shifted teachers from being struggle removers to struggle facilitators by equipping them with specific strategies; questioning, wait time, encouragement, and acknowledgement. As teachers implemented these strategies, students transformed from passive recipients of procedures to active, persevering problem solvers who viewed mistakes normal rather than barriers. Crucially, it was the tangible evidence of this student success that ultimately transformed the teachers’ core beliefs, validating Guskey’s assertion.

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APPENDIX

Appendix A: PD session Materials

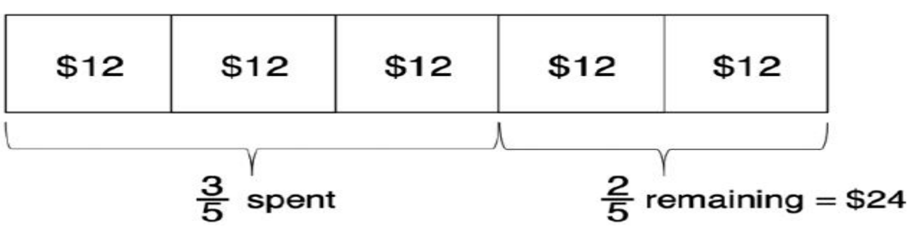
A.1. Sample Handouts

Hand out 1: Shopping Trip Task

ጆሴፍ ለልደት ቀኑ ያገኘውን ገንዘብ ለማውጣት ከጓደኞቹ ጋር ወደ የገበያ ማዕከል ሄደ። ወደ ቤት ሲመለስ 24 ዶላር ቀርቶት ነበር። በገበያ ማዕከሉ ውስጥ ለቪዲዮ ጨዋታዎች እና ለምግብ የልደት ገንዘቡን 3/5ኛ አውጥቶ ነበር። 1) ምን ያህል ገንዘብ አወጣ? 2) ለልደቱ ምን ያህል ገንዘብ አግኝቶ ነበር?

መ/ርት ማህሌት እና መ/ር ዳንኤል ችግሩን በክፍላቸው ሲያቀርቡ፣ ሁለቱም መምህራን ተማሪዎች ለመጀመር ሲታገሉ ያያሉ። በሁለቱም ክፍሎች ውስጥ ያሉ አንዳንድ ተማሪዎች ወዲያውኑ እጃቸውን በማውጣት "አልገባኝም" ወይም "ምን ማድረግ እንዳለብኝ አላውቅም" ይላሉ።

መ/ርት ማህሌት ለተማሪዎቿ በምትሰጠው ምላሽ በጣም ቀጥተኛ (directive) ነዉ። አራት ማዕዘን እንዲስሉ ትነግራቸዋለች እና ጆሴፍ ያወጣውን እና የተረፈውን ለመወከል እንዴት በአምስት እንደሚከፍሉት ታሳያቸዋለች። ከዚያም ከዚህ በታች እንደሚታየው እያንዳንዱን የአራት ማዕዘኑን አንድ አምስተኛ ክፍል 12 ዶላር ብለው እስኪሰይሙ ድረስ ተማሪዎቿን ደረጃ በደረጃ ትመራቸዋለች። በመጨረሻም፣ ተማሪዎቹ በስዕሉ ላይ ያለውን መረጃ ተጠቅመው ለጥያቄዎቹ መልስ እንዲያገኙ ትነግራቸዋለች።



መ/ር ዳንኤል የተማሪዎቹን ትግል የሚይዝበት መንገድ በጣም የተለየ ነው። ሲታገሉ ካያቻው በኋላ፣ በችግሩ ላይ መስራታቸውን እንዲያቆሙ ያደርጋል እና ሁሉም ተማሪዎች ስለ ችግሩ የሚያውቋቸውን ሁለት ነገሮች እና ስራቸውን ለማስቀጠል ይረዳቸው ዘንድ እንዲያውቁ የሚመኙትን አንድ ነገር እንዲጽፉ ይጠይቃቸዋል። ከዚያም መ/ሩ ቀጥሎ ምን መደረግ እንዳለበት በርካታ ሃሳቦች የሚቀርቡበት አጭር

የክፍል ውይይት ያስጀምራል። የቀረቡት ጥቆማዎች አምስተኛዎችን የሚያሳይ የቴፕ ዲያግራም ወይም የቁጥር መስመር መሳል፣ ወይም ዝም ብሎ እንደ 50 ዶላር ያለ ቁጥር መርጦ በሙከራ እና ስህተት (trial and error) መቀጠልን ያካትታሉ። መ/ር ዳንኤል ተማሪዎቹ ስራቸውን ሲቀጥሉ የተጋሩትን የተለያዩ ሃሳቦች እንዲያጤኑ ያበረታታል።

የሁለቱን መ/ራን የተማሪዎችን ትግል የደገፉበት መንገድ እንዴት ታዩታላችሁ የትኛዉ መ/ር የተሻለ ድጋፍ ያደረግ

<u>የመምህር እና የተማሪ ተግባራት</u>

Q: የመምህራንን የተማሪዎች ተግባራት ምን መሆን አለበት?

ከታች ያለው ሰንጠረዥ በሂሳብ ክፍል ውስጥ ትግልን እንደ ተፈጥሯዊ የመማር ሂደት አካል አድርገው የሚቀበሉ የመምህር እና የተማሪ ተግባራትን ያጠቃልላል።

መምህራን ምን እያደረጉ ነው?	ተማሪዎች ምን እያደረጉ ነው?
ተማሪዎች በትምህርቱ ወቅት ምን ሊያታግላቸው እንደሚችል አስቀድሞ መገመት እና በትግሉ ውስጥ ውጤታማ በሆነ መንገድ ለመደገፍ ዝግጁ መሆን።	አንዳንድ ጊዜ በሂሳብ ተግባራት ላይ መታገል፣ ነገር ግን ትልቅ ግኝቶች ብዙ ጊዜ ከግራ መጋባት እና ከትግል እንደሚመጡ ማወቅ።
ተማሪዎች ከተግባራት ጋር የሚታገሉበት ጊዜ መስጠት፣ እና ስራውን ለእነሱ ከመስራት ይልቅ የአስተሳሰብ ሂደታቸውን የሚደግፉ ጥያቄዎችን መጠየቅ።	ከትግላቸው ምንጭ ጋር የተያያዙ እና ተግባራቱን ለመረዳትና ለመፍታት እድገት እንዲያሳዩ የሚረዷቸውን ጥያቄዎች መጠየቅ።
ስለ ስህተቶች፣ የተሳሳቱ ግንዛቤዎች እና ትግሎች ውይይቶችን በማመቻቸት፣ ግራ መጋባት እና ስህተቶች የመማር ተፈጥሯዊ አካል መሆናቸውን ተማሪዎች እንዲገነዘቡ መርዳት።	በችግር አፈታት ሂደት ውስጥ መጽናት እና "እዚህ ጋር እንዴት መቀጠል እንዳለብኝ አላውቅም" ማለት ተቀባይነት እንዳለው፣ ነገር ግን ተስፋ መቁረጥ ተቀባይነት እንደሌለው መገንዘብ።
ተማሪዎች ለሂሳብ ሃሳቦች ትርጉም ለመስጠት ለሚያደርጉት ጥረት እና በችግሮች ውስጥ ለሚያሳዩት የአመክንዮ ጽናት ማድነቅ።	መልሱን ወይም ችግሩን እንዴት እንደሚፈቱ ሳይነግሯቸው እርስ በእርስ መረዳዳት።

Hand out 2

Q: What kind of struggle student face?

Research identifies four main types of student struggle.

1. Getting Started:- occur initially.
- ✓ Confusion about what the task is asking: "I don't know what to do" or "I'm very confused."
- ✓ Claiming to forget a prior concept: "I have absolutely no idea….I don't remember that far."
- ✓ Gestures of uncertainty and resignation: Looks, thinks, sits back, and then says "I don't know."
- ✓ No work on paper, indicating an inability to begin.
2. Carrying Out a Process:- occurs when students have a plan but encounter an impasse while attempting to execute a procedure.
- ✓ Encountering an impasse despite a formulated plan.
- ✓ Unable to carry out an algorithm or process effectively.
- ✓ Forgetting facts or a formula necessary for the step.
- ✓ Mistakes that hinder the continuation of the process.
3. Experiencing Uncertainty in Explaining and Sense-Making:- Occur as students are asked to share, justify, or reflect on their work in small groups or with the whole class.
- ✓ Difficulty in explaining their work: "I know what I'm thinking, I just can't show the exact way."
- ✓ Expressing uncertainty about their solution or steps.
- ✓ Unclear about the reasons for their choice of strategy.
- ✓ Unable to make sense of their own work or other's solutions.
4. Expressing Misconception and Errors: Two types:-
- ✓ Conceptual errors: These arise when students misunderstand.
- ✓ Careless errors: These are unintended and avoidable.

Hand out 3

4 Teacher Strategies to Support PS (Warshauer, 2015)

How to respond when you see a student struggling, without lowering the cognitive demand of the task?

1.Question: Ask questions that help students focus on their thinking and identify the source of their struggle. Examples:

- What have you tried so far?
- Why does that make sense?
- Can you draw a picture to show your thinking?

2. Encourage: Affirm the student's effort and persistence, rather than praising them for getting the right answer quickly. Examples:

- You haven't given up even though this is hard.
- Your method of sharing is correct, keep going.
- I can see you are really thinking hard about this.

3. Give Time (Wait Time): Provide adequate time and space for students to manage their struggle. Do not step in too soon and too much. Examples: Silently walking away after asking a question. Saying,

- Take your time; I will come back in a few minutes.
- Keep trying; I want to see what you can figure out on your own.

4. Acknowledge: Explicitly communicate that confusion and mistakes are a normal, expected, and necessary part of learning mathematics. Examples:

- Don't worry if you don't get it right away; mistakes help our brains grow.
- I see you erased that; it's good to revise our thinking.

A.3. **Experiential task for teachers**

Suppose we have a 48 gallon rain barrel containing 24 gallons of water and a 5 gallon water jug containing 3 gallons of water. Which container is said to be fuller? If we drain a gallon of water from each container, does this change your answer about which container is fuller? Explain.

After teacher solve this the following handout given to them:

Hand out 4

The Rain Barrel Problem

Suppose we have a 48 gallon rain barrel containing 24 gallons of water and a 5 gallon water jug containing 3 gallons of water. Which container is said to be fuller? If we drain a gallon of water from each container, does this change your answer about which container is fuller? Explain.

The following interaction occurred:

Helina is a student who faces challenge in the above task.

Helina: I don't know what to do.

Teacher: OK. How can we compare the amounts in the barrel to the jug?

Helina: I'm not sure.

Teacher: Well, you have part and the whole for each, don't you?

Helina: Yeah.

Teacher: What kind of numbers can we make with part and a whole?

Helina: Fractions?

Teacher: Excellent! So, what do your fractions look like?

Helina: Um . . . 24/48 and 3/5 . . . but. . . .

Teacher: Good. Now what do we do with fractions when they have different denominators?

Helina: Get common denominators? Oh, OK. I think I can do that.

This series of teacher questioning alters a challenging problem, requiring students to think and reason deeply into a procedural problem structured by the teacher.

Strategy used by teachers in supporting PS

Strategy 1: Teachers ask questions that help students focus on their thinking and identify the source of their struggle. They encourage students to build on their thinking or look at other ways to approach the problem without solving the problem for them.

Sample dialogue for this "If we drain a gallon of water from each container, does this change your answer about which container is fuller?"

Abel is student

Teacher: OK. What do you think? [Going over to Abel's table of four students]

Abel: I put "no" because it would be the same as before because you have taken a gallon from both.

Teacher: OK. So show me what that would look like. Show me what your gallons would look like. If you take a gallon from each, what are you looking at?

The teacher began by asking Abel questions and giving him an opportunity to explain his solution.

Those in the square brackets are physical actions

Abel: [Writing on his paper]

Teacher: OK. So what you're telling me [pointing to Abel's work], you have 23 gallons out of 48 and 2 gallons out of 5, that you're still going to have that the 2 gallons out of 5 will be . . .

Abel: Lower, less full.

Teacher: But didn't you tell me 3/5 was fuller?

Abel: Wait, wait.

Teacher: So would it change?

Abel: Oh, OK. Yes, yes because, see . . .

Teacher: Because why?

Abel: They would be the same as before because you're taking a gallon from both.

Teacher: OK, which one are you telling me is fuller?

Strategy 2: Teachers encourage their students to reflect on their work and support student struggle in their effort to explain their thinking and not just in getting correct answers.

Abel: 2/5 . . . but isn't that fuller now? [Looking up at the teacher questioningly]

Teacher: Hmmmm, why would that one is fuller now, do you think?

Abel: Because the other one isn't a half.

Teacher: OK, it's not half. That means it's not fuller?

Abel: It's more than 0.5 now.

Teacher: Tell me that one more time.

Strategy 3: Teachers give time and help students manage their struggles through adversity and failure by not stepping in too soon or helping too much and thus taking the intellectual work away from the students.

Abel: I said it's more than 0.5 now. Because this one is no longer 0.5 because [squints and thinks], you have to subtract; you have taken away a gallon.

Teacher: OK.

Abel: And this one is no longer 0.6 because you have taken a gallon. And this one would no longer be a half. . . . I know what I'm saying.

Teacher: Be patient [gives Abel time and listens intently].

Abel: [Prepares to compute long division of his fractions that he has on his paper]

Teacher: Get your percents and call me back over. Keep working.

Abel: Back off [said to another student in his group who is looking at his paper while Abel is doing his computation. Abel continues to work while the teacher checks on other students' work.]

Strategy 4: Teachers acknowledge that struggle is an important and natural part of learning and doing mathematics.

Abel: OK. I have it. Ring, ring, ring [makes a bell like sound].

Teacher: What have you got? I'm here. Glad you called.

Abel: Now I have got the percentage, drum roll please.

Teacher: [Taps on Abel's desk] go.

Abel: This one is now 47 percent [pointing to his work for the 48 gallon barrel]. This is 40 percent [pointing to his work for the water jug].

Teacher: Which one's fuller?

Abel: The 48 gallon barrel.

Appendix B: Instructional Tasks Used in the Study

B.1. Baseline task

Teacher A: One carton can pack 4 books. To pack 342 books,

A. How many cartons are needed?
B. How many books will be left unpacked?
C. If there is a carton that can pack 3 books and a carton that can pack 5 books, which carton will pack all the books without missing any book?

መምህር ሀ፡-አንድ ካርቶን 4 መጻሕፍትን መያዝ ይችላል። 342 መጻሕፍትን ለማሸግ፡-

ሀ. ስንት ካርቶኖች ያስፈልጋሉ?

ለ. ሳይታሸጉ የሚቀሩ መጻሕፍት ብዛት ስንት ነው?

ሐ. 3 መጻሕፍትን የሚይዝ ካርቶን እና 5 መጻሕፍትን የሚይዝ ካርቶን ቢኖር፣ ምንም መጽሐፍ ሳይተርፍ ሁሉንም መጻሕፍት መያዝ የሚችለው የትኛው ካርቶን ነው?

Teacher B: One carton can pack 4 books. To pack 134 books,

A. How many cartons are needed?
B. How many books will be left unpacked?
C. If there is a carton that can pack 2 books and a carton that can pack 3 books, which carton will pack all the books without missing any book?

መምህር ለ፡-አንድ ካርቶን 4 መጻሕፍትን መያዝ ይችላል። 134 መጻሕፍትን ለማሸግ፡-

ሀ. ስንት ካርቶኖች ያስፈልጋሉ?

ለ. ሳይታሸጉ የሚቀሩ መጻሕፍት ብዛት ስንት ነው?

ሐ. 2 መጻሕፍትን የሚይዝ ካርቶን እና 3 መጻሕፍትን የሚይዝ ካርቶን ቢኖር፣ ምንም መጽሐፍ ሳይተርፍ ሁሉንም መጻሕፍት መያዝ የሚችለው የትኛው ካርቶን ነው?

B.2. Cycle 1 task

Ali, Beza, and Leul were given one bread that has equal size for each. Ali divided his bread into two equal parts and ate one. Beza divided her bread into four equal parts and ate two. Leul divided his bread into three equal parts and ate two.

A. From Ali and Beza, who ate more? Explain.
B. Of the three children, who ate the most?
C. If the leftover bread is combined, would it be less than or more than one whole bread? Explain.

ለአሊ፣ ለቤዛ እና ለልዑል ለእያንዳንዳቸው እኩል መጠን ያለው አንድ አንድ ዳቦ ተሰጣቸው። አሊ ዳቦውን ለሁለት እኩል ቦታ ከፍሎ አንዱን በላ። ቤዛ ዳቦዋን ለአራት እኩል ቦታ ከፍላ ሁለቱን በላች። ልዑል ዳቦውን ለሦስት እኩል ቦታ ከፍሎ ሁለቱን በላ።

ሀ. ከአሊ እና ከቤዛ ብዙ የበላው ማን ነው? አስረዱ።

ለ. ከሦስቱ ልጆች ብዙ የበላው ማን ነው?

ሐ. የተረፈው ዳቦ ቢደመር ፣ ከአንድ ሙሉ ዳቦ ያንሳል ወይስ ይበልጣል? አስረዱ።

B.3. Cycle 2 task

How can five injera be shared equally among three children? How much will each child receive? Show your thinking using drawings, words, fractions, or another method.

አምስት እንጀራዎችን ለሦስት ልጆች እኩል እንዴት ማካፈል ይቻላል? እያንዳንዱ ልጅ ምን ያህል ይደርሰዋል? አሰራራችሁን ስዕሎችን፣ ቃላትን፣ ክፍልፋዮችን ወይም ሌላ ዘዴን በመጠቀም አሳዩ።

Appendix C: Data collection Instruments

C.1. Baseline Interview / መነሻ ቃለ-መጠይቅ

1. Let's talk about your math instruction in the classroom; specifically, how do you introduce challenging questions to your students? What kind of support do you provide so they can work independently?
   እስቲ ስለ ክፍል ውስጥ የሒሳብ ት/ት አሰጣጥዎ እናውራ፤ በተለይ ለተማሪዎች ትንሽ ከበድ ያሉ (ፈታኝ) ጥያቄዎችን ሲሰጡ እንዴት ነው የሚያስተዋውቋቸው? ተማሪዎቹ በራሳቸው እንዲሰሩስ ምን ዓይነት ድጋፍ ያደርጋሉ?
2. What do you usually do when you observe students struggling during a math task? Why did you choose that approach/strategy?
   ተማሪዎች በሒሳብ ስራ ወቅት ሲቸገሩ ወይም ሲጨነቁ ሲያዩ አብዛኛውን ጊዜ ምን ያደርጋሉ? ያንን መንገድ (ስልት) ለምን መረጡ? (ወዲያውኑ መልሱን ይነግሯቸዋል?)
3. What do you believe are the benefits of students struggling and working hard to find an answer in math? How does this belief influence your teaching? (Do you think that struggling to find the answer helps the student truly understand the subject?)
   ተማሪዎች በሒሳብ ትምህርት ውስጥ ለፍተውና ደክመው መልስ ማግኘታቸው ጥቅሙ ምንድነው ብለው ያምናሉ? ይህ እምነትዎስ በት/ት አሰጣጥዎ ላይ ምን ተጽዕኖ አለው? (ተማሪው ተቸግሮ መልሱን ማግኘቱ በትክክል ትምህርቱን እንዲረዳ ያደርገዋል ብለው ያስባሉ?)
4. When you see your students successfully solve a difficult question (e.g., when their understanding increases), how has it affected your attitude towards giving students the opportunity to struggle? (Did the students' improved performance make you more confident in your own teaching methods?)
   ተማሪዎችዎ አንድን አስቸጋሪ ጥያቄ ተጋፍጠው ውጤታማ ሲሆኑ ሲያዩ (ለምሳሌ ግንዛቤያቸው ሲጨምር)፣ እርስዎ ተማሪዎች እንዲለፉና እንዲታገሉ በሚሰጡት ዕድል ላይ ምን ዓይነት አመለካከት እንዲኖርዎት አድርጓል? (የተማሪዎቹ ውጤት ማደግ እርስዎ በራስዎ የማስተማር ዘዴ ላይ የበለጠ እንዲተማመኑ አድርጎዎታል?)

5. How do you view your students' motivation and resilience when they face challenging math questions? (Are they eager to work on the question, or do they give up immediately?)
ተማሪዎችዎ ፈታኝ የሆኑ የሒሳብ ጥያቄዎች ሲገጥሟቸው ያላቸውን ተነሳሽነት እና ጥንካሬ እንዴት ያዩታል? (ተማሪዎቹ ጥያቄውን ለመስራት ይጓጓሉ ወይስ ወዲያውኑ ተስፋ ይቆርጣሉ?)
6. Could you tell me about a time you observed students struggling to solve a math problem? (I would appreciate it if you share the outcome, whether good or bad). (For example, the joy seen when a student succeeds after struggling, or a time they became very frustrated?)
ተማሪዎች አንድን የሒሳብ ጥያቄ ለመስራት ሲታገሉ የታዘቡት አጋጣሚ ካለ ቢነግሩኝ? ጥሩም ይሁን መጥፎ ያዩትን ውጤት ቢያጋሩኝ ደስ ይለኛል። (ለምሳሌ ተማሪው ተቸግሮ መጨረሻ ላይ ሲሳካለት የታየበት ደስታ፣ ወይም በጣም ተቸግሮ የተበሳጨበት አጋጣሚ አለ?)
7. In the current context of Ethiopian primary schools, what obstacles have you faced in encouraging students to explore and struggle in math? What kind of resources or training would help you? What new teaching methods would you like to learn?
በኢትዮጵያ ነባራዊ ሁኔታ በመጀመሪያ ደረጃ ትምህርት ቤቶች ተማሪዎች በሒሳብ እንዲመራመሩና እንዲለፉ ለማድረግ ምን እንቅፋቶች ገጠሙዎት? ምን ዓይነት ግብዓት ወይም ስልጠና ቢሰጥዎት ይረዳዎታል?

C.2. Post-cycle 1 Interview / የዑደት 1 ድህረ-ትግበራ ቃለ-መጠይቅ

1. How did you support students during today's lesson? Specifically, how did you use questioning, wait time, encouragement, and acknowledgement? In what ways was this different from your usual teaching?
በዛሬው የክፍል ውስጥ ትምህርት ተማሪዎችን እንዴት ደገፉ? በተለይ ጥያቄዎችን መጠየቅን፣ ጊዜን መስጠትን፣ ማበረታታትን እና እውቅና መስጠትን እንዴት ተጠቀሙ? ይህስ ከተለመደው የማስተማር ዘዴዎ በምን ይለያል?
2. Compared to your previous lessons, what changes did you observe in students' participation and perseverance? Which students showed noticeable change, and why?
ካለፉት ትምህርቶችዎ ጋር ሲነጻጸር፣ በተማሪዎች ተሳትፎ እና ጽናት ላይ ምን ለውጦችን ታዝበዋል? የትኞቹ ተማሪዎች ላይ ጉልህ ለውጥ ታየ? ለምን?

3. Looking at students' work, what evidence shows their perseverance and engagement (e.g., multiple attempts, use of different strategies, drawings, corrections, or explanations)?
የተማሪዎችን ስራ ሲመለከቱ፣ ጽናታቸውን እና ተሳትፏቸውን የሚያሳየው ማስረጃ ምንድነው (ለምሳሌ፡- ደጋግሞ መሞከር፣ የተለያዩ ስልቶችን መጠቀም፣ ስዕሎች፣ እርማቶች ወይም ማብራሪያዎች)?
4. After observing students working through challenges how has your belief about PS changed, if at all? Explain with examples from the lesson.
ተማሪዎች ፈታኝ ሁኔታዎችን ተጋፍጠው ሲሰሩ ከተመለከቱ በኋላ፣ በዚህ ዙሪያ ያለዎት እምነት እንዴት ተቀየረ? (ለውጥ ካለ) ከክፍል ውስጥ ትምህርቱ በምሳሌ ያስረዱ?
5. How did students' participation and perseverance influence your decision to allow students more time to struggle in mathematics?
የተማሪዎች ተሳትፎ እና ጽናት ተማሪዎች በሂሳብ ትምህርት ውስጥ ይበልጥ እንዲታገሉ ተጨማሪ ጊዜ ለመስጠት በወሰኑት ውሳኔ ላይ ምን ተጽዕኖ አሳደረ?
6. Which aspects of the PD helped you support PS effectively? What challenges did you face during implementation?
ከሙያ ማሻሻያ ስልጠናው ውስጥ ፍሬያማ ትግልን ውጤታማ በሆነ መንገድ ለመደገፍ የረዳዎት የትኛው ክፍል ነው? በምትገበሩበት ወቅት ምን ተግዳሮቶች ገጠሙዎት?
7. Based on your experience in this cycle, what specific changes would you suggest improving the support of PS in the next cycle?
በዚህ ዑደት ባገኙት ልምድ ላይ በመመስረት፣ በቀጣዩ ዑደት የተማሪዎችን ፍሬያማ ትግል ድጋፍ ለማሻሻል ምን የተለዩ ለውጦችን ይጠቁማሉ?

C.3. Post-Cycle 2 interview / የዑደት 2 ድህረ-ትግበራ ቃለ-መጠይቅ

1. How did you support students during this Cycle 2 lesson while they were working on the task? Please describe how you used questioning, wait time, encouragement, and acknowledgement, and whether your use of these strategies improved from Cycle 1.
   በዚህ የዑደት 2 ትምህርት ተማሪዎች የተሰጣቸውን ስራ በሚሰሩበት ጊዜ እንዴት ደገፉአቸው? ጥያቄዎችን መጠየቅን፣ ጊዜን መስጠትን፣ ማበረታታትን እና እውቅና መስጠትን እንዴት እንደተጠቀሙ፣ እና የእነዚህ ስልቶች አጠቃቀምዎ ከዑደት 1 የተሻሻለ መሆን አለመሆኑን ይግለጹ።
2. Compared to Cycle 1 or your usual teaching methods, what did you do differently in this Cycle 2 lesson? Why did you make these changes?
   ከዑደት 1 ወይም ከተለመደው የማስተማር ዘዴዎ ጋር ሲነጻጸር፣ በዚህ የዑደት 2 ትምህርት ላይ ምን የተለየ ነገር አደረጉ? እነዚህንስ ለውጦች ለምን አደረጉ?
3. What changes did you observe in students' participation, engagement, confidence, and willingness to attempt the task during Cycle 2? For example, did more students try different methods, ask questions, or explain their thinking?
   በዑደት 2 ወቅት በተማሪዎች ተሳትፎ፣ ትኩረት፣ በራስ መተማመን እና ስራውን ለመሞከር ባላቸው ፍላጎት ላይ ምን ለውጦችን ታዝበዋል? ለምሳሌ፣ በርካታ ተማሪዎች የተለያዩ ዘዴዎችን ሞክረዋል፣ ጥያቄዎችን ጠይቀዋል፣ ወይስ ሀሳባቸውን አብራርተዋል?
4. How did students respond when they faced difficulty in Cycle 2? Did they continue working, revise their ideas, try different strategies, ask for help appropriately, or give up?
   በዑደት 2 ላይ ተማሪዎች ችግር ሲያጋጥማቸው ምን ምላሽ ሰጡ? መስራታቸውን ቀጠሉ፣ ሀሳባቸውን አስተካከሉ፣ የተለያዩ ስልቶችን ሞከሩ፣ ተገቢውን እርዳታ ጠየቁ፣ ወይስ ተስፋ ቆረጡ?
5. Looking at students' work (their papers, drawings, written explanations, or solutions), what evidence shows that students were thinking independently and engaging in PS during Cycle 2?
   የተማሪዎችን ስራ (ወረቀቶቻቸውን፣ ስዕሎቻቸውን፣ የጽሁፍ ማብራሪያዎቻቸውን ወይም መፍትሄዎቻቸውን) ሲመለከቱ፣ ተማሪዎች በራሳቸው እያሰቡ እና በዑደት 2 ውስጥ በፍሬያማ ትግል ላይ ተሳትፈው እንደነበር የሚያሳየው ማስረጃ ምንድነው?

6. After Cycle 2, how has your belief and attitude toward allowing PS in mathematics changed? Please explain how your thinking may be different from before the intervention or Cycle 1.

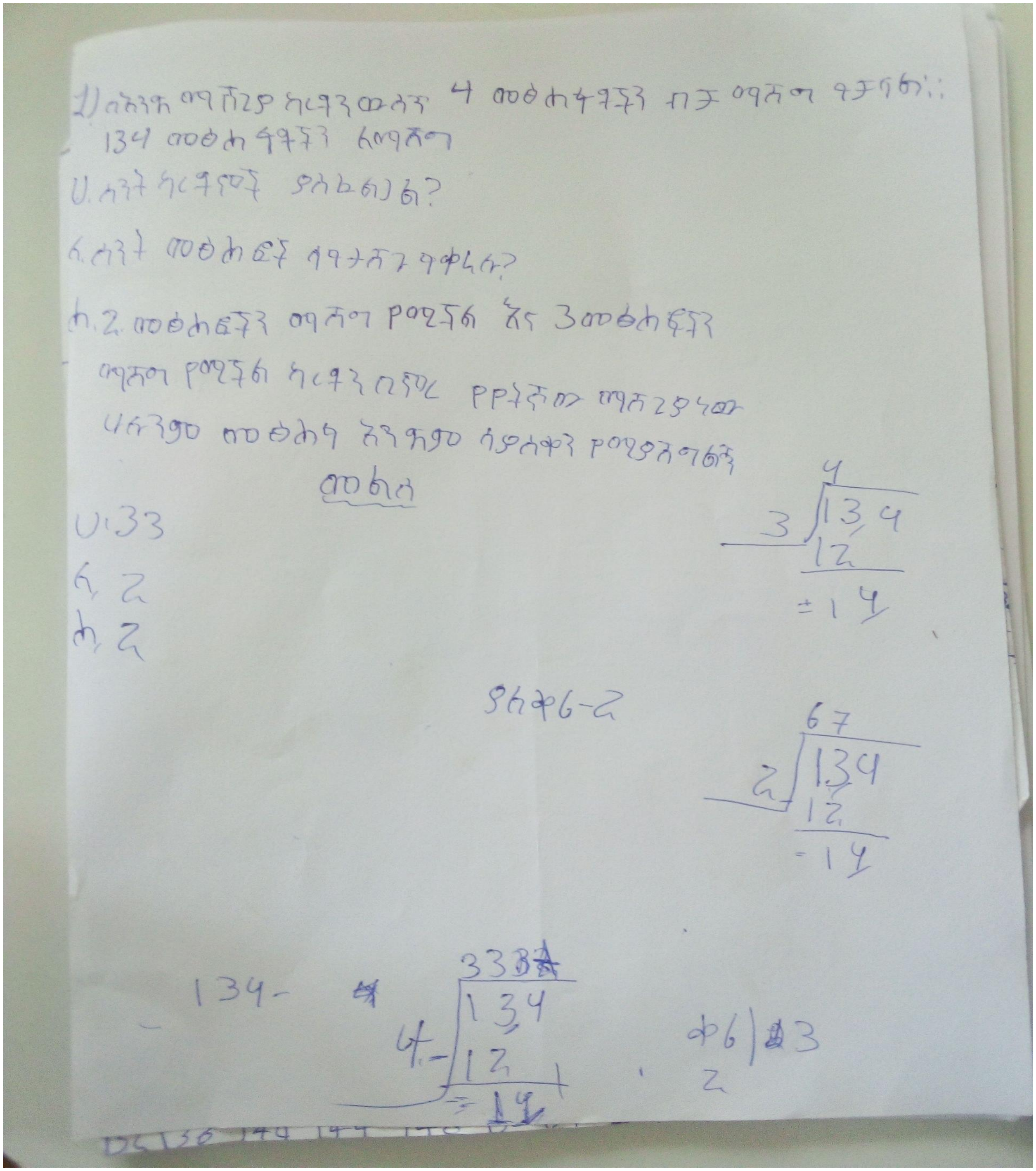
   በዑደት 2 የተማሪዎችን ምላሽ ከተመለከቱ በኋላ፣ በሂሳብ ትምህርት ውስጥ ፍሬያማ ትግልን ከመፍቀድ አንጻር ያለዎት እምነት እና አመለካከት እንዴት ተቀይሯል? አስተሳሰብዎ ከስልጠናው በፊት ወይም ከዑደት 1 ጋር ሲነጻጸር በምን ሊለይ እንደሚችል ያብራሩ።

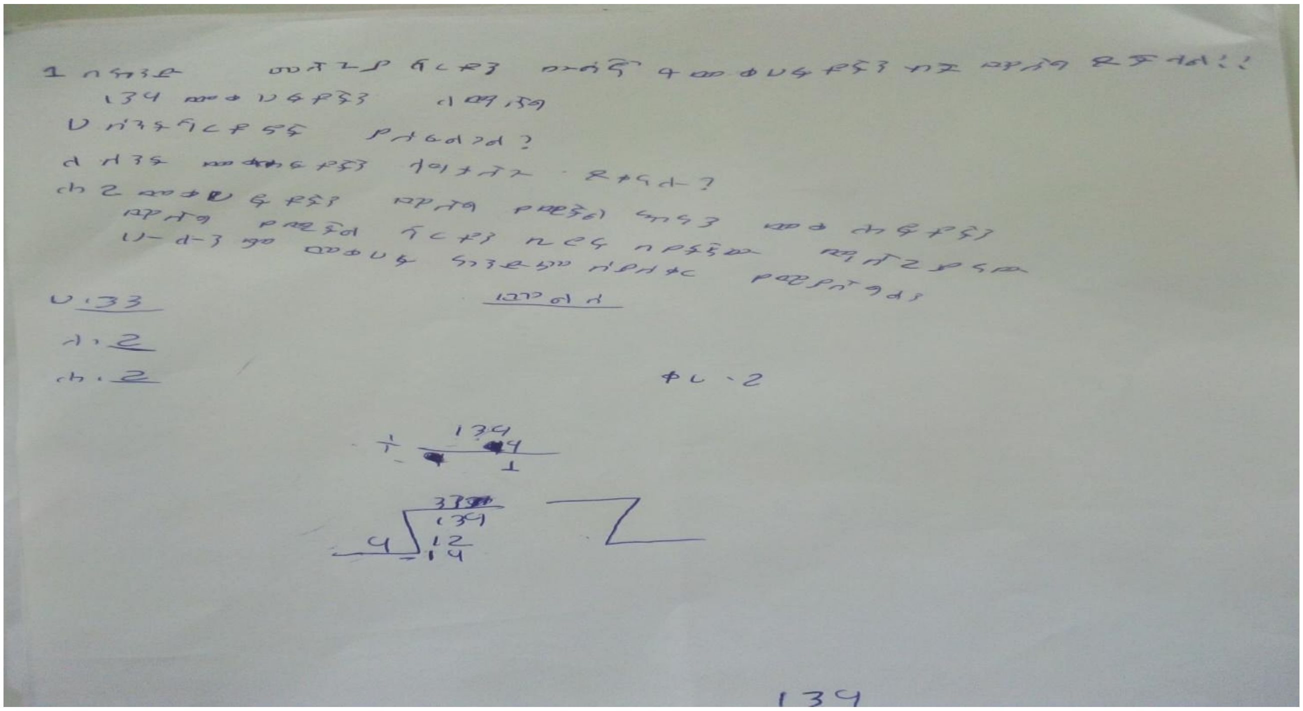
7. Reflecting on Cycle 2, what worked well in supporting PS, what challenges still remained, and what recommendations would you make for future lessons or PD?

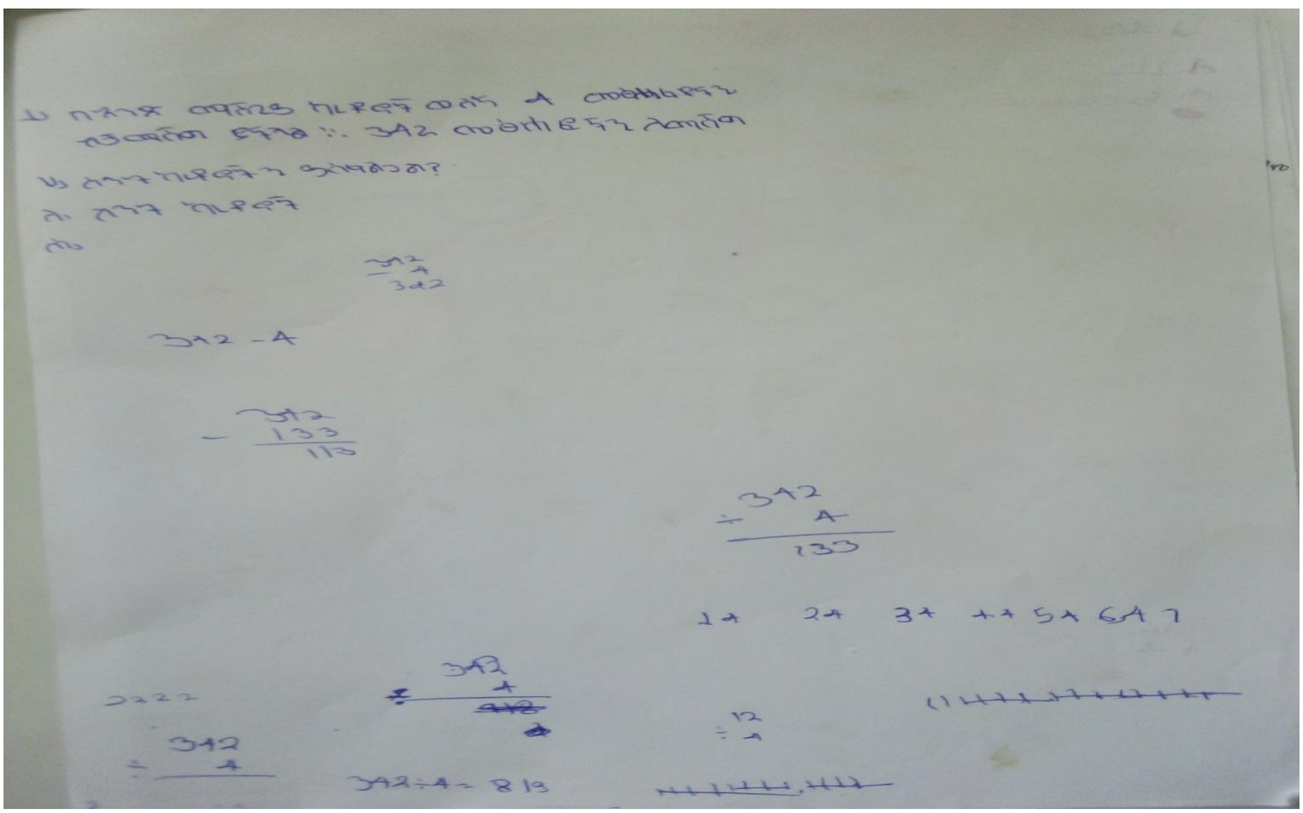
   ዑደት 2ን መለስ ብለው ሲገመግሙ፣ ፍሬያማ ትግልን በመደገፍ ረገድ በጥሩ ሁኔታ የሰራው ምንድነው፣ አሁንም የቀሩት ተግዳሮቶች ምንድን ናቸው፣ እና ለወደፊት ትምህርቶች ወይም የሙያ ማሻሻያ ስልጠናዎች ምን ምክረ-ሀሳብ ይሰጣሉ?

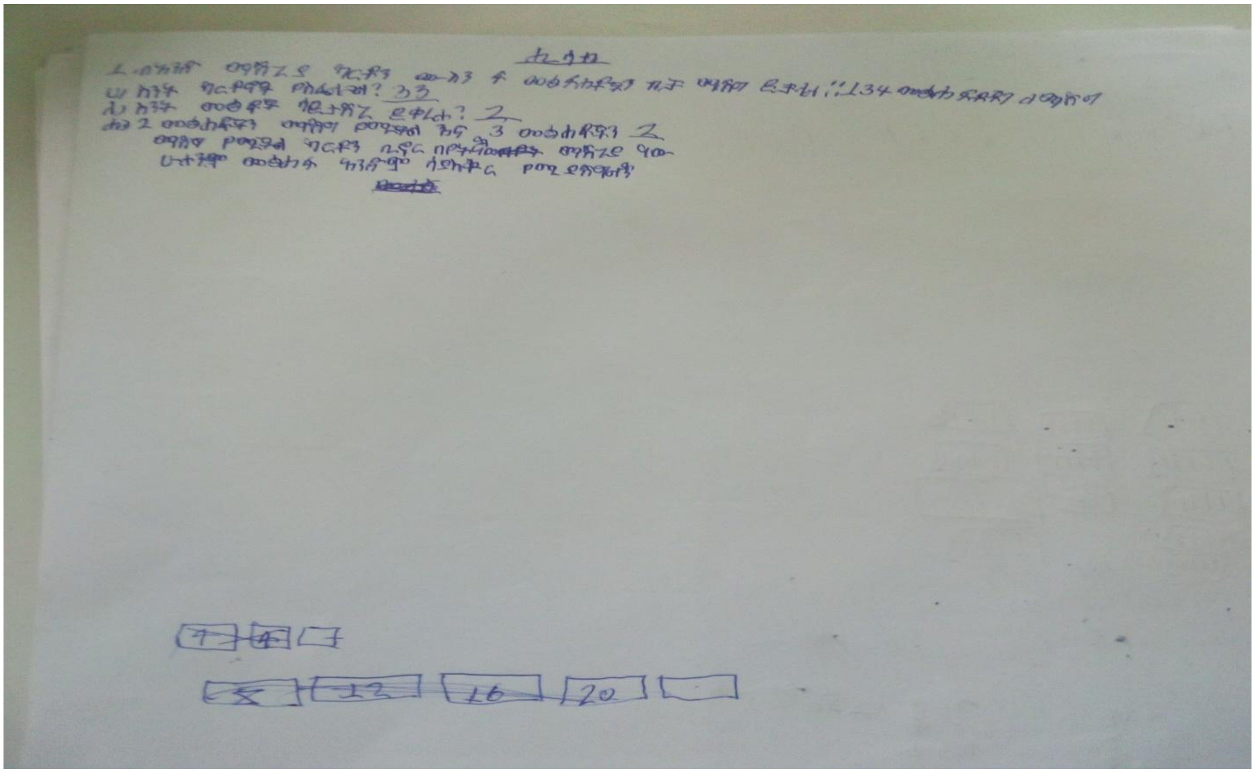
Appendix D: Sample Student Work and Artifacts

D.1. Sample student work and artifacts collected during Baseline

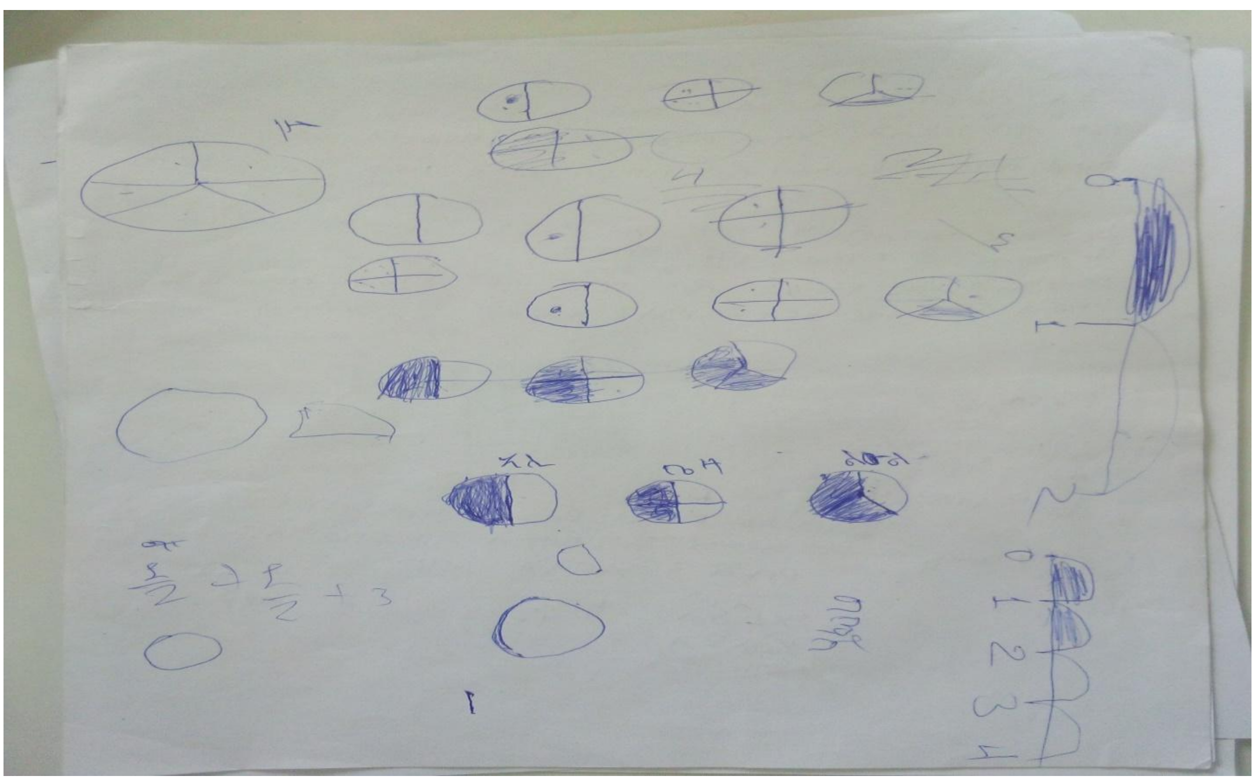


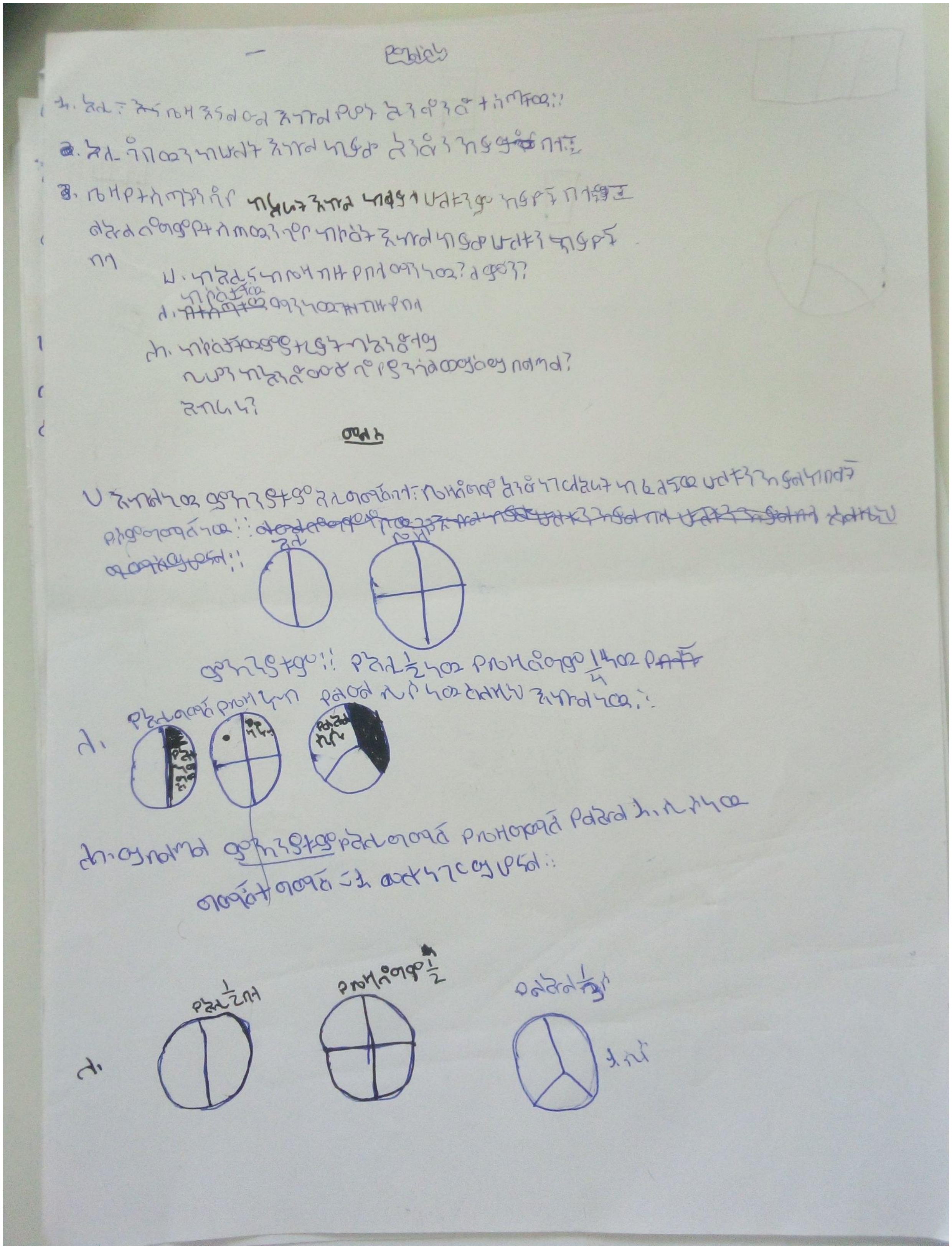


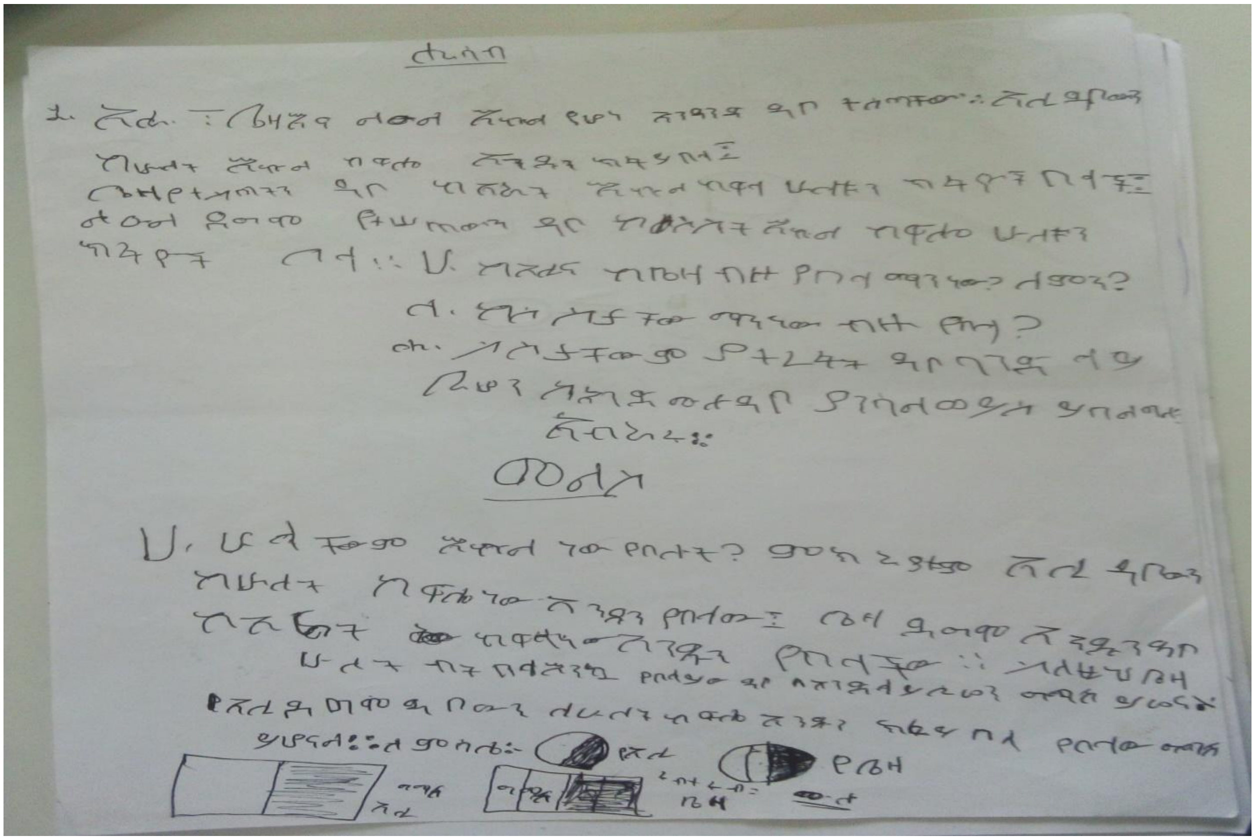


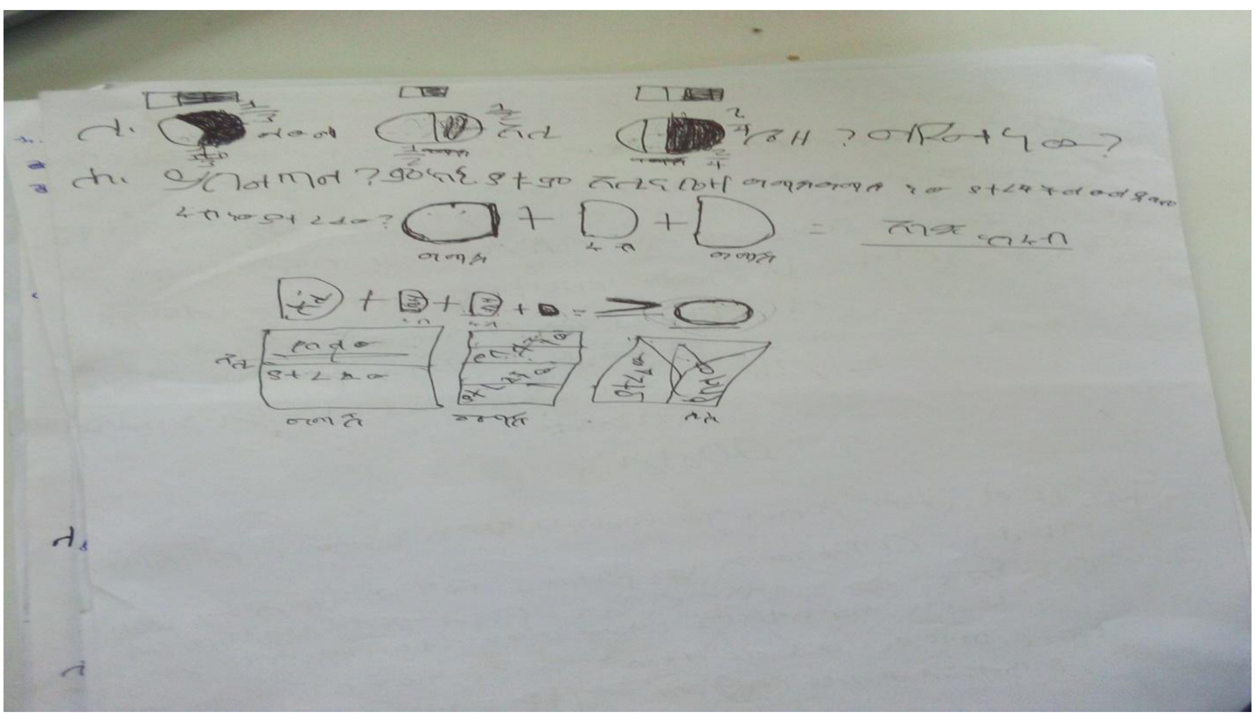


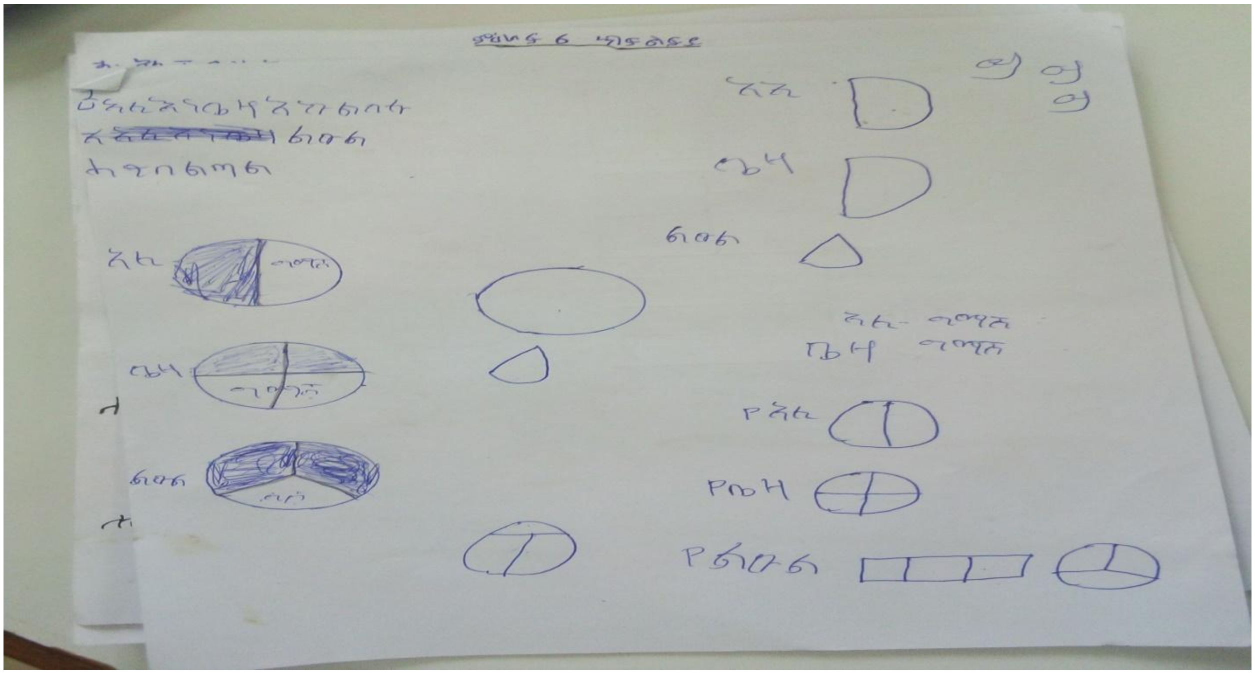
D.2. Sample student work and artifacts collected during Cycle 1











D.3. Sample student work and artifacts collected during Cycle 2