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KINEMATICAL AND DYNAMIC ANALYSIS OF PLANAR FOUR BAR LINKAGE FOR PATH GENERATION

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KINEMATICAL AND DYNAMIC ANALYSIS OF PLANAR FOUR BAR LINKAGE FOR PATH GENERATION

by

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a Thesis

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Advisor

Dr. Alem Bazezw

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Kinematic and dynamic analysis of planar four bar linkage for path generation

A Thesis in partial fulfillment of the requirements for the Degree of Master of Science in "Mechanical Design"

Presented to the School of Mechanical and Industrial Engineering, Bahir Dar Institute of Technology, Bahir Dar University

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DECLARATION

I, Yared Abraha, declare that the thesis comprises my own work. In compliance with internationally accepted practices, I have dually acknowledged and referred all materials used in this work. I understand that non-adherence to the principles of academic honesty and integrity, misrepresentation/ fabrication of any idea/data/fact/source will constitute sufficient ground for disciplinary action by the university and can also evoke penal action from the sources which have not been properly cited or acknowledged.

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ABSTRACT

Mechanisms are means of Power Transmission as well as motion transformers. A Four-bar Mechanism consists mainly of four planar links connected with four revolute joints. Input usually given as rotary motion of a link and output can obtain from the motion of another link or a coupler point. Kinematic and Dynamic Analyses presented in this thesis. Path generation method has applied to generate planar four-bar mechanism. The task of analyses refers to a problem in which a coupler point desired to generate a given path. coupler point has traces a set of predefined points, the position of coupler point at each stage using analytical methods to approximate a linkage dimensions required to achieve a set of prescribed path points in order to study the Kinematic (velocity and acceleration) and Dynamic equation requirements and to satisfy the detailed steps in formulating the Planar four-bar mechanism solutions. MATLAB coding has written to find the solution of Velocity and Acceleration, and Dynamic force at various positon of the four bar mechanism producing by the linkages joints. Modelling of Planar Four-Bar Mechanisms is accomplished using the CATIA V5R21 in order to simplify the process and simulate the desired path is generated by coupler point. For the input example of predefined point P_X & P_Y the coupler path point generated. velocity and acceleration graphs of the coupler point CATIA V5R21 and reaction force on the pin joints for prescribed input point has simulated with MATLAB R2016b. the coupler point can have used to generate prescribed points of practical applications.

Acknowledgement

I like to express my first grateful and appreciation to my advisor, Dr. Alem Bazezew, that Who have helped me in different but valuable ways in the preparation of this thesis work I also tank to Prof. Rashid Ahmed Kahin who have gave me a suggestions on how to make this thesis more effective.

All those who have support me this success was not possible without their advice, constructive criticism and ideas they have been generous with their time.

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NOMENCLATURE

x_0	X - Position of Drive Crank Pivot
y_0,	Y - Position of Drive Crank Pivot
r_2	Drive Crank Length
r_3	Connecting Rod Length
r_4	Follower Crank Length
r_1	Ground Pivot Length
r_p	Trace Point Length
β	Trace Point Angle
θ_1	Ground Pivot Angle
θ_2	Drive Crank Angle
θ_3	Connecting Rod Angle
θ_4	Follower Crank Angle
s	length of the shortest link
l	length of longest link
p & q	lengths of the two remaining linksω_2
ω_2	Velocity of link 2
ω_3	Velocity of link 3
ω_4	Velocity of link 4
α_2	Angular acceleration of link 2
α_3	Angular acceleration of link 3
α_4	Angular acceleration of link 4
F_{12},F_{32}, F_{23}, F_{43}, F_{14} and F_{34}	joint forces acting on links
F_g & T_g	Inertia force and inertia moment of links
m	Mass of link
T_1	Torque of external load
T_g	Inertia moment of link 3
T_s	Driving torque

CHAPTER ONE

INTRODUCTION

1.1. Background of the Study

Mechanism is a combination of rigid or resistant bodies so formed and connected that they move up on each other with defined relative motion. One of the main objects of designing a mechanism is to develop a system that transforms motion in a specific way to provide mechanical advantage. Typical problem in mechanism design is coordinating the input and output motions.

In kinematic analysis, a particular given mechanism is investigated based on the mechanism geometry plus possibly other known characteristics (such as input angular velocity, angular acceleration, etc.). Kinematic synthesis, on the other hand, is the process of designing a mechanism to accomplish a desired task.

There are three main tasks in kinematic synthesis; function generation, path generation and motion generation.

- **Function Generation:** where the functional requirement is the relative motion between links, which are generally connected to ground pivots (Figure 2), or where a specific input position $f(c)$ crank will result in a specific output position $g(c)$ follower;

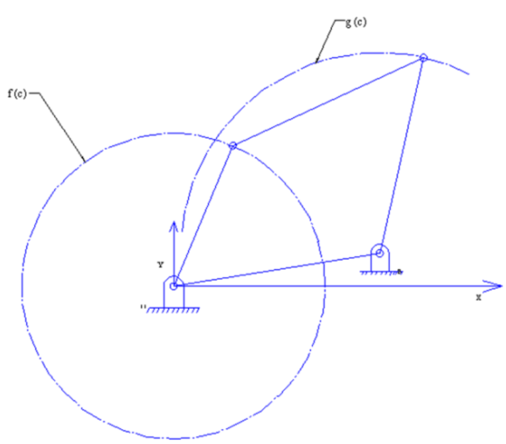


Figure 1: Function generation

- **Path Generation:** where the functional requirement is the output path of a trace point, which is typically on a coupler link (Figure 3).

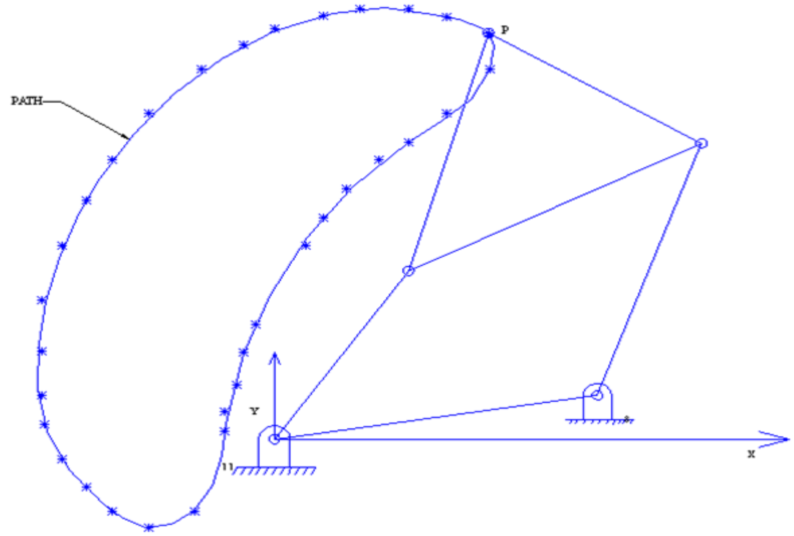


Figure 2: Path generation

- **Motion Generation:** where the functional requirement is the entire motion, path and angle of a trace point on a coupler link (***Figure 4***).

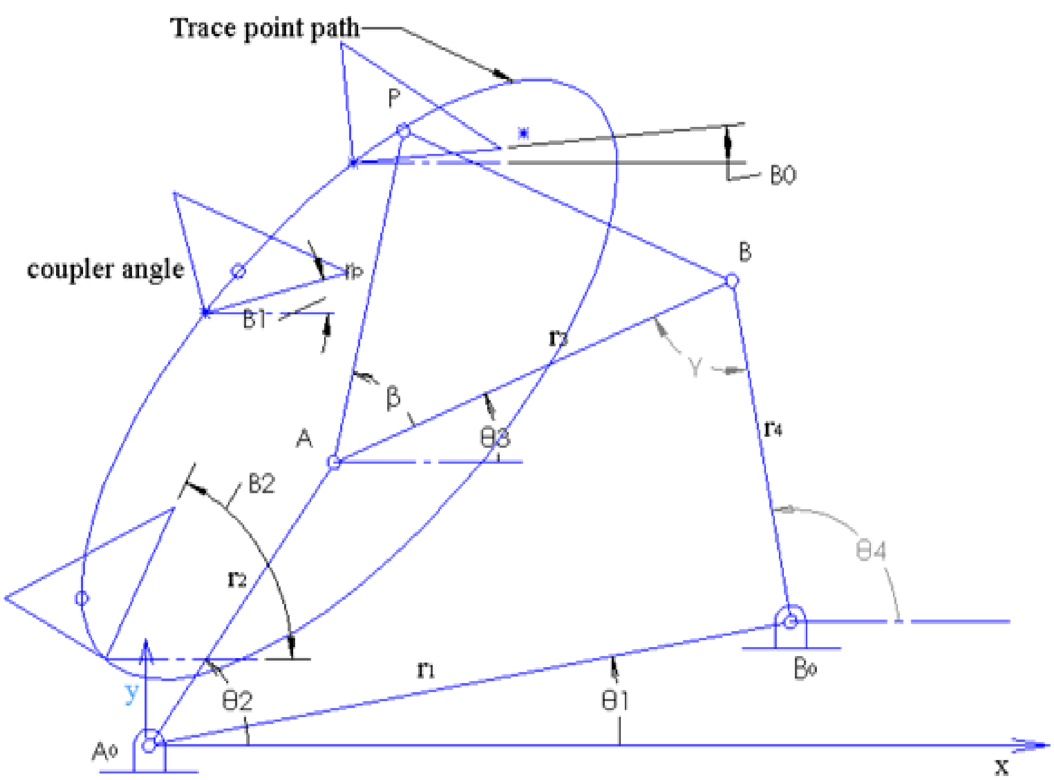


Figure 3: Motion generation

As mentioned before Path generation is an example of constraint condition which describes desired position and orientation for a certain point on the mechanism with respect to the fixed coordinate system. According to the given path of the point on the coupler, the construction parameters can obtained by writing objective functions of the mechanism [17]. But without the possibility of controlling a structural (linkage mechanism) error on a path out of those points [18].

In this study, path generation kinematic synthesis carried out for the planar four-bar mechanism to obtain the construction parameters of mechanism. In order to define the position of the four-bar mechanism with respect to the fixed frame, an extra loop closure equation formulated to achieve solutions. Nonlinear system of equations has written for each independent loop. Precision points are determined by using Chebyshev spacing [17]. The first question in solving this problem is usually: What kind of a mechanism (if any) can used to perform this function. In addition, the second question is: How does one design such a mechanism? There are various methods of performing kinematic analysis of mechanisms, including graphical, analytical, and numerical. The choice of a method depends on the problem at hand and on available computational means [26].

The objective of kinematics analysis is to develop various means of transforming motion to achieve a specific kind needed in applications. For example, an object is to be moved from point to point along some path [26].

The objective of dynamics analysis is to study the behavior of a given machine or mechanism when subjected to dynamic forces. For the above example, when the mechanism already known then external load is applied and its motion is studied. The analysis of forces induced in joints by the motion is part of this work.

One of the most useful and most common mechanism is the four bar linkage a sketch shown in **figure.4** Link 1(r_1) is the frame or ground, and is generally stationary; Link 2 (r_2) is the driver, which may rotate completely or may oscillate; link 3 $(r_3)$is called the coupler and link 4 $(r_4)$is follower or output link motion.

The motion or path of a trace point on a coupling link can expressed as a function of the dimensions of the mechanism and the angle of the input link (**Figure 5**).

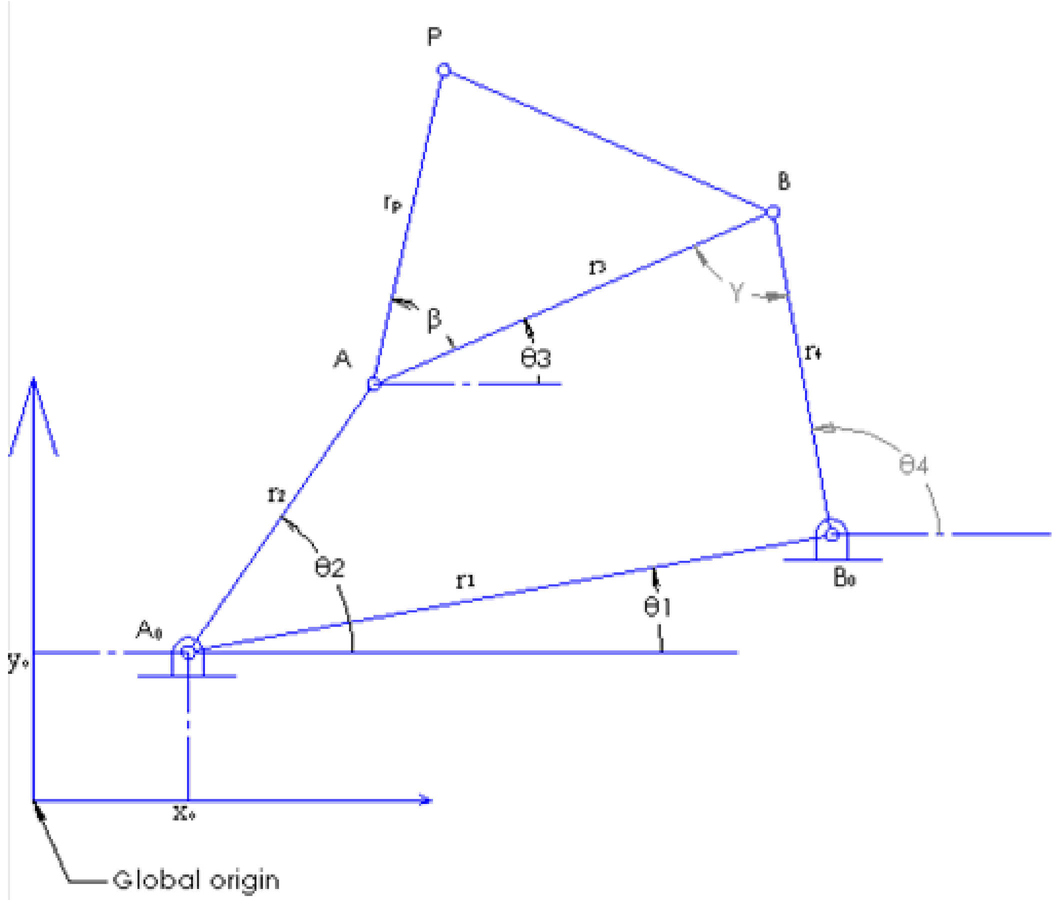


Figure 4: planar Four-bar mechanism variables

$$Path = F\,(r, \theta_2) \qquad [1.1]$$

$$r = (x_0,\ y_0, r_2\ , r_3, r_4, r_1, r_p, \theta_1, \beta)$$

Where θ_2 = Drive Crank Angle

x_0 = X Position of Drive Crank Pivot

y_0, = Y Position of Drive Crank Pivot

r_2 = Drive Crank Length

r_3 = Connecting Rod Length

r_4 = Follower Crank Length

r_1 = Ground Pivot Length

r_p= Trace Point Length

θ_1 = Ground Pivot Angle

β = Trace Point Angle

The following angles may easily derive during the analysis of the position and motion of the four-bar mechanism.

θ_3 = Connecting Rod Angle

θ_4 = Follower Crank Angle

For planar motion there are three independent variables (degrees of freedom) associated with each link in a plane: these can be taken as the X position of the center of gravity, the Y position of the center of gravity, and the angle of the link. The degrees of freedom (DOF) of a four bar linkage with four pin joints can found by applying **Kutzbach** Criterion:

$$\mathrm{DOF} = 3(\mathrm{n}-\mathrm{l}) - 2\times \mathrm{f}_1 - \mathrm{f}_2 \qquad [1.2]$$

n = Number of links,

f_1 = Sum of the number of pin joints, slider joints or pure rolling joints,

f_2 = Number of roll-slide joints

Therefore, the degrees of freedom of a four-bar planar mechanism are:

$$\mathrm{DOF} = 3(4-1) - 2\times 4 - 0 = 1 \qquad [1.3]$$

The importance of having one degree of freedom is that by defining the motion of one of the links, which is typically a link attached to ground (i.e. the drive crank) driven by a device (e.g. motor, solenoid, hydraulic cylinder, etc.), the motion of the entire mechanism is defined. When synthesizing mechanisms, the output function, path, or motion can solved based upon a specified input.

1.2. Problem Statement of study

Design a four bar Linkage which will move a path on its coupler link, Find the lengths and angles of the four links, the coupler link dimensions and determine of kinematic and dynamic conditions for the prescribed position. Additional Today, there is a need to reduce cost and save time for early stages of design namely for preliminary design. Therefore, a program, which synthesizes mechanism according to user inputs and does trial-error for numerous requirements and conditions, will be advantageous especially for pick and place industries products and loader that have lots of requirements and conditions.

1.3. Objective

1.3.1 General objective

The main objectives of this thesis are kinematic and dynamic analysis of Planar Four-Bar Linkage for Path Generation.

1.3.2. Specific objective

Four-Bar Mechanism that shown in ***Figure.1*** will be analysis where a coupler point desired to generate a given path defining the input prescribed point for coupler point is the first task for Kinematic and Dynamic Analysis in this thesis work. The desired path must be found and simplified with a suitable algebraic method to study the Motion, (velocity and acceleration) of the links in order to satisfy the kinematic requirements and detailed steps in formulating the Dynamic equation [17]

Formulation of Dynamic Force problem is the key to calculate the joint reactions, i.e., the constraint forces and moments due to the presence of kinematic joints that couple the links, involves length, velocity and acceleration calculations such as matrix of simultaneous equations for solving of the dynamic performance of a linkage. The reactions at joints have calculated using the Newtonian approach [16].

1.4. Methodology

1.4.1. Formulation of Kinematic and Dynamic Equations

In this thesis work, position of the four-bar mechanism with respect to the fixed frame is carried out for the planar four-bar mechanism to obtain path generation kinematic synthesis the construction parameters of mechanism. In order to define the velocity and acceleration of the coupler prescribed point of the planar four-bar linkage.

Dynamic formulation for linkages has presented for the determination of constraint forces and moments at the joints. Based on this analytical matrix formulation, the four-bar linkage has chosen to illustrate this approach. First, closed-loop linkage is opened by virtually cutting the joints of the independent closed loops. For example, four-bar linkage has only one closed-loop and opened by virtually cutting a joint Then, the uncoupled Newton–Euler equations of motion for the resulting open system written from the free-body diagrams in the Cartesian coordinates used to reduce the dimension of the equations motion.

This leads to a set of linear equations in terms of the constraint forces of the cut joint plus the driving torque for known motion of a mechanism. The number of constrained equations of motion depends on the number of unknown forces and moments associated with the cut-joints and driving torques in the opened system. These constrained equations solved first for the unknown forces and moments associated with the cut joints and driving forces (torques).

For a planar four-bar linkage, to find the values of linkage length, velocity or acceleration, and dynamic force analysis at various positon of the four bar mechanism MATLAB coding has written. to simplify the modeling process and simulate the desired path which generated with the coupler point CATIA V5R21 is used to accomplished the process in less time.

CHAPTER TWO

LITERATURE REVIEW

The previous work done in kinematic synthesis that is of interest in this thesis falls into two categories; namely, optimal synthesis and synthesis involving the dynamics and forces of the linkage. An overview of relevant works follows.

Tong, Y "Four-Bar Linkage Synthesis for A Combination of Motion and Path-Point Generation" This paper developed techniques to design planar four-bar linkages for tasks common to pick-and-place devices that include a combination of motion and path requirements. Geometric Constraint Programming (GCP) techniques used, Location and orientation triangles were constructed and constrained to satisfy the target positions and precision points. Numerical techniques also presented. Using continuation methods, solutions for the polynomial system of synthesis equations obtained for tasks that have a finite number of solutions. Iteration methods were also discussed for a combination of tasks that have a single parameter set of solutions Examples were presented that illustrate linkage synthesis for the 4-position, 2-point case, 4-position, 1-point case and the 3-position, 3-point case [1].

Galal Ali Hassaan "Synthesis of Planar Mechanisms, Part Iii: Four-Bar Mechanisms for Three Coupler-Positions Generation" this paper trying to approaching almost accurate and reliable in synthesizing 4-bar planar mechanisms for 3-specific positions of its coupler. The assumptions are only one dimension (r1) giving easy and straight forward design of the 4-bar mechanism. The coupler traces exactly the desired 3-positions and A MATLAB code is written to solve & satisfying for the output equations [2].

Wen-Yi Lin "A GA–DE ''Hybrid Evolutionary Algorithm for Path Synthesis of Four-Bar Linkage'' In This paper the proposed method is verified by studying four cases (for more than five target points) by using A new real-coded evolutionary algorithm for application to path synthesis of a four-bar linkage. Findings show that much more accurate solutions for three cases obtained with the method of A GA–DE hybrid evolutionary algorithm combining differential evolution (DE) with the real-valued genetic algorithm (RGA). Which called "GA–DE hybrid algorithm." applied to the problem of four-bar linkage synthesis additional Problem formulation start solving with Position equations of the tracing point [3].

H. Zhou, Kwun-Lon Ting ''Adjustable Slider–Crank Linkages for Multiple Path Generation'' in this paper Adjustable slider–crank linkages with a simple adjustment of the position of the slider guider multiple paths has generating A difficulty is to select the corresponding comparison points on the two desired and generated paths over a number of points. The optimal synthesis model is set up based on the position structural error of the slider guider introduced to reflect the overall difference between the desired and the generated paths [4].

H. Zhou a, Edmund H.M. Cheung '' Adjustable Four-Bar Linkages for Multi-Phase Motion Generation'' in this paper an optimal synthesis method of adjustable four-bar linkages for multi-phase motion generation is put forward. the position of a driven side-link fixed pivot is adjusted to generated multi-phase motion by the same four-bar linkage and The closed-form synthesis procedures of driving and driven side-links are presented through adjusting the position of the driven side-link fixed pivot to generated multi-phase motion with the same four-bar linkage. The closed-form synthesis solutions for the driving and driven side links have derived and results of two-synthesis example's [5].

Yu Hongying, Tang Dewei, Wang Zhixing ''Study on a New Computer Path Synthesis Method of A Four-Bar Linkage''. In this paper, a two-DOF (degree of freedom) auxiliary mechanism is adopted to transform coupler-points of the given path to a coupler-angle function curve The method effectively solves the approximate path synthesis problem for the more than 9 coupler-points given set of software is developed to realize the design and visualization of path synthesis for a four-bar linkage [6].

S.K. Acharyya M. Mandal '' Performance of EAs for Four-Bar Linkage Synthesis'' In this paper applies three evolutionary algorithms namely GA, PSO and DE to three cases of path synthesis of four-bar Mechanisms to minimizing the error between desired and obtained coupler curve. A comparative study has done among different algorithms to reach the goal functions. This observed that the DE shows fast convergence to the optimal result and very low error of adjustment to target points [7].

Monish P. Wasnik1, M. K. Son pimple, Undirwade ‘‘Optimal Synthesis of a Path Generator Linkage Using Non-Conventional Approach’’ This paper, studies the solution methods of optimal synthesis of a Path Generator Linkage using Non – Conventional Approach. by using Harmony Search Method and a common kind of goal function which is used to find the appropriate dimension and to minimize the error and find mechanism. With accurate Solution without consider input angle as design variable. Three cases are explained: (1) 6 points (15 variables) (2) 10 points (19 variables). The first case is a path-synthesized problem with given six target points arranged in a vertical line without prescribed timing [8].

Ren-Chung Soong * and Sun-Li Wu‘‘Design of Variable Coupler Curve Four-Bar Mechanisms’’ This paper presents a method for designing a variable coupler curve four-bar mechanism with one link replaced by an adjustable screw-nut link and driven by a servomotor. The different desired couple of curves can generate by controlling the angular displacement of the driving link and changing the length of the adjustable link [9].

Hong Zhou ‘‘Dimensional Synthesis of Adjustable Path Generation Linkages Using the Optimal Slider Adjustment’’ This paper, presents A precisely continuous desired path can be generated by an adjustable four-bar linkage. The linkage feasibility conditions and path generation flexibilities has analyzed with two demonstrated example the slider is used to adjust the pivot location of the driven side link. In which one is for adjustable crank-rocker linkage to generate a rectangular path, and another one is for adjustable double-crank linkage to generate a pentagon path. The synthesis model established in the paper based on the required optimal adjustment of the adjustable P joint when the desired continuous path is generated precisely. The global optimal synthesis solution searched using a real-coded GA [10].

Hong-Sen Yan, Ren-Chung Soong ‘‘Kinematic and Dynamic Design of Four-Bar Linkages by Links Counterweighing with Variable Input Speed’’ Integrates kinematic and dynamic design with variable input speeds is introduced. In this paper work a novel method for four-bar linkages, that satisfies kinematic design requirements and also attains trade-off of dynamic balance, by properly designing the speed trajectory of the input link, the disk counterweight of moving links, and link dimensions of the given or desired mechanisms, the expected output motion characteristics and dynamic balancing performance are obtained. Demonstrate the design procedures for this proposed approach with by four examples. The input motion characteristics designed with Bezier curves with undetermined control points [11].

N. Nariman-Zadeh, M. Felezi ‘’ Pareto Optimal Synthesis of Four-Bar Mechanisms for Path Generation’’ optimally design four-bar mechanisms hybrid multi-objective genetic algorithms (GAs) are used. It was demonstrated that two effective four-bar mechanisms in terms of both tracking error and transmission angle error could be found among the non-dominated Pareto fronts compromising those conflicting objective functions for Pareto optimum synthesis of four-bar linkages considering the minimization of two objective functions simultaneously. The important conflicting objective functions that have been considered in this work are tracking error (TE) and transmission angle’s deviation from 90 (TA). The obtained Pareto fronts demonstrate that trade-offs between these two objectives can be recognized so that a designer can optimally compromise for the selection of a desired four-bar linkage [12].

Mekonnen Gebreslasie and Alem Bazezew ‘’ Synthesis, Analysis and Simulation of a Four-Bar Mechanism Using MATLAB Programming’’ in this paper kinematic synthesis and analysis of four-bar mechanisms for the motion-generation problem of three and four precision points is presented. The dimensional synthesis is based on the complex number method approach while analysis of the motion characteristics is carried out by solution of a set of vector loop equations derived from the synthesized mechanism. Newton-Raphson iteration algorithm has used to solve the non-linear system of equations. In addition, A

MATLAB computer program written to implement the solutions to problems described [13].

Gordon R. Pennock, Ali Israr ‘’ Kinematic Analysis and Synthesis of an Adjustable Six-Bar Linkage’’ This paper investigates the kinematics of an adjustable six-bar planar linkage used as a variable-speed transmission mechanism where the input crank rotates at a constant speed and the output link consists of an overrunning clutch mounted on the output shaft. The analysis uses a novel technique in which kinematic coefficients are obtained with respect to an independent variable. Then kinematic inversion is used to express the kinematic coefficients with respect to the input variable of the linkage. This technique decouples the position equations and provides additional insight into the geometry of the adjustable linkage. A control arm can adjust the angle that the output link oscillates, for each revolution of the input crank. The paper shows how to determine the angle of oscillation of the output link for a specified position of the fixed pivot and investigates the extreme positions of the output link corresponding to the extreme positions of a point on the coupler link. For this reason, the paper includes a study of the geometry of the path traced by a coupler point and determines the location of the ground pivot of the control arm, which will cause the output link to remain stationary during a complete rotation of the input crank [14].

H. Zhou, Edmund H.M. Cheung ‘‘Analysis and Optimal Synthesis of Adjustable Linkages for Path Generation’’ In this paper, through adjusting the position of the driven side-link fixed pivot to generate different paths with four-bar linkage Structural error of the driven side-link and optimal synthesis model is set up based on the link-length. difference between desired and the generated paths, can avoid difficulty by selecting corresponding comparison points on the two paths [15].

Himanshu Chaudhary, S. Kumar Saha, ‘‘Balancing of Four-Bar Linkages Using Maximum Recursive Dynamic Algorithm’’ This paper presents dynamic balancing of four-bar linkages and solving methodology based on the maximum reclusiveness of the dynamic equations for the evaluation of bearing forces [16].

Mehmet ismet Can Dede, Duygu Comen, ‘‘Kinematic Synthesis of Path Generation of Planer Mechanism’’ In this paper Kinematic synthesis of planar mechanisms is studied. Path generation method applied to generate planar four-bar mechanism. in which a coupler point is desired to generate a given path. Independent loop closure equations written for each constraint of the mechanism to obtain the objective functions and nonlinear systems of equations are solved to finding the construction parameters of the mechanism, which satisfies the desired path [17].

Radovan R. Bulatovic ,Stevan R. Djordjevic, ‘‘Optimal Synthesis of a Four-Bar Linkage by Method of Controlled Deviation’’ This paper describes the procedure of optimal synthesis of a four-bar linkage by the method of controlled deviations, the coupler point in relation to the projected path can be followed at any moment. The method is illustrated on the example of rectilinear motion of the coupler point, although the method applied when the path of the point is any algebraic curve [18].

Han Jianyou, Qian Weixiang, Zhao Huishe, ‘‘Study On Synthesis Method of Λ-Formed 4-Bar Linkages Approximating A Straight Line’’ This paper presents an analytical synthesis of 4-bar linkages approximating to generate coupler curve with approximate straight line through three pairs of coincident points using displacement–matrix method. Results of given examples are proved that the relative formulas are correct [19].

CHAPTER THREE

FOUR-BAR MECHANISM AND ITS CLASSIFICATIONS

An important property of a classification system would be the aid it could furnish a designer in finding the forms and arrangements best suited to satisfying certain conditions. The planar four- bar mechanism, which consists of four pin-connected rigid links gains its importance as a basic mechanism because it is one of the simplest of all mechanisms to produce. The four-bar linkage derives its renown from the fact that the members of a three bar linkage are incapable of relative motion and a linkage composed of more than four bars has indeterminate motion with a single input. Though it may assume many forms, often with little resemblance to the usual representation, a four-bar linkage consists of two members in pure rotation about fixed axes, called the driving and follower crank; a coupler in combined motion, which joins the moving ends of the cranks; and a fixed frame, which establishes the relative position of the stationary crank centers.

3.1 Chebyshev Linkage

Chebyshev Linkage is a four-bar double rocker mechanism where the coupler rotates through 360. Below Figure shows a schematic diagram of the Chebyshev linkage. In Chebyshev's Linkage, either the crank or the coupler can use as the input link. The output obtained from the middle of the coupler. The limiting positions of the crank in relation to the frame for this mechanism are shown. in ***Figure 5 &6.***

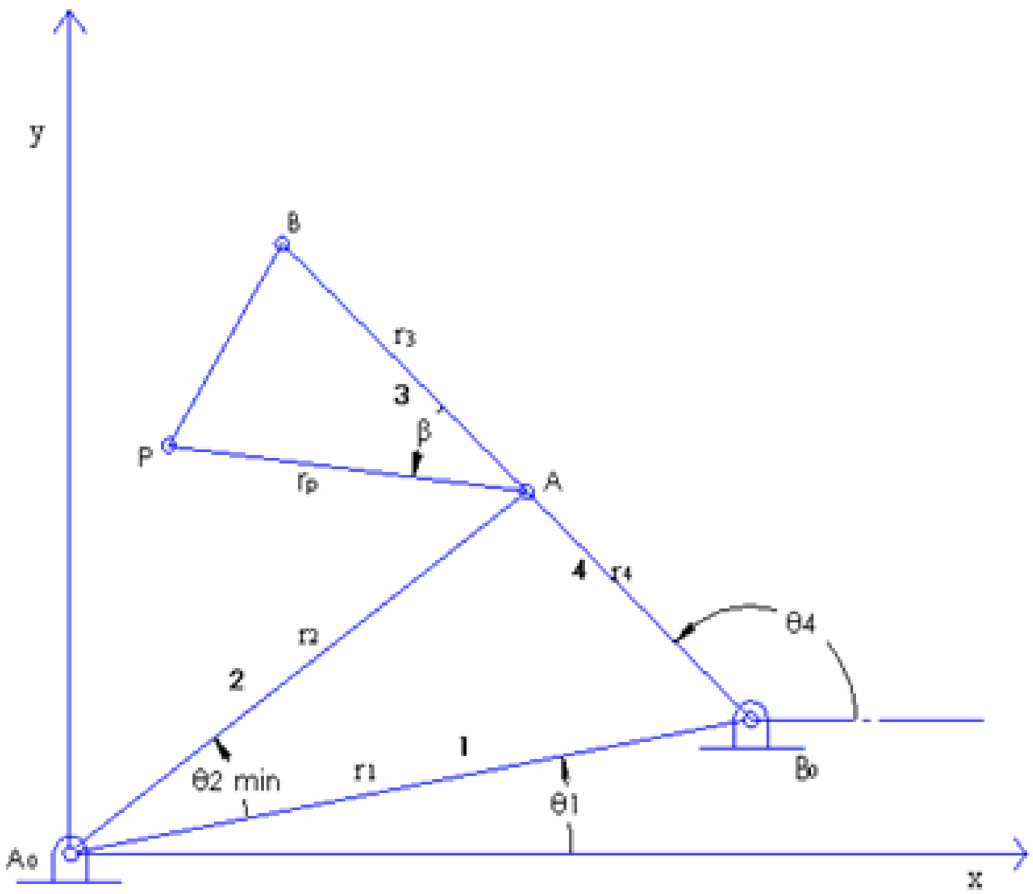


Figure 5: Limiting Angle of Chebyshev Linkage

Limiting Angle of Chebyshev Linkage

By cosine law: for $\theta_1 = 0$

$$(r_4 - r_3)^2 = {r_2}^2 + {r_1}^2 - 2r_2 r_1 cos(\theta_{min})$$

$$\theta_{min} = \cos^{-1}\{({r_2}^2 + {r_1}^2 - (r_4 - r_3)^2)\ /2r_2 r_1\ \}$$

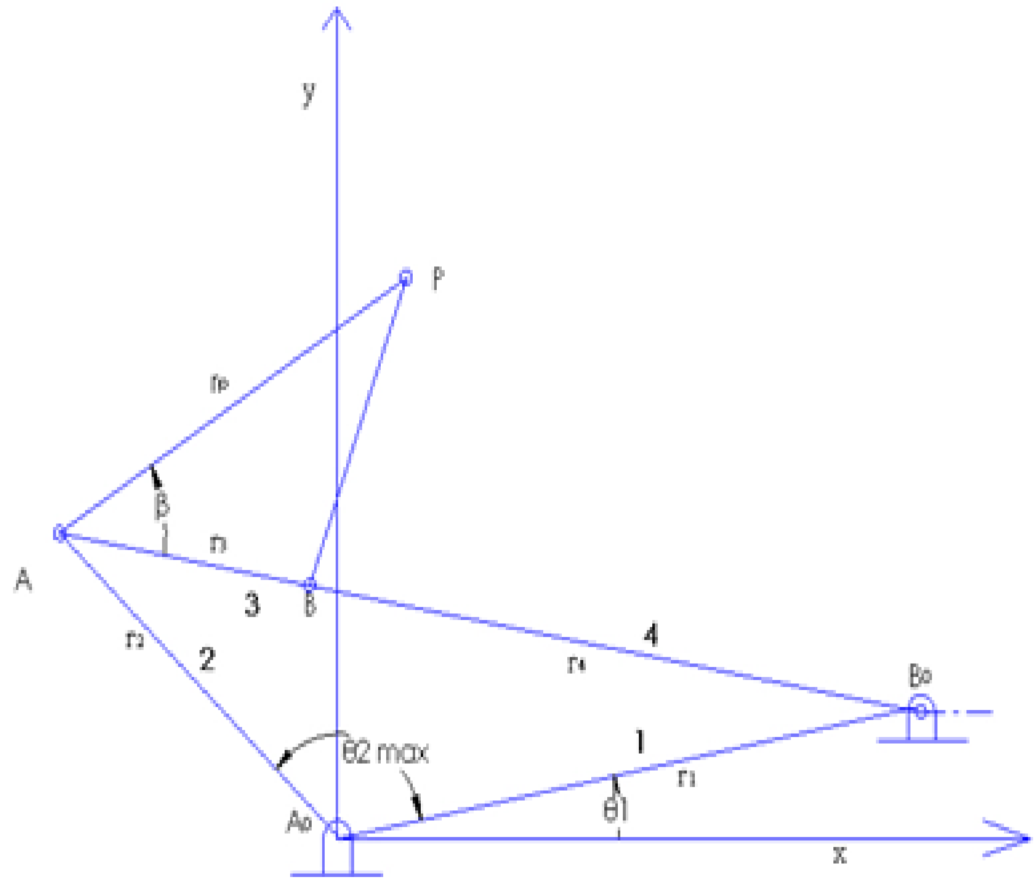


Figure 6: Another Limiting Angle of Chebyshev Linkage

$$(r_4 + r_3)^2 = {r_2}^2 + {r_1}^2 - 2 * r_2 r_1 cos(\theta_{max})$$

$$\theta_{max} = \cos^{-1}\{({r_2}^2 + {r_1}^2 - (r_4 + r_3)^2) / 2r_2 r_1 \}$$

3.2 Grashof Criteria

Classifications and the Grashof Criteria

There are two main classes of four bar mechanisms based on the rotational and dimensional limitations of its links called Grashof's criterion, which are:

Grashof Linkages, which is comprised of:

- crank rocker mechanism
- double rocker mechanism
- Rocker-Crank (drag link) mechanism
- Crank-Crank (crossover-position or change point) mechanism

Non-Grashof Linkages, which includes

Double rocker mechanisms (second kind triple rocker mechanisms).

- Inward/Inward Triple rocker
- Outward/Inward Triple rocker
- Inward/Outward Triple rocker
- Outward/Outward Triple rocker

3.2.1 Grashof Linkages

A Grashof linkage is a planar four-bar linkage with (see fig: 5 for $\theta_1 = 0$)

$$s + l \leq p + q$$

Where

s = length of the shortest link

l = length of longest link

p & *q* = lengths of the two remaining links.

For linkages of this type continuous relative motion between the shortest link and its adjacent links is possible.

1. **Crank-Rocker**

This type of Grashof linkage obtained when the shortest link is the input link(r_2). The input link has full motion. The output link has a limited range of motion that defined as follows:

Lower limit: $\theta_4 = 180 + \theta_1 - \theta_4''$

Upper limit: $\theta_4 = 180 + \theta_1 - \theta_4'$

Where

$$\theta_{min} = \theta_4{}' = \text{acos}\left[\frac{(r_1{}^2 + r_4{}^2 - (r_3 - r_2)^2)}{(2 \times r_1 \times r_4)}\right]$$

And

$$\theta_{max} = \theta_4{}'' = \text{acos}\left[\frac{(r_1{}^2 + r_4{}^2 - (r_3 + r_2)^2)}{(2 \times r_1 \times r_4)}\right]$$

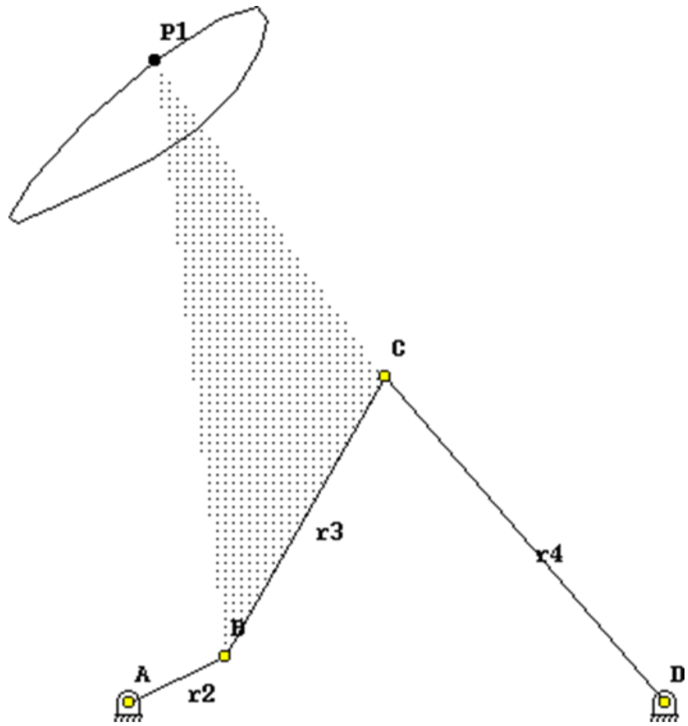


Figure 7: Grashof Crank-Rocker

2. **Double Rocker**

This type of Grashof linkage is obtained when the shortest link is the floating link (r_3). Note that the complete relative motion between the shortest link and its adjacent links are still possible. Both the input and output links have limited ranges of motion that defined as follows:

Input link

Lower limit: $\theta_2 = \theta_1 + \theta_2'$

Upper limit: $\theta_2 = \theta_1 + \theta_2''$

Where

$$\theta_{min} = \theta_2' = \text{acos}\left[\frac{(r_1^2 + r_2^2 - (r_3 - r_4)^2)}{(2 \times r_1 \times r_4)}\right]$$

And

$$\theta_{max} = \theta_2'' = \text{acos}\left[\frac{(r_1^2 + r_2^2 - (r_3 + r_4)^2)}{(2 \times r_1 \times r_4)}\right]$$

Output Link

Lower limit: $\theta_4 = 180 + \theta_1 - \theta_4''$

Upper limit: $\theta_4 = 180 + \theta_1 - \theta_4'$

Where

$$\theta_4' = \operatorname{acos}\left[\frac{(r_1^2 + r_4^2 - (r_3 - r_2)^2)}{(2 \times r_1 \times r_4)}\right]$$

and

$$\theta_4'' = \operatorname{acos}\left[\frac{(r_1^2 + r_4^2 - (r_3 + r_2)^2)}{(2 \times r_1 \times r_4)}\right]$$

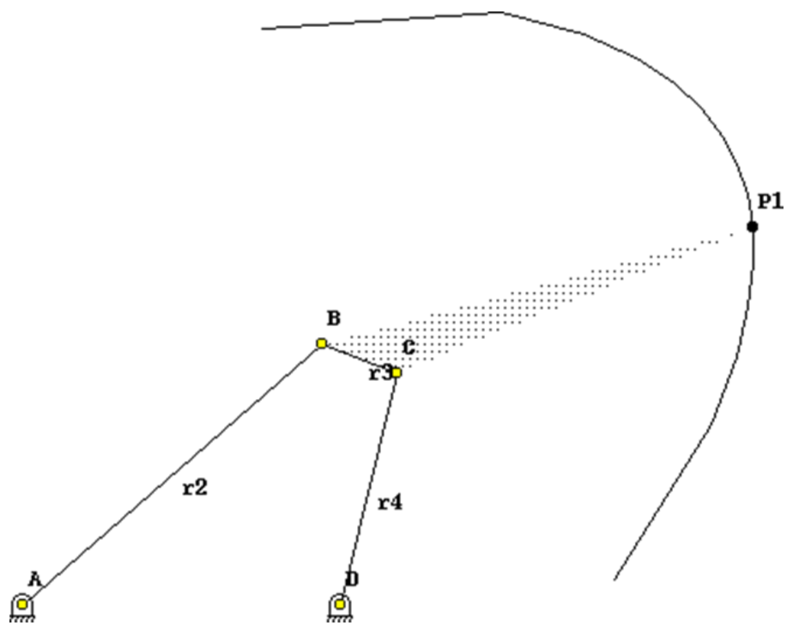


Figure 8: Grashof Double Rocker

3. **Rocker-Crank**

This type of Grashof linkage obtained when the shortest link is the output link (r4). The output link has full motion. The input link has a limited range of motion that defined as follows:

Lower limit: $\theta_2 = \theta_1 - \theta_2'$

Upper limit: $\theta_2 = \theta_1 - \theta_2''$

Where

$$\theta_2{}' = \text{acos}\left[\frac{(r_1{}^2 + r_2{}^2 - (r_3 - r_2)^2)}{(2 \times r_1 \times r_4)}\right]$$

and

$$\theta_2{}'' = \text{acos}\left[\frac{(r_1{}^2 + r_2{}^2 - (r_3 + r_2)^2)}{(2 \times r_1 \times r_4)}\right]$$

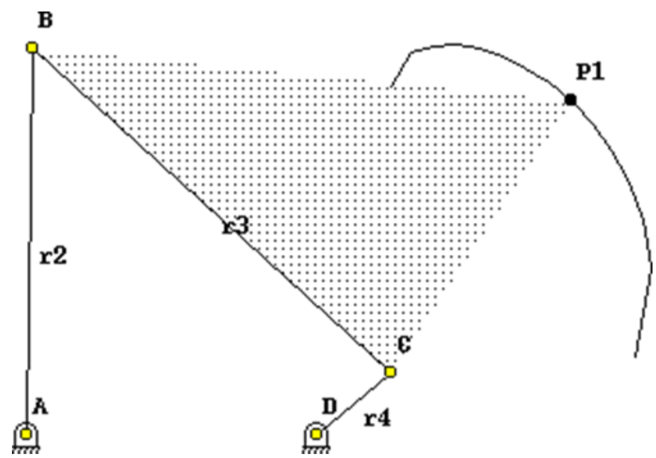


Figure 9: Grashof Rocker-Crank

4. Crank-Crank

This type of Grashof linkage obtained when the shortest link is the ground link(r_1). Both the input and output links have full motion. **Fig:** (10)

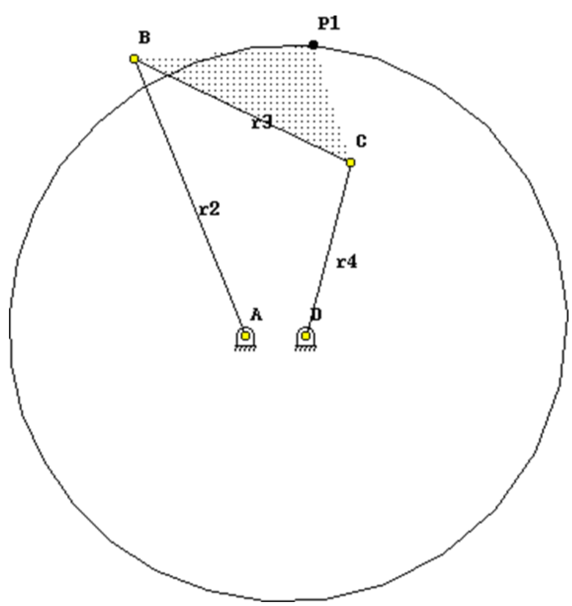


Figure 10: Grashof Crank-Crank

3.2.2 Non-Grashof Linkages

A non-Grashof linkage is a planar four-bar linkage with

$$s + l > p + q$$

Where

s = length of the shortest link

l = length of longest link

P and *q* = lengths of the two remaining links.

For linkages of this type continuous relative motion between any, two of its links is not possible.

1. Inward/Inward Triple rocker: input inward limited, output inward limited

When

$$\mathrm{r}_1 + \mathrm{r}_2 < \mathrm{r}_3 + \mathrm{r}_4$$

The input link (r_2) of the linkage is said to be inward limited.

When

$$\mathrm{r}_1 + \mathrm{r}_2 < \mathrm{r}_3 + \mathrm{r}_4$$

The input link (r_2) of the linkage is said to be inward limited.

When

$$\mathrm{r}_1 + \mathrm{r}_4 < \mathrm{r}_2 + \mathrm{r}_3$$

The output link (r_4) of the linkage is said to be inward limited.

When a link is inward limited there are limits on the possible values of the angle of the link that the actual angle must remain outside (the link is limited when moving in the inward direction).

The limits angle defined as follows.

Input Link

Lower limit $\theta_2 = \theta_1 - \theta_2{}'$

Upper limit: $\theta_2 = \theta_1 + 360 - \theta_2{}'$

Where

$$\theta_2{}' = \text{acos}\left[\frac{(r_1{}^2 + r_2{}^2 - (r_4 - r_3)^2)}{(2 \times r_1 \times r_2)}\right]$$

Output Link

Lower limit: $\theta_4 = \theta_1 - 180 + \theta_4{}'$

Upper limit: $\theta_4 = \theta_1 + 180 - \theta_4{}'$

Where

$$\theta_4{}' = \text{acos}\left[\frac{(r_1{}^2 + r_4{}^2 - (r_2 - r_3)^2)}{(2 \times r_1 \times r_4)}\right]$$

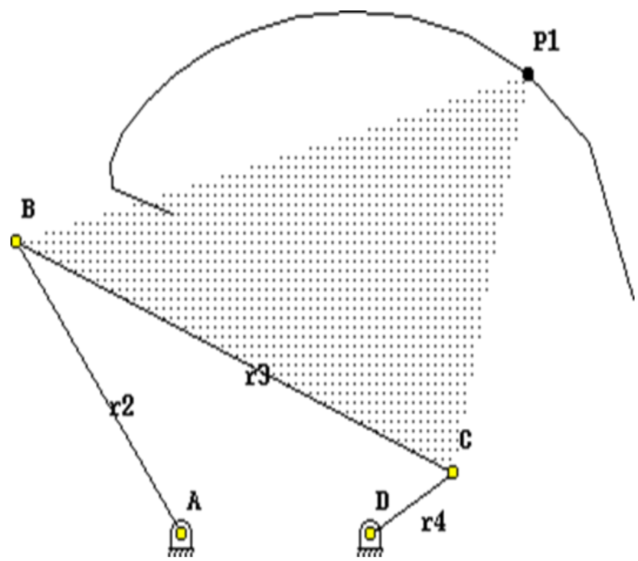


Figure 11: Non-Grashof Inward/Inward Limited Triple Rocker

2. **Outward/Inward** Triple rocker: input outward limited, output inward limited

When

$$r_1 + r_2 < r_3 + r_4$$

The input link (r_2) of the linkage is said to be outward limited.

When

$$r_1 + r_4 < r_2 + r_3$$

The output link (r_4) of the linkage is said to be inward limited.

When a link is outward limited there are limits on the possible values of the angle of the link that the actual angle must remain inside (the link is limited when moving in the outward direction).

When a link is inward limited there are limits on the possible values of the angle of the link that the actual angle must remain outside (the link is limited when moving in the inward direction).

These limits angle defined as follows.

Input Link

Lower limit: $\theta_2 = \theta_1 - \theta_2''$

Upper limit: $\theta_2 = \theta_1 + \theta_2''$

Where

$$\theta_2'' = \operatorname{acos}\left[\frac{(r_1^2 + r_2^2 - (r_4 + r_3)^2)}{(2 \times r_1 \times r_2)}\right]$$

Output Link

Lower limit: $\theta_4 = \theta_1 - 180 + \theta_4'$

Upper limit: $\theta_4 = \theta_1 + 180 - \theta_4'$

Where

$$\theta_4'' = \operatorname{acos}\left[\frac{(r_1^2 + r_4^2 - (r_2 - r_3)^2)}{(2 \times r_1 \times r_4)}\right]$$

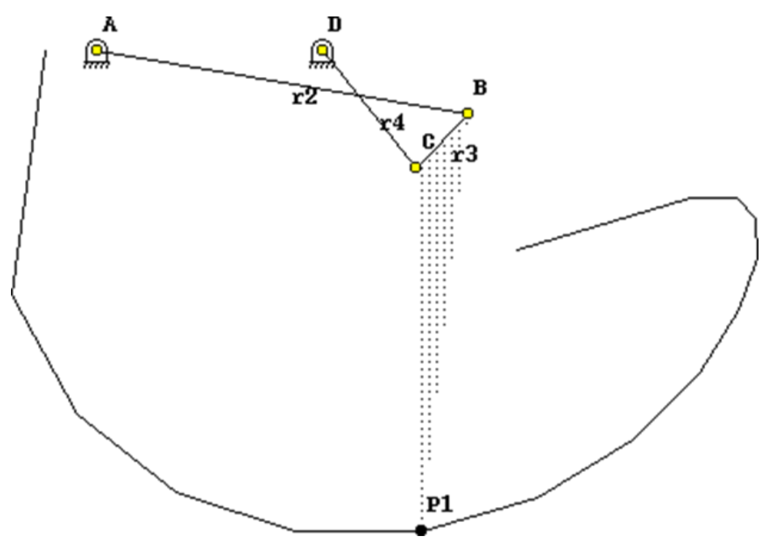


Figure 12: Non-Grashof Outward/Inward Limited Triple Rocker

3. **Inward/Outward** Triple rocker: input inward limited, output outward limited

When

$$r_1 + r_2 < r_3 + r_4$$

the input link (r_2) of the linkage is said to be inward limited.

When

$$r_1 + r_4 \geq r_2 + r_3$$

The output link (r_4) of the linkage is said to be outward limited.

When a link is inward limited there are limits on the possible values of the angle of the link that the actual angle must remain outside (the link is limited when moving in the inward direction).

When a link is outward limited there are limits on the possible values of the angle of the link that the actual angle must remain inside (the link is limited when moving in the outward direction).

These limits angle defined as follows.

Input Link

Lower limit: $\theta_2 = \theta_1 + \theta_2{}'$

Upper limit: $\theta_4 = \theta_1 + 180 + \theta_4{}''$

Where

$$\theta_2{}' = \operatorname{acos}\left[\frac{(r_1{}^2 + r_2{}^2 - (r_4 - r_3)^2)}{(2 \times r_1 \times r_2)}\right]$$

Output Link

Lower limit: $\theta_4 = \theta_1 + 180 - \theta_4{}''$

Upper limit: $\theta_4 = \theta_1 + 180 + \theta_4{}''$

Where

$$\theta_4{}'' = \operatorname{acos}\left[\frac{(r_1{}^2 + r_4{}^2 - (r_2 + r_3)^2)}{(2 \times r_1 \times r_4)}\right]$$

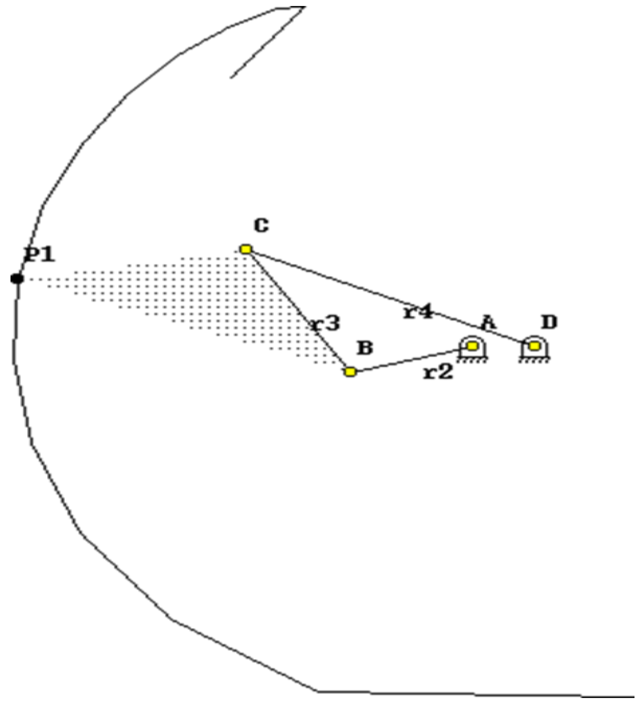


Figure 13: Non-Grashof Inward/Outward Limited Triple Rocker

4. **Outward/Outward** Triple rocker: input outward limited, output outward limited

When

$$r_1 + r_2 \geq r_3 + r_4$$

The input link (r_2) of the linkage is said to be outward limited.

When

$$r_1 + r_4 \geq r_2 + r_3$$

The output link (r_4) of the linkage is said to be outward limited.

When a link is outward limited there are limits on the possible values of the angle of the link that the actual angle must remain inside (the link is limited when moving in the outward direction).

These limits angle defined as follows.

Input Link

Lower limit: $\theta_2 = \theta_1 - \theta_2''$

Upper limit: $\theta_2 = \theta_1 + \theta_2''$

Where

$$\theta_2'' = \text{acos}\left[\frac{(\mathrm{r_1}^2 + \mathrm{r_2}^2 - (\mathrm{r_4} + \mathrm{r_3})^2)}{(2 \times \mathrm{r_1} \times \mathrm{r_2})}\right]$$

Output Link

Lower limit: $\theta_4 = \theta_1 + 180 - \theta_4''$

Upper limit: $\theta_4 = \theta_1 + 180 + \theta_4''$

Where

$$\theta_4'' = \text{acos}\left[\frac{(\mathrm{r_1}^2 + \mathrm{r_4}^2 - (\mathrm{r_2} + \mathrm{r_3})^2)}{(2 \times \mathrm{r_1} \times \mathrm{r_4})}\right]$$

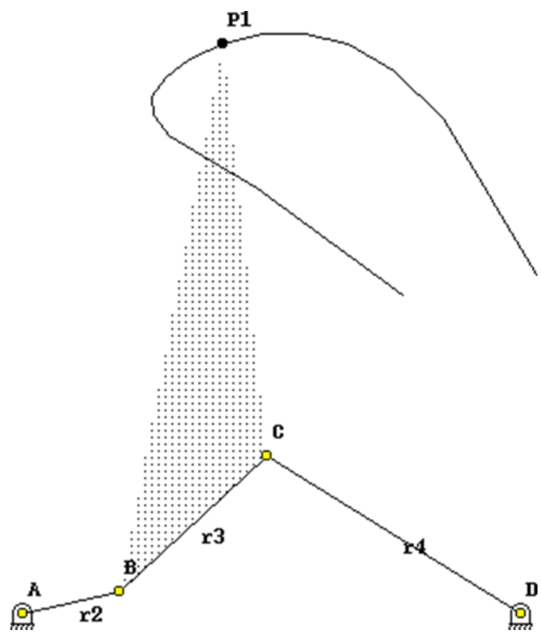


Figure 14: Non-Grashof Outward/Outward Limited Triple Rocker

S shortest link l ***l =*** longest link

p and ***q*** links of intermediate links

Table 1: General Classification of Mechanism

Type of Mechanism	Shortest Link	Relationship Between Links
GRASHOF	Any	$s + l \le p + q$
Crank rocker	Driver crank	$s + l < p + q$
Drag link	Fixed link	$s + l < p + q$
Double-rocker	Coupler	$s + l < p + q$
Crossover-position	Any	$s + l = p + q$
NON-GRASHOF		
Triple-rocker	Any	$s + l > p + q$

CHAPTER FOUR

KINEMATIC ANALYSIS OF FOUR BAR MECHANISMS

In the design of mechanisms, one must know the direction of motion, velocity, accelerations .and the forces of each machine member at various positions during the operating cycle. When this information is available, the designer can then select the optimum size, material, and design the machine members to meet the specifications. The needed information has obtained from the kinematic analysis of the motion of the machine members.

This work deals with the analysis of four-bar linkage mechanisms that have planar rigid-body motion. Based on an evaluation of a defined mechanism's geometry kinematic synthesis is the process of designing a mechanism to accomplish a desired task.

4.1 Position Analysis of Planar Four-bar Mechanisms

The connecting rod of a four-bar mechanism ls also known as the Coupler, and the points belonging to it are called Coupler points.

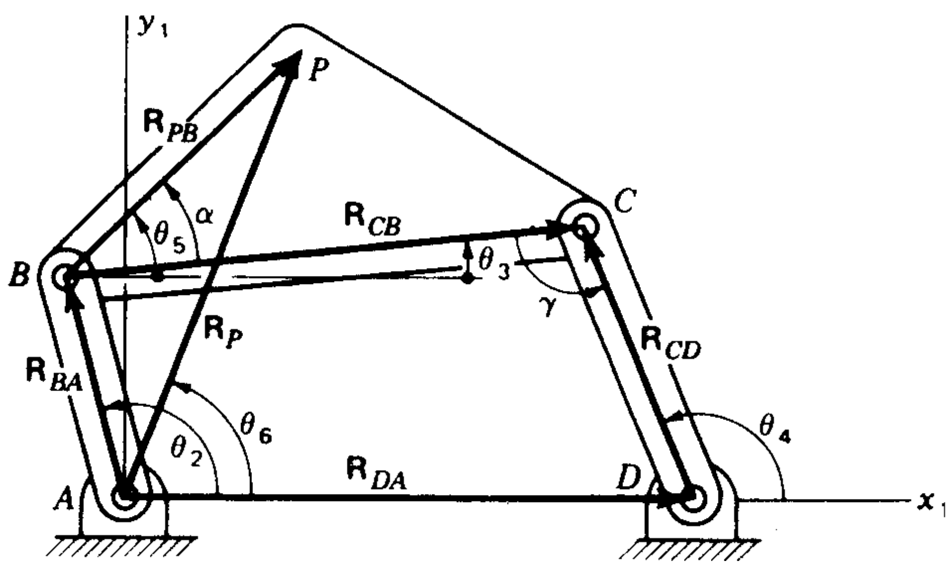


Figure 15: Four-bar linkage

Consider the four-bar linkage shown in **Fig. 16**. Here we may wish to find the position of the **coupler point P**, which corresponds to some given crank angle $\boldsymbol{\theta_2}$ the loop-closure equation is

$$R_{BA} + R_{CB} = R_{DA} + R_{CD} \qquad 4.1$$

In addition, the position of point **P** given by the position-difference equation

$$R_P = R_{BA} + R_{PB} \qquad 4.2$$

Although it appears that this equation has three unknowns, it can be reduced to two once the loop-closure equation (4.1) is solved by noticing the constant angular relationship between $\boldsymbol{R_{PB}}$ and$\boldsymbol{R_{CB}}$.

$$\boldsymbol{\theta_5 = \theta_3 + \alpha} \qquad 4.3$$

For *i =1, 2, 3*

$$\boldsymbol{\theta_{5i} = \theta_{3i} + \alpha}$$

The graphical solution for this problem is started by combining the two known terms of Eq. (4.1), thus locating the positions of points **B** and **D** as shown in **Fig. 2-14**

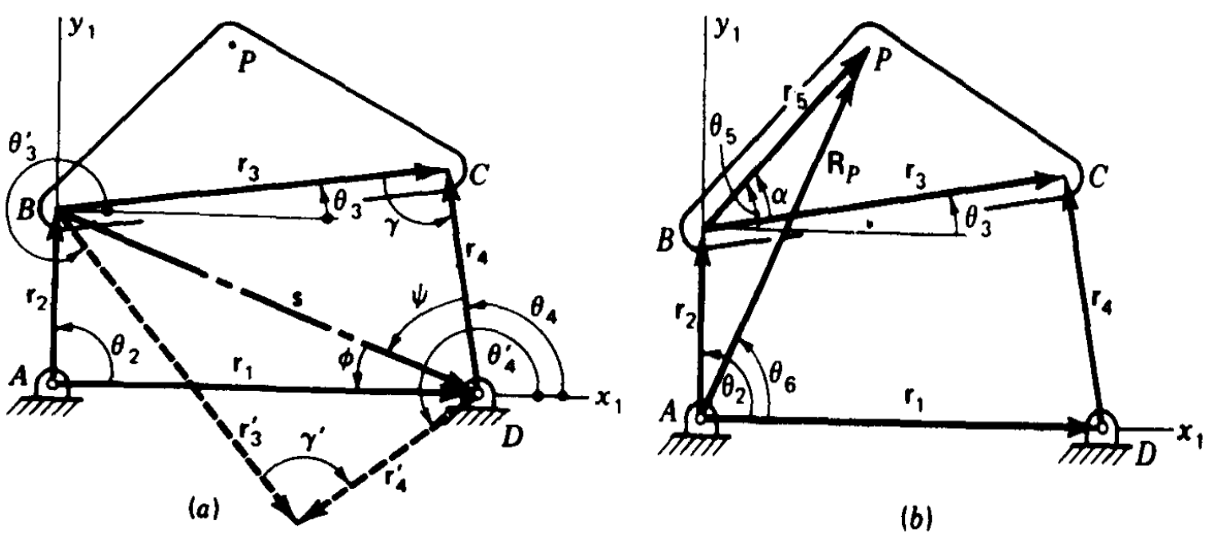


Figure 16: position analysis of the four-bar linkage

$$S = R_{DA} - R_{BA} = R_{CB} - R_{CD} \qquad 4.4$$

The analysis of the four-bar linkage is a classic problem with solution dating back over a century Note from the figure that s is the diagonal distance **BD**. The law of cosines can be written for the triangle **BAD** and again for the triangle **BCD**. In terms of the link lengths and angles defined in the figure, we have

$$S = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2} \qquad 4.5$$

$$S_i = \sqrt{r_{1i}^2 + r_{2i}^2 - 2r_{1i} r_{2i} \cos\theta_{2i}}$$

$$\gamma = \pm\cos^{-1}\frac{r_3^2 + r_4^2 - S^2}{2r_3 r_4} \qquad 4.6$$

$$\gamma_i = \pm\cos^{-1}\frac{r_{3i}^2 + r_{4i}^2 - S_i^2}{2r_{3i} r_{4i}}$$

Where, the plus-or-minus signs refer to the two solutions of the transmission angle $\boldsymbol{\gamma}$ and $\boldsymbol{\gamma'}$ respectively. The law of cosines can be written again for the same two triangles to find the angles $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$

$$\phi = \cos^{-1}\frac{r_1^2 + S^2 - r_2^2}{2r_1 S} \qquad 4.7$$

$$\phi_i = \cos^{-1}\frac{r_{1i}^2 + S_i^2 - r_{2i}^2}{2r_1 S_i}$$

$$\psi = \cos^{-1}\frac{r_4^2 + S^2 - r_3^2}{2r_4 S} \qquad 4.8$$

$$\psi_i = \cos^{-1}\frac{r_{4i}^2 + S_i^2 - r_{3i}^2}{2r_{4i} S_i}$$

Where it is noted from the figure that the magnitudes of $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$ are both less than 180^0 and that $\boldsymbol{\psi}$ is always positive while $\mathbf{sin}\,\boldsymbol{\phi}$ is of the same sign as $\mathbf{sin}\,\boldsymbol{\theta_2}$ From these we find the unknown angles $\boldsymbol{\theta_3}$ and $\boldsymbol{\theta_4}$**.**

$$\theta_4 = 180^0 - \phi \mp \psi \qquad 4.9$$

$$\theta_{4i} = 180^0 - \phi_i \mp \psi_i$$

$$\theta_3 = \theta_4 - \gamma \qquad 4.10$$

$$\theta_{3i} = \theta_{4i} - \gamma_i$$

We start by writing the loop closure equation in complex polar form. Using the notation of **Fig. 17**, we have

$$\mathrm{r}_2 e^{j\theta_2} + \mathrm{r}_3 e^{j\theta_3} = \mathrm{r}_1 e^{j\theta_1} + \mathrm{r}_4 e^{j\theta_4} \qquad 4.11$$

$$\mathrm{r}_{2i} e^{j\theta_{2i}} + \mathrm{r}_{3i} e^{j\theta_{3i}} = \mathrm{r}_{1i} e^{j\theta_{1i}} + \mathrm{r}_{4i} e^{j\theta_{4i}}$$

Where x_i chosen as the real axis. Using Euler's formula, we separate the real and imaginary parts of the equation

$$\mathrm{r}_2 \cos\theta_2 + \mathrm{r}_3 \cos\theta_3 = \mathrm{r}_1 + \mathrm{r}_4 \cos\theta_4 \qquad 4.12$$

$$\mathrm{r}_{2i} \cos\theta_{2i} + \mathrm{r}_{3i} \cos\theta_{3i} = \mathrm{r}_{1i} + \mathrm{r}_{4i} \cos\theta_{4i}$$

$$\mathrm{r}_2 \sin\theta_2 + \mathrm{r}_3 \sin\theta_3 = \mathrm{r}_4 \sin\theta_4 \qquad 4,13$$

$$\mathrm{r}_{2i} \sin\theta_{2i} + \mathrm{r}_{3i} \sin\theta_{3i} = \mathrm{r}_{4i} \sin\theta_{4i}$$

Where, angles $\boldsymbol{\theta_3}$ and $\boldsymbol{\theta_4}$ are the two unknowns. Next, we rearrange these equations to isolate the $\boldsymbol{\theta_3}$ terms

$$\mathrm{r}_{3i} \cos\theta_{3i} = \mathrm{r}_{1i} + \mathrm{r}_{4i} \cos\theta_{4i} - \mathrm{r}_{2i} \cos\theta_{2i}$$

$$\mathrm{r}_{3i} \sin\theta_{3i} = \mathrm{r}_{4i} \sin\theta_{4i} - \mathrm{r}_{2i} \sin\theta_{2i}$$

Square and add the two equations

$$\mathrm{r}_{3i}^2 = \mathrm{r}_{4i}^2 + \mathrm{r}_{2i}^2 + \mathrm{r}_{1i}^2 + 2\mathrm{r}_{1i}\mathrm{r}_{4i} \cos\theta_{4i} - 2\mathrm{r}_{1i}\mathrm{r}_{2i} \cos\theta_{2i} - 2\mathrm{r}_{2i}\mathrm{r}_{4i} \cos(\theta_{4i} - \theta_{2i}) \qquad 4.14$$

This eliminating the unknown $\boldsymbol{\theta_3}$

We can combine a ·number of the known quantities of this equation and reduce its complexity by noting from the figure that

$$S^{xi} = \mathrm{r}_{1i} - \mathrm{r}_{2i} \cos\theta_{2i} \qquad 4.15$$

$$S^{yi} = -\mathrm{r}_{2i} \sin\theta_{2i} \qquad 4.16$$

$$\gamma_i = \cos^{-1}\frac{\mathrm{r}_{3i}^2 + \mathrm{r}_{4i}^2 - \mathrm{r}_{1i}^2 - \mathrm{r}_{2i}^2 + 2\mathrm{r}_{1i}\mathrm{r}_{2i}\cos\theta_{2i}}{2\mathrm{r}_{3i}\mathrm{r}_{4i}} \qquad 4.17$$

where this last equation is equivalent to Eqn. (4.5) and (4.6) above. After making these substitutions and rearranging, Eq. (4.14) reduces to

$$S^{xi}\cos\theta_{4i} + S^{yi}\sin\theta_{4i} - \mathrm{r}_{3i}\cos\gamma_i + \mathrm{r}_{4i} = 0 \qquad 4.18$$

When dealing with both sine and cosine of the same unknown angle in a single equation, it is sometimes helpful to substitute the half-angle identities from trigonometry

$$\cos\theta_{4i} = \frac{1 - \tan^2(\theta_{4i}/2)}{1 + \tan^2(\theta_{4i}/2)} \qquad \sin\eta = \frac{2\tan(\theta_{4i}/2)}{1 + \tan^2(\theta_{4i}/2)} \qquad 4.19$$

Substituting these into Eq. (4.18), clearing fractions, and rearranging terms, we obtain a quadratic equation

$$(\mathrm{r}_{4i} - \mathrm{r}_{3i}\cos\gamma_i - S^{xi})\tan^2\frac{\theta_{4i}}{2} + 2\,S^{yi}\tan\frac{\theta_{4i}}{2} + (\mathrm{r}_{4i} - \mathrm{r}_{3i}\cos\gamma_i + S^{xi}) = 0 \qquad 4.20$$

From which we obtain two solutions

$$\tan\frac{\theta_{4i}}{2} = \frac{-\,S^{yi} \mp \sqrt{(\,S^{yi})^2 - \mathrm{r}_{4i}^2 + 2\mathrm{r}_{3i}\mathrm{r}_{4i}\cos\gamma_i - \mathrm{r}_{3i}^2\cos^2\gamma_i + (S^{xi})^2}}{\mathrm{r}_{4i} - \mathrm{r}_{3i}\cos\gamma_i - S^{xi}} \qquad 4.21$$

When we substitute from Eqn. (4.15), (4.16) and (4.17), this reduces to

$$\tan\frac{\theta_{4i}}{2} = \frac{-\,S^{yi} \mp \mathrm{r}_3\sqrt{1 - \cos^2\gamma_i}}{\mathrm{r}_{4i} - \mathrm{r}_{3i}\cos\gamma_i - S^{xi}} \qquad 4.22$$

Therefore

$$\theta_{4i} = 2\tan^{-1}\frac{\mathrm{r}_{2i}\sin\theta_{2i} \mp \mathrm{r}_{3i}\sin^2\gamma_i}{\mathrm{r}_{4i} - \mathrm{r}_{3i}\cos\gamma_i - \mathrm{r}_{1i} - \mathrm{r}_{2i}\cos\theta_{2i}} \qquad 4.23$$

The solution for the other unknown, the angle θ_3, can found by a completely analogous procedure. Isolating the θ_4 terms of Eqn. (o) and (p) the same procedure as before squaring and adding eliminates θ_4 and leaves a quadratic equation which can be solved for $\boldsymbol{\theta_3}$ the solution is

$$\theta_{3i} = 2\tan^{-1}\frac{-\mathrm{r}_{2i}\sin\theta_{2i} \pm \mathrm{r}_{4i}\sin^2\gamma_i}{\mathrm{r}_{3i} - \mathrm{r}_{4i}\cos\gamma_i + \mathrm{r}_{1i} - \mathrm{r}_{2i}\cos\theta_{2i}} \qquad 4.24$$

Having solved the basic four-bar linkage, we now seek an expression for the position of the coupler point P. From Fig. 17, in complex polar notation we write

$$\mathrm{R}_p = \mathrm{R}_P e^{i\theta_6} = \mathrm{r}_2 e^{i\theta_2} + \mathrm{r}_5 e^{i(\theta_3+\alpha)} \qquad 4.25$$

$$\mathrm{R}_{pi} = \mathrm{R}_{Pi} e^{i\theta_{6i}} = \mathrm{r}_2 e^{i\theta_{2i}} + \mathrm{r}_5 e^{i(\theta_{3i}+\alpha)}$$

we obtain

$$\mathrm{R}_{Pi}(\cos\theta_{6i} + j\sin\theta_{6i})$$
$$= \mathrm{r}_{2i}(\cos\theta_{2i} + j\sin\theta_{2i}) + \mathrm{r}_{5i}(\cos(\theta_{3i}+\alpha) + j\sin(\theta_{3i}+\alpha) \qquad 4.26$$

On equating the real terms and the imaginary terms separately, we obtain two real equations corresponding to the horizontal and vertical components of the two-dimensional vector equation

$$\mathrm{R}_{Pxi}\cos\theta_{6i} = \mathrm{r}_2\cos\theta_{2i} + \mathrm{r}_{5i}\cos(\theta_{3i}+\alpha) \qquad 4.27$$

$$\mathrm{R}_{Pyi}\sin\theta_{6i} = \mathrm{r}_{2i}\sin\theta_{2i} + \mathrm{r}_{5i}\sin(\theta_{3i}+\alpha) \qquad 4.28$$

By squaring and adding Eqn. (4.27) & (4.28), $\boldsymbol{\theta_6}$ is eliminated and a solution is found for $\mathbf{R_P}$

$$\mathrm{R}_{pi} = \sqrt{\mathrm{r}_{2i}^2 + \mathrm{r}_{5i}^2 + 2\mathrm{r}_{2i}\mathrm{r}_{5i}\cos(\theta_{3i}+\alpha-\theta_{2i})} \qquad 4.29$$

The angle $\boldsymbol{\theta_6}$ found from

$$\theta_{6i} = \tan^{-1}\frac{\mathrm{r}_{2i}\sin\theta_{2i} + \mathrm{r}_{5i}\sin(\theta_{3i}+\alpha-\theta_{2i})}{\mathrm{r}_{2i}\cos\theta_{2i} + \mathrm{r}_{5i}\cos(\theta_{3i}+\alpha-\theta_{2i})} \qquad 4.30$$

Since $\mathbf{R_p}$ and $\boldsymbol{\theta_6}$ are the two unknowns. The solutions can found directly.

4.2 Velocity Analysis of Planar Four-bar Mechanisms

Velocity analysis begins with formulating the loop-closure equation for the four-bar mechanism shown below. Taking the time derivative of the position analyze relation, we obtain the velocity relation.

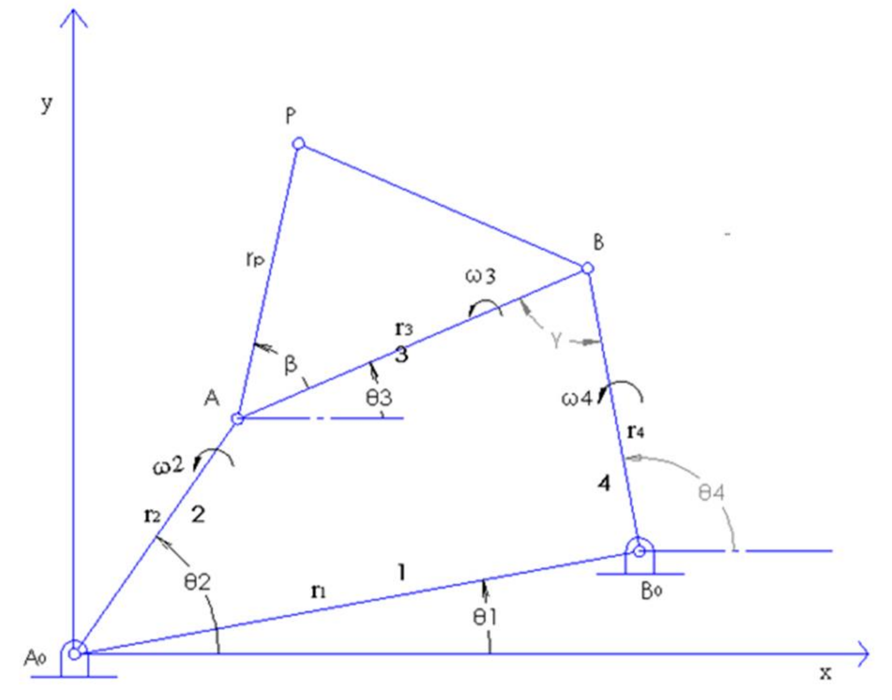


Figure 17: Angular Velocity Analysis

$$\mathrm{r}_{2i}+\mathrm{r}_{3i}=\mathrm{r}_{1i}+\mathrm{r}_{4i} \qquad 4.31$$

$$\frac{d}{dt}(\mathrm{r}_{3i}\mathrm{e}^{\mathrm{i}\theta_{3i}}-\mathrm{r}_{4i}\mathrm{e}^{\mathrm{i}\theta_{4i}}=\mathrm{r}_{1i}\mathrm{e}^{\mathrm{i}\theta_{1i}}-\mathrm{r}_{2i}\mathrm{e}^{\mathrm{i}\theta_{2i}})=0$$

$$\omega_{3i}\mathrm{r}_{3i}\mathrm{e}^{\mathrm{i}\theta_{3i}}-\omega_{4i}\mathrm{r}_{4i}\mathrm{e}^{\mathrm{i}\theta_{4i}}=-\omega_{2i}\mathrm{r}_{2i}\mathrm{e}^{\mathrm{i}\theta_{2}i} \qquad 4.32$$

To derive analytical solutions for ω_3 and ω_4 we will assume the link lengths $(\mathrm{r}_2,\mathrm{r}_3,\mathrm{r}_1,\mathrm{r}_4$, the angular positions $\theta_1,\theta_2,\theta_3,\theta_4$ and the angular velocity of link 2 ω_2 is known. Multiplying Eqn. (4.32) by $\mathrm{e}^{-\mathrm{i}\theta_4}$ and taking the imaginary part, we get.

$$\omega_{3i}\mathrm{r}_{3i}\sin(\theta_{3i}-\theta_{4i})=-\omega_{2i}\mathrm{r}_{2i}\sin(\theta_{2i}-\theta_{4i}) \qquad 4.33$$

$$\omega_{3i}=\frac{\omega_{2i}\mathrm{r}_{2i}\sin(\theta_{4i}-\theta_{2i})}{\mathrm{r}_{3i}\sin(\theta_{4i}-\theta_{3i})} \qquad 4.34$$

Similarly, the following formula for ω_4 can be derived multiplying Eqn.(4.32) by $(\mathrm{e}^{-\mathrm{i}\theta_3})$

$$\omega_{4i}=\frac{\omega_{2i}\mathrm{r}_{2i}\sin(\theta_{3i}-\theta_{2i})}{\mathrm{r}_{4i}\sin(\theta_{3i}-\theta_{4i})} \qquad 4.35$$

4.3 Acceleration Analysis of Planar Four-bar Mechanisms

Acceleration analysis begins with formulating the loop-closure equation for the four-bar mechanism shown below. Taking the time derivative of the velocity analyze relation, we obtain the acceleration relation.

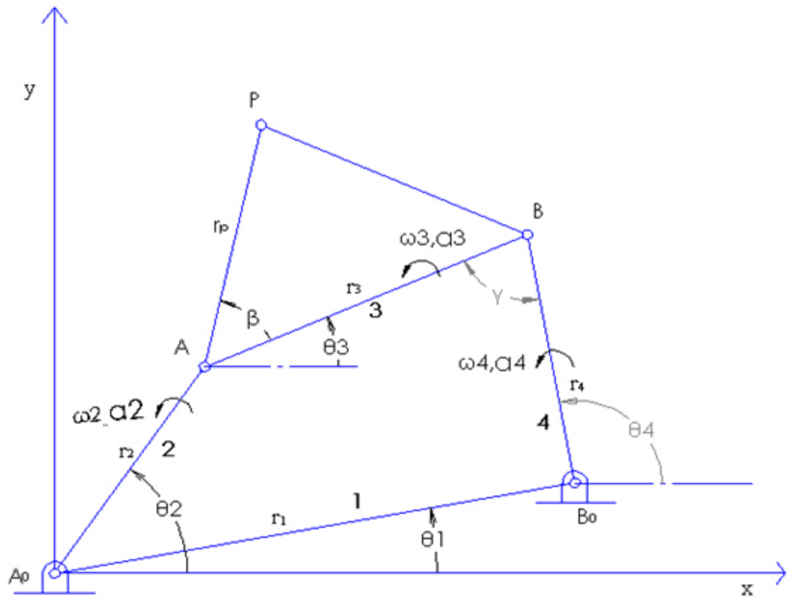


Figure 18: Angular Acceleration Analysis

$$\frac{d^2}{dt^2}(\mathrm{r}_{3i}\mathrm{e}^{\mathrm{j}\theta_{3i}} - \mathrm{r}_{4i}\mathrm{e}^{\mathrm{j}\theta_{4i}} = \mathrm{r}_{1i}\mathrm{e}^{\mathrm{j}\theta_i} - \mathrm{r}_{2i}\mathrm{e}^{\mathrm{j}\theta_2 i}) = 0$$

Time derivatives of the above relation, we can obtain the acceleration relation

$$j\alpha_{3i}\mathrm{r}_{3i}\mathrm{e}^{\mathrm{j}\theta_{3i}} - \omega_{3i}{}^2\mathrm{r}_{3i}\mathrm{e}^{\mathrm{j}\theta_{3i}}$$
$$-j\alpha_{4i}\mathrm{r}_{4i}\mathrm{e}^{\mathrm{j}\theta_{4i}} + \omega_{4i}{}^2\mathrm{r}_{4i}\mathrm{e}^{\mathrm{j}\theta_4} = j\alpha_{2i}\mathrm{r}_{2i}\mathrm{e}^{\mathrm{j}\theta_{2i}} + \omega_{2i}{}^2\mathrm{r}_{2i}\mathrm{e}^{\mathrm{j}\theta_2} \qquad 4.36$$

To derive analytical solutions for α_3 and α_4 we will assume the link lengths$(\mathrm{r}_2, \mathrm{r}_3, \mathrm{r}_1, \mathrm{r}_4)$, the angular positions $(\theta_1, \theta_2, \theta_3, \theta_4)$ the angular velocities $(\omega_1, \omega_2, \omega_3, \omega_4)$ and at least one angular acceleration is known. After multiplying Eqn.(4,36) by $\mathrm{e}^{-\mathrm{i}\theta_4}$ taking the real part and simplifying, we get

$$\alpha_{3i} = \frac{\begin{pmatrix} -r_{2i}\alpha_2 sin(\theta_{4i} - \theta_{2i}) + r_{2i}{\omega_{2i}}^2 cos(\theta_{4i} - \theta_{2i}) \\ +r_{3i}{\omega_{3i}}^2 cos(\theta_{4i} - \theta_{3i}) - r_{4i}{\omega_{4i}}^2 \end{pmatrix}}{r_{3i}\sin(\theta_{4i} - \theta_{3i})} \qquad 4.37$$

Similarly, the following formula for α_4 can be derived by multiplying Eqn.(4.36) by $e^{-i\theta_3}$

$$\alpha_{4i} = \frac{\begin{matrix} r_{2i}\alpha_{2i} sin(\theta_{3i} - \theta_{2i}) - r_{2i}{\omega_{2i}}^2 cos(\theta_{3i} - \theta_{2i}) \\ +\ r_{4i}{\omega_{4i}}^2 cos(\theta_{3i} - \theta_{4i}) - r_{3i}{\omega_{3i}}^2 \end{matrix}}{r_{4i}\sin(\theta_{3i} - \theta_{4i})} \qquad 4.38$$

Having found the angular velocity of link 3 we can compute the linear velocity of point P on the link as shown below. Starting from positing vector

$$R_p = R_P e^{j\theta_6} = r_2 e^{j\theta_{2i}} + r_5 e^{j(\theta_{3i}+\alpha)}$$

$$R_p = W_1 e^{j\theta_{2i}} + Z_1 e^{j(\theta_{5i})}$$

The Velocity of point p as follows;

$$\vec{V}_{Rp} = iW_1\omega_{2i}e^{j\theta_{2i}} + iZ_1\omega_{3i}e^{j(\theta_{5i})} \qquad 4.39$$

The real and imaginary components are then

Real

$$\vec{V}_{Rpx} = -W_1\omega_{2i}\sin\theta_{2i} - Z_1\omega_{3i}\sin\theta_{5i}$$

Imaginary

$$\vec{V}_{Rpx} = W_1\omega_{2i}\cos\theta_{2i} + Z_1\omega_{3i}\cos\theta_{5i}$$

The magnitude and direction of $\vec{V}_{Rp}$ are.

$$V_{Rp} = \sqrt{{V_{Rpx}}^2 + \left(iV_{Rpy}\right)^2}$$

$$\theta_{Rp} = \tan^{-1}\frac{\left(V_{Rpy}\right)}{\left(V_{Rpx}\right)}$$

To find acceleration at point p, differentiating velocity equation with respect to time.

$$\vec{a}_{Rp} = (-{\omega_{2i}}^2 - i\alpha_{2i})W_1 e^{j\theta_{2i}} + (-{\omega_{3i}}^2 + i\alpha_{3i})Z_1 e^{j\theta_{5i}}$$

The real and imaginary components are then

Real

$$\vec{a}_{Rpx} = -\mathrm{W}_1({\omega_{2i}}^2 \cos\theta_{2i} + \alpha_{2i}\sin\theta_{2i}) - \mathrm{Z}_1({\omega_{3i}}^2 \cos\theta_{5i} + \alpha_{3i}\sin\theta_{5i})$$

Imaginary

$$\vec{a}_{Rpy} = -\mathrm{W}_1({\omega_{2i}}^2 \sin\theta_{2i} - \alpha_{2i}\cos\theta_{2i}) - \mathrm{Z}_1({\omega_{3i}}^2 \sin\theta_{5i} - \alpha_{3i}\cos\theta_{5i})$$

The magnitude and direction of $\vec{a}_{Rp}$ are.

$$a_{Rp} = \sqrt{{a_{Rpx}}^2 + \left(ia_{Rpy}\right)^2}$$

$$\theta_{Rp} = \tan^{-1}\frac{(a_{Rpy})}{(a_{Rpx})}$$

CHAPTER FIVE

DYNAMIC ANALYSIS OF PLANER FOUR BAR LINKAGE

5.1 Dynamic Force Analysis

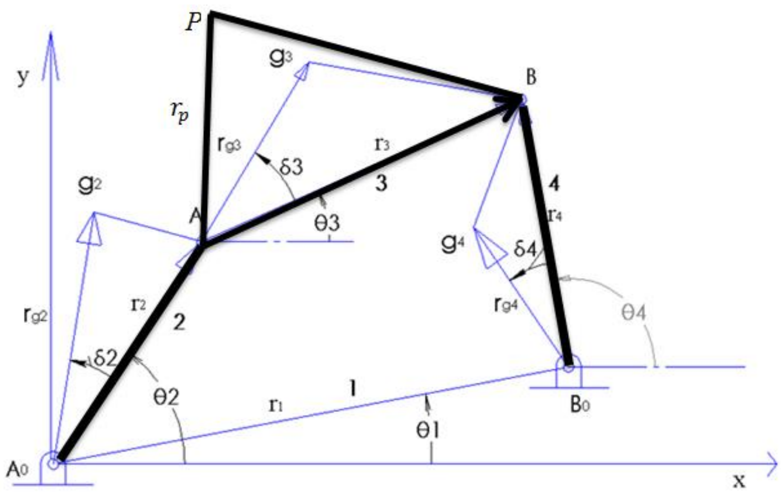


Figure 19: Four-Bar Linkage Dynamic Force Analysis

Three static equilibrium equations is evaluated in terms of

Forces in the X and Y directions and

Moment about the center of gravity of the link can write for each link.

Linkages dynamic force free body diagram

Link 2

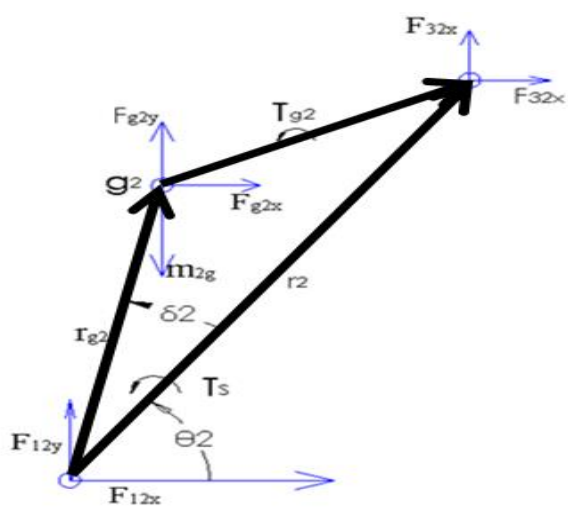


Figure 20: Crank linkage direction of force.

Three static equilibrium equations is evaluated in terms of forces in the X and Y directions

$$\sum F_x = 0$$

$$F_{12x} + F_{32x} + F_{g2x} = 0 \qquad 5.1$$

$$\sum F_y = 0$$

$$-m_{2g} + F_{12y} + F_{32y} + F_{g2y} = 0 \qquad 5.2$$

Moment about the center of gravity of the link 2

$$\sum M_{g_2} = 0$$

$$T_s + -r_{g2} F_{12} + (r_2 - r_{g2})F_{32} + T_{g2} = 0 \qquad 5.3$$

Equations (5.3), can be expressed as

$$T_S - r_{g2}\cos(\theta_2 + \delta_2) F_{12y} + r_{g2} sin(\theta_2 + \delta_2) F_{12x} + [r_2 \cos(\theta_2) - r_{g2}\cos((\theta_2 + \delta_2)] F_{32y} - [r_2 \sin(\theta_2)) - r_{g2} cos(\theta_2 + \delta_2)] F_{32x} + T_{g2} = 0$$

Where

- $r_{g2} = r_{g2} * \mathrm{e}^{\mathrm{i}(\theta_2+\delta_2)}$ Position vector from joint A_0 to center gravity of link 2
- F_{12} & F_{32} F_{23} and $F_{43} F_{14}$ and F_{34} are the joint forces acting on link 2.
- F_{g2} & T_{g2} are the inertia force and inertia moment of link 2.
- m_2 Mass of link 2 & T_s driving torque

Link 3

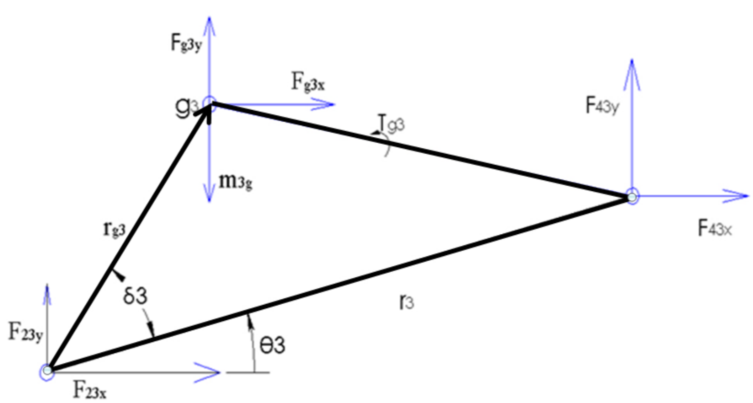


Figure 21: Coupler linkage direction of force

Three static equilibrium equations is evaluated in terms of forces in the X and Y directions

$\sum F_x = 0$

$$F_{23x} + F_{43x} + F_{g3x} = 0 \qquad 5.44$$

$\sum F_y = 0$

$$-m_{3g} + F_{23y} + F_{43y} + F_{g3y} = 0 \qquad 5.5$$

Moment about the center of gravity of the link 3

$\sum M_{g_3} = 0 = 0$

$$-r_{g3} F_{23} + (r_3 - r_{g3})F_{43} + T_{g3} = 0 \qquad 5.6$$

Equations (5.6), can be expressed as

$$r_{g3}\cos(\theta_3 + \delta_3)F_{23y} + r_{g3}sin(\theta_3 + \delta_3)F_{23x} + [r_3\cos(\theta_3) - r_{g3}\cos((\theta_3 + \delta_3)]F_{43y} - [r_3\sin(\theta_3)) - r_{g3}\cos(\theta_3 + \delta_3)]F_{43x} + T_{g3} = 0$$

Where

- $r_{g3} = r_{g3} * e^{i(\theta_3+\delta_3)}$ position vector from joint A_0 to the center of gravity link3
- F_{23} & F_{43} are the joint forces acting on link 3.
- F_{g3} & T_{g2} are the inertia force and inertia moment of link 2.
- m_3 mass of link 3 and T_{g3} inertia moment of link 3

Link 4

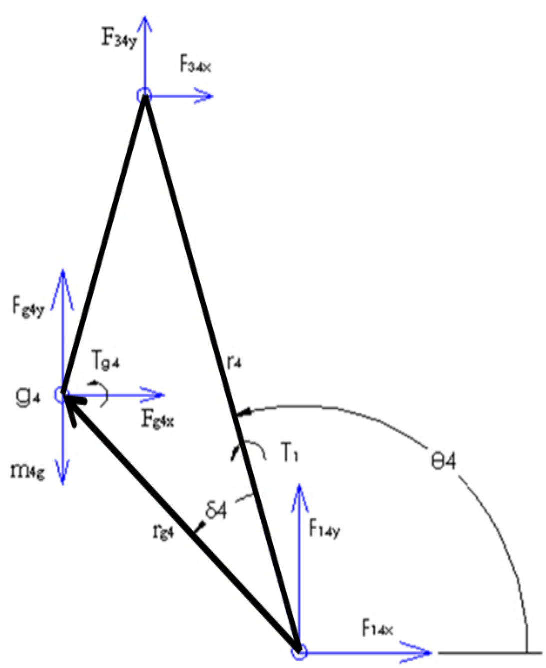


Figure 22: rocker (follower) linkage direction of forces

Three static equilibrium equations is evaluated in terms of forces in the X and Y directions

$$\sum F_x = 0$$

$$F_{34x} + F_{14x} + F_{g4x} = 0 \quad 5.7$$

$$\sum F_y = 0$$

$$-m_{4g} + F_{34y} + F_{14y} + F_{g4y} = 0 \quad 5.8$$

Moment about the center of gravity of the link 4

$$\sum M_{g_4} = 0$$

$$\left(-r_{g4}\right) F_{14} + \left(r_4 - r_{g4}\right) F_{34} + T_{g4} + T_1 = 0 \quad 5.9$$

Equations (9), can be expressed as

$$r_{g4}\cos(\theta_4 + \delta_4)F_{14y} + r_{g4}sin(\theta_4 + \delta_4)F_{14x}$$
$$+ \left[r_4 \cos(\theta_4) - r_{g4} \cos((\theta_4 + \delta_4)\right]F_{34y} - \left[r_4 \sin(\theta_4)\right)$$
$$- r_{g3}cos(\theta_4 + \delta_4)]F_{34x} + T_{g4} + T_1 = 0$$

Where

- $r_{g4} = r_{g4} * \mathrm{e}^{\mathrm{i}(\theta_4 + \delta_4)}$ Position vector from joint B_0 to the center of gravity link 4,
- F_{14}& F_{34} are the joint forces acting on link 4
- F_{g4} & T_{g4} are the inertia force and inertia moment of link 4.
- m_4 Mass of link 4 and T_1 torque of external load

Equations (1-9) can write as nine linear equations in terms of nine unknowns.

They can expressed in a symbolic form

$$-(F_{12x} + F_{32x}) = F_{g2x}$$

$$-(F_{12y} + F_{32y}) = F_{g2y} - m_{2g}$$

$$-\left[T_s + \left(-r_{g2}\right) * F_{12} + \left(r_2 - r_{g2}\right) * F_{32} \right] = T_{g2}$$

$$-(F_{23x} + F_{43x}) = F_{g3x}$$

$$-(F_{23y} + F_{43y}) = F_{g3y} - m_{3g}$$

$$-[(-r_{g3}) * F_{23} + (r_3 - r_{g3}) * F_{43}] = T_{g3}$$

$$-(F_{34x} + F_{14x}) = F_{g4x}$$

$$-(F_{34y} + F_{14y}) = F_{g4y} - m_{4g}$$

$$-[(-r_{g4}) * F_{14} + (r_4 - r_{g4}) * F_{34}] = T_{g4} + T_1$$

From Eqn. (4) & (7)

$$F_{43x} - F_{14x} = F_{g4x}$$

From Eqn. (5) & (8)

$$F_{43y} - F_{14y} = F_{g4y} - m_{4g}$$

When

$F_{ijx} = -F_{jix}$ And $F_{ijy} = -F_{jiy}$,

They can expressed in a symbolic form

$$\boldsymbol{Ax} = \boldsymbol{b} \qquad 5.10$$

x = the transpose of $(F_{12x}, F_{12y}, F_{23x}, F_{23y}, F_{34x}, F_{34y}, F_{14x}, F_{14y}, T_s)$ and is a vector consisting of the unknown forces and input torque,

b = the transpose of $(F_{g2x}, F_{g2y} - m_{2g}, T_{g2}, F_{g3x}, F_{g3y} - m_{3g}, T_{g3}, F_{g4x}, F_{g4y} - m_{4g}, T_{g4} + T_1)$ and is a vector that contains external load plus inertia forces and inertia torques.

By rearrange terms in equations put all known constant terms on the left side and then put them in matrix form

Let

$M = r_2 sin\theta_2 - r_{g2} cos\theta_2$ $\qquad N = -r_2 \cos\theta_2 + r_{g2} cos\theta_2$

$L = r_3 \sin\theta_3 - r_{g3} \cos\theta_g$ $\qquad S = -r_3 \cos\theta_3 + r_{g3} \cos\theta_g$

$Q = -r_4 \sin\theta_4 + r_{g4} \cos\theta_4$ $\qquad T = r_4 \cos\theta_4 + r_{g4} \cos\theta_4$

$$\begin{bmatrix} F_{g2x} \\ F_{g2y}-m_{2g} \\ T_{g2} \\ F_{g3x} \\ F_{g3y}-m_{3g} \\ T_{g3} \\ F_{g4x} \\ F_{g4y-m_{4g}} \\ T_{g4}+T_1 \end{bmatrix} = \begin{Bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ r_{g2}sin\theta_2 & -r_{g2}cos\theta_2 & M & N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & r_{g3}\sin\theta_g & -r_{g3}\sin\theta_g & L & S & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & Q & T & r_{g4}sin\theta_4 & -r_{g4}sin\theta_4 & 0 \end{Bmatrix} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{34y} \\ F_{14x} \\ F_{14y} \\ T_s \end{bmatrix}$$

CHAPTER SIX

THREE POSITION ANALYTICAL EXAMPLE

6.1. Kinematical Analysis of Planar Four-bar Mechanisms

6.1.1 Linkage Dimension Analysis

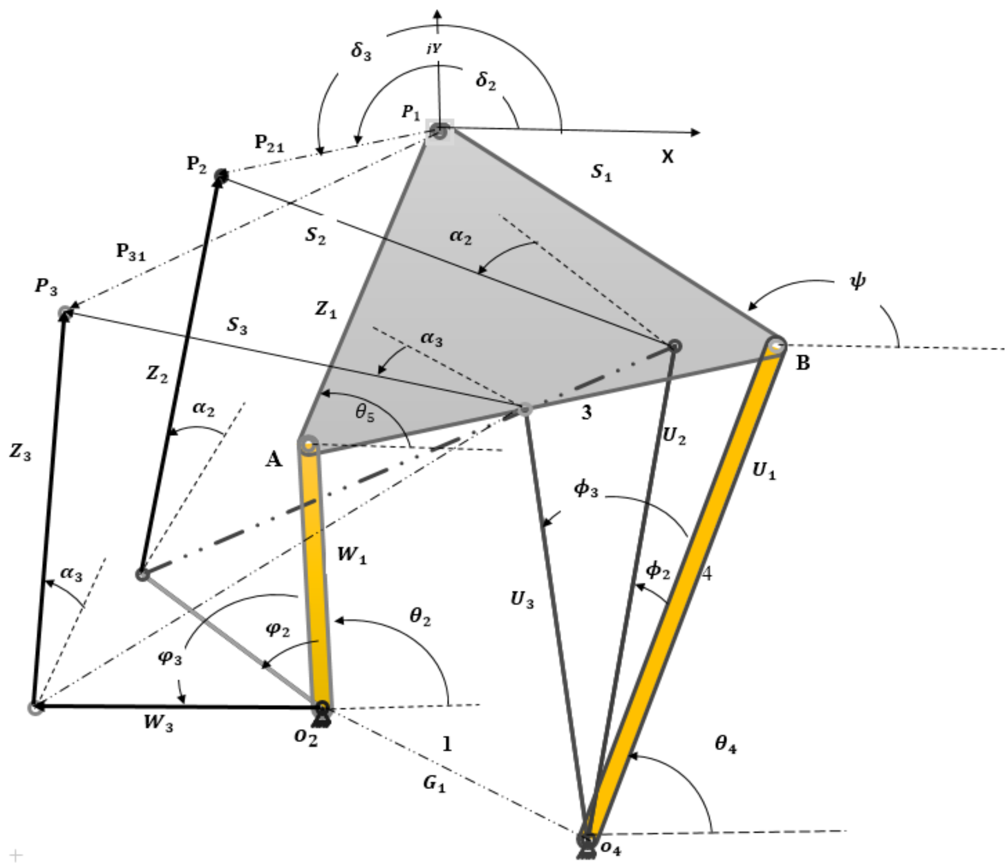


Figure 23: Three-positioning analytical synthesis

Assumed prescribe points from fixed point 0_2

	X - axis distance from 0_2	Y- axis distance from 0_2
p_1	40	120
p_2	50	300
p_3	80	270

Input Assume

$$\varphi_2 = 35^0 \qquad \alpha_2 = 5^0$$

$$\varphi_3 = 50^0 \qquad \alpha_3 = 10^0$$

$\theta_{p21} = \tan^{-1}[\frac{y_2-y_1}{x_2-x_1}] = \tan^{-1}[\frac{300-120}{50-40}] = 86.8^0$=

$$\theta_{p31} = \tan^{-1}[\frac{y_3 - y_1}{x_3 - x_1}] = \tan^{-1}[\frac{270-120}{80-40}] = 75.1^0$$

$$\delta_{21} = 180^0 + \theta_{p21} = 266.8^0 = 4.656\,rad$$

$$\delta_{31} = 180^0 + \theta_{p31} = 255.1^0 = 4.45\,rad$$

$$p_{21} = \sqrt{(y_2-y_1)^2 + (x_2-x_1)^2} = \sqrt{(300-120)^2 + (50-40)^2} = 180.3$$

$$p_{31} = \sqrt{(y_3-y_1)^2 + (x_3-x_1)^2} = \sqrt{(270-120)^2 + (80-40)^2} = 155.2$$

	Rad			Rad			Rad
$\cos\varphi_2$	-0.904		$\cos\alpha_2$	0.284		$\cos\delta_{21}$	-0.97
$\cos\varphi_3$	0.965		$\cos\alpha_3$	-0.8391		$\cos\delta_{31}$	-0.81
$\sin\varphi_2$	-0.4282		$\sin\alpha_2$	-0.9589		$\sin\delta_{21}$	0.233
$\sin\varphi_3$	-0.2634		$\sin\alpha_3$	-0.544		$\sin\delta_{31}$	-0.59

$$\begin{bmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{bmatrix} \begin{Bmatrix} \cos\varphi_2 - 1 & -\sin\varphi_2 & \cos\alpha_2 - 1 & -\sin\alpha_2 \\ \cos\varphi_3 - 1 & -\sin\varphi_3 & \cos\alpha_3 - 1 & -\sin\alpha_3 \\ \sin\varphi_2 & \cos\varphi_2 - 1 & \sin\alpha_2 & \cos\alpha_2 - 1 \\ \sin\varphi_3 & \cos\varphi_3 - 1 & \sin\alpha_3 & \cos\alpha_3 - 1 \end{Bmatrix} = \begin{pmatrix} p_{21} * \cos\delta_{21} \\ p_{31} * \cos\delta_{31} \\ p_{21} * \sin\delta_{21} \\ p_{31} * \sin\delta_{31} \end{pmatrix}$$

$$\begin{bmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{bmatrix} \begin{Bmatrix} -1.904 & 0.4282 & -0.7163 & -0.9589 \\ -.035 & 0.2634 & -1.839 & -0.544 \\ -0.4282 & -1.904 & -0.9589 & -0.7163 \\ -0.2634 & -.035 & -0.544 & -1.839 \end{Bmatrix} = \begin{pmatrix} -175.3 \\ -125.3 \\ 42 \\ -91.56 \end{pmatrix}$$

This is in the form of

$$R = [A]^{-1}(P)$$

$$[A]^{-1} = \begin{Bmatrix} -0.5 & 0.2619 & -0.0746 & -0.0951 \\ 0.067 & 0.1429 & -0.496 & 0.3027 \\ 0.048 & -0.4758 & -0.051 & -0.203 \\ 0.0558 & 0.1006 & 0.03518 & -0.4758 \end{Bmatrix}$$

$$\begin{bmatrix} W_{1x} \\ W_{1y} \\ Z_{1x} \\ Z_{1y} \end{bmatrix} = \begin{Bmatrix} -0.5 & 0.2619 & -0.0746 & -0.0951 \\ 0.067 & 0.1429 & -0.496 & 0.3027 \\ 0.048 & -0.4758 & -0.051 & -0.203 \\ 0.0558 & 0.1006 & 0.03518 & -0.4758 \end{Bmatrix} \begin{pmatrix} -175.3 \\ -125.3 \\ 42 \\ -91.56 \end{pmatrix}$$

$W_{1x} = 0.5 * 175.3 - 0.2619 * 125.3 - 0.0746 * 42 + 0.0951 * 91.56$

$W_{1y} = -0.067 * 175.3 - 0.1429 * 125.3 - 0.496 * 42 - 0.3027 * 91.56$

$Z_{1x} = -0.048 * 175.3 + 0.4758 * 125.3 - 0.051 * 42 + 0.203 * 91.56$

$Z_{1y} = -0.0558 * 175.3 - 0.1006 * 125.3 - 0.03518 * 42 + 0.4758 * 91.56$

From this

$W_{1x} = 60.41$ $\qquad$ $Z_{1x} = 67.6$

$W_{1y} = -78.2$ $\qquad$ $Z_{1y} = 20$

$$W_1 = R_2 = \sqrt{{W_{1x}}^2 + {W_{1y}}^2} = 98.82$$

$$\theta_{21} = \tan^{-1}\left(\frac{W_{1y}}{W_{1x}}\right) = -52.3^0 = -2.997\,rad$$

$$\theta_{22} = \theta_{21} + 35^0 = -17.3^0 = -0.3\,rad$$

$$\theta_{23} = \theta_{21} + 50^0 = -2.3^0 = -0.04\,rad$$

$$Z_1 = R_p = \sqrt{{Z_{1x}}^2 + {Z_{1y}}^2} = 70.5$$

$$\theta_{Z1} = \theta_{51} = \tan^{-1}\left(\frac{Z_{1y}}{Z_{1x}}\right) = 16.48^0 = 0.287\ \ rad$$

$$\theta_{52} = \theta_{51} + \alpha_2 = 42.9^0 + 5^0 = 21.5^0 = 0.375\,rad$$

$$\theta_{53} = \theta_{51} + \alpha_3 = 42.9^0 + 10^0 = 31.5^0 = 0.549\,rad$$

In addition, dimension determine of the left dyad or linkages.

$$\begin{bmatrix} U_{1_x} \\ U_{1_Y} \\ S_{1_x} \\ S_{1_Y} \end{bmatrix} * \begin{bmatrix} \cos\phi_2 - 1 & -\sin\phi_2 & \cos\alpha_2 - 1 & -\sin\alpha_2 \\ \cos\phi_3 - 1 & -\sin\phi_3 & \cos\alpha_3 - 1 & -\sin\alpha_3 \\ \sin\phi_2 & \cos\phi_2 - 1 & \sin\alpha_2 & \cos\alpha_2 - 1 \\ \sin\phi_3 & \cos\phi_3 - 1 & \sin\alpha_3 & \cos\alpha_3 - 1 \end{bmatrix} = \begin{bmatrix} p_2 * cos\,\delta_2 \\ p_3 * cos\,\delta_3 \\ p_2 * sin\,\delta_2 \\ p_3 * sin\,\delta_3 \end{bmatrix}$$

Assume Out Put Link Rotate Respect to Input Link Rotation

$$\phi_2 = 15^0 \qquad \text{And} \qquad \phi_2 = 30^0$$

	Rad			Rad			Rad
$\cos\phi_2$	-0.76		$\cos\alpha_2$	0.284		$\cos\delta_2$	-0.97
$\cos\phi_3$	0.99		$\cos\alpha_3$	-0.8391		$\cos\delta_3$	-0.81
$\sin\phi_2$	0.65		$\sin\alpha_2$	-0.9589		$\sin\delta_2$	0.233
$\sin\phi_3$	-0.13		$\sin\alpha_3$	-0.544		$\sin\delta_3$	-0.59

$$\begin{bmatrix} U_{1_x} \\ U_{1_Y} \\ S_{1_x} \\ S_{1_Y} \end{bmatrix} * \begin{bmatrix} -1.76 & -0.65 & -0.716 & 0.9589 \\ -0.8457 & 0.988 & -1.839 & 0.9589 \\ 0.6503 & -1.76 & -0.959 & -0.716 \\ -0.988 & -0.8457 & -0.544 & -1.839 \end{bmatrix} = \begin{bmatrix} -175.3 \\ -125.3 \\ 42 \\ -91.56 \end{bmatrix}$$

This matrix can written

$$D * [C] = P$$

$$D = [C]^{-1} * P$$

$$\begin{bmatrix} U_{1_x} \\ U_{1_Y} \\ S_{1_x} \\ S_{1_Y} \end{bmatrix} = \begin{bmatrix} -0.3457 & 0.0617 & 0.2874 & -0.26 \\ -0.3226 & 0.2745 & -0.3481 & 0.1105 \\ 0.1385 & -0.4399 & -0.274 & -0.0504 \\ 0.2931 & -0.0292 & 0.0867 & -0.4399 \end{bmatrix} \begin{bmatrix} -175.3 \\ -125.3 \\ 42 \\ -91.56 \end{bmatrix}$$

$$U_{1_x} = 0.3457 * 175.3 - 0.0617 * 125.3 + 0.2874 * 42 + 0.26 * 91.56$$

$$U_{1_Y} = 0.3226 * 175.3 - 0.2745 * 125.3 - 0.3481 * 42 - 0.1105 * 91.56$$

$$S_{1_x} = -0.1385 * 175.3 + 0.4399 * 125.3 - 0.274 * 42 + 0.0504 * 91.56$$

$$S_{1_Y} = -0.2931 * 175.3 + 0.0292 * 125.3 + 0.0867 * 42 + 0.4399 * 91.56$$

$$U_{1_x} = 88.75 \qquad S_{1_x} = 23.95$$

$$U_{1_Y} = -2.6 \qquad S_{1_Y} = -3.8$$

$$U_1 = R_4 = \sqrt{{U_{1_x}}^2 + {U_{1_Y}}^2} = 88.8$$

$$\theta_{41} = \tan^{-1}\left(\frac{U_{1_y}}{U_{1_x}}\right) = -1.68^0 = -0.0293\, rad$$

$$\theta_{42} = \theta_{41} + \phi_2 = 13.32^0 = 0.232\, rad$$

$$\theta_{43} = \theta_{21} + \phi_3 = 28.32^0 = 0.49\, rad$$

$$S_1 = \sqrt{{S_{1_x}}^2 + {S_{1_Y}}^2} = 24.25$$

$$\psi_{S1} = tan^{-1}\left(\frac{S_{1_y}}{S_{1_x}}\right) = -9.02^0 = -0.157\, rad$$

$$\psi_{S2} = \psi_{S1} + \alpha_2 = -4.02^0 = -0.07\, rad$$

$$\psi_{S3} = \psi_{S1} + \alpha_3 = 0.98^0 = 0.02\, rad$$

Link 3 V_1 (R_3) is defined in terms of vectors Z_1 and S_1.

$$V_1 = Z_1 - S_1$$

$$W_1 = R_2 = 98.82 \qquad \theta_{21} = -52.3^0 = -2.997\, rad$$

$$U_1 = R_4 = 88.8 \qquad \theta_{41} = -1.68^0 = -0.0293\, rad$$

$$Z_1 = R_p = 70.5 \qquad \theta_{Z1} = \theta_{51} = 16.48^0 = 0.287\ \ rad$$

$$S_1 = 24.25 \qquad \psi_{S1} = -9.02^0 = -0.157\, rad$$

$$V_1 = R_3 = \sqrt{{Z_1}^2 + {S_1}^2 - 2Z_1S_1\cos(\theta_{Z1} - \psi_{S1})} = 48.6$$

$$\theta_{31} = \tan^{-1}(W_1)$$

Angle $< Z_1V_1$

$$\beta = \cos^{-1}\left(\frac{{V_1}^2+{Z_1}^2-S^2}{2Z_1V_1}\right) = \cos^{-1}\left(\frac{48.6^2+70.5^2-24.25}{2*70.5*48.6}\right)=$$

$$\beta = \cos^{-1}(0.985) = 9.969^0 = 0.174\, rad$$

$$\theta_{31} = \theta_{51} - \beta = 16.5^0 - 9.969^0 = 6.81^0$$

$$\theta_{32} = \theta_{52} - \beta = 21.5^0 - 9.969^0 = 11.81^0$$

$$\theta_{33} = \theta_{53} - \beta = 26.5^0 - 9.969^0 = 16.51^0$$

The ground link defined by equation

$G_1 = W_1 + V_1 - U_1$

$G_1 = W_1 + V_1 - U_1$

$$G_1 e^{j(\theta_1)} = W_2 e^{j(\theta_2)} + V_1 e^{j(\theta_3)} - U_1 e^{j(\theta_4)}$$

Real Part:-

$$G_1 \cos\theta_1 = W_1 \cos\theta_2 + V_1 \cos\theta_3 - U_1 \cos\theta_4$$

Imaginary Part:-

$$G_1 \sin\theta_1 = W_1 \sin\theta_2 + V_1 \sin\theta_3 - U_1 \sin\theta_4$$

Square and add above equations to find.

$$G_1 = \sqrt{\begin{matrix} W_1{}^2 + V_1{}^2 + U_1{}^2 + 2W_1V_1\cos(\theta_2 - \theta_3) \\ - \ 2W_1U_1\cos(\theta_2 - \theta_4) - 2V_1U_1\cos(\theta_3 - \theta_4) \end{matrix}}$$

$$G_1 = \sqrt{\begin{matrix} 98.82^2 + 48.6^2 + 88.8^2 \\ +2 * 98.82 * 48.6 * cos(-52.3^0 - 6.81^0) \\ -2 * 98.82 * 88.8 * cos(-52.3^0 - (180 + 1.68^0)) \\ -2 * 48.6 * 88.8 * cos(6.81^0 - (180 + 1.68^0)) \end{matrix}}$$

$$G_1 = \sqrt{20012.8 + 4931.3 - 1203.94 - 4225.74}$$

$$G_1 = \sqrt{19514.42}$$

$$G_1 = R_1 = 139.69$$

6.12 Velocity Analysis of Planar Four-bar Mechanisms

Let link 2 input ω_2 =0.05 rad/s

Link 3 ω_{3i}

$$\omega_{3i} = \frac{\omega_2 \mathrm{r}_2 \sin(\theta_{4i} - \theta_{2i})}{\mathrm{r}_3 \sin(\theta_{3i} - \theta_{4i})}$$

$$\omega_{31} = \frac{0.05 * 98.82 * \sin(-1.68 + 52.3)}{48.6 * \sin(6.81 + 1.68)} = 0.044\, rad/s$$

$$\omega_{32} = \frac{0.05 * 98.82 * \sin(13.32 + 17.3)}{48.6 * \sin(11.81 - 13.32)} = 0.073\, rad/s$$

$$\omega_{33} = \frac{0.05 * 98.82 * \sin(28.32 + 2.3)}{48.6 * \sin(16.51 - 28.32)} = 0.106\, rad/s$$

Link 4 $\boldsymbol{\omega_{4i}}$

$$\omega_{4i} = \frac{\omega_2 r_2 \, sin(\theta_{2i} - \theta_{3i})}{r_4 \, sin(\theta_{4i} - \theta_{3i})}$$

$$\omega_{41} = \frac{0.05 * 98.82 * sin(-52.3 - 6.81)}{88.8 * sin(-1.68 - 6.81)} = 0.0379 \, rad/s$$

$$\omega_{42} = \frac{0.05 * 98.82 * sin(-17.3 - 11.81)}{88.8 * sin(13.32 - 11.81)} = 0.042 \, rad/s$$

$$\omega_{43} = \frac{0.05 * 98.82 * sin(-2.3 - 15.51)}{88.8 * sin(28.32 - 16.51)} = -0.0699 \, rad/s$$

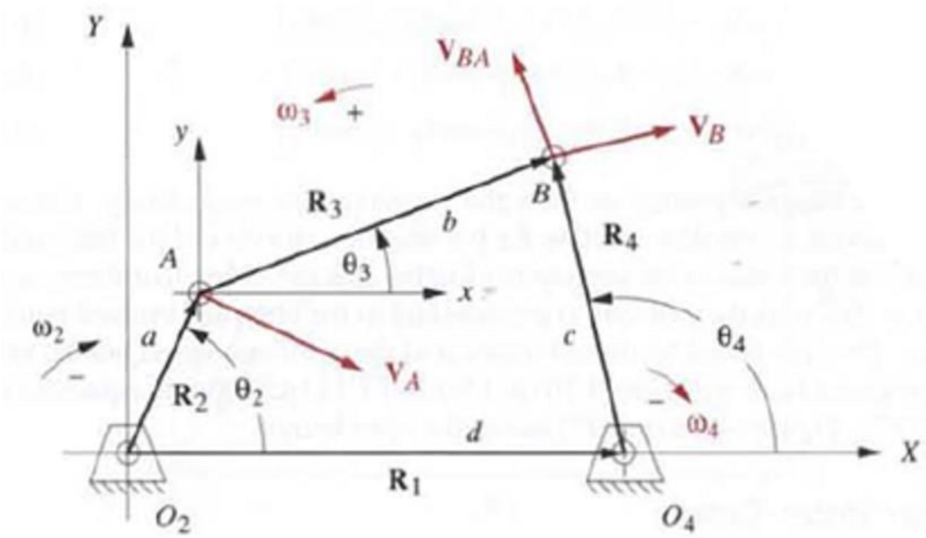


Figure 24: Position vector loop showing velocity vector [25]

The relative velocity or velocity difference equation

$$V_A + V_{B/A} - V_B = 0$$

Where

$$V_A = \omega_2 r_2 e^{i\theta_2 i} = \omega_2 r_2 (cos \, \theta_{2i} + j \, sin \, \theta_{2i})$$

$$V_{A1} = 0.05 * 98.82(cos(-52.3) + j \, sin(-52.3))$$

$$V_{A1} = (-2.21 - j4.42) = in \; vector$$

$$V_{A1} = \sqrt{{V_{A1x}}^2 + {V_{A1y}}^2} = 4.94 \, m/s$$

$$V_{A2} = 0.05 * 98.82(cos(-17.3) + j \, sin(-17.3))$$

$$V_{A2} = (0.105 - j4.94) = in \; vector$$

$$V_{A2} = 4.94 \, m/s$$

$$V_{A3} = 0.05 * 98.82(cos(-2.3) + j\,sin(-2.3))$$

$$V_{A3} = (-3.29 - j3.68)$$

$$V_{A3} = 4.94\ m/s$$

$V_{A1} = V_{A2} = V_{A3} = 4.94\ m/s$ Constant input

$$V_{B/A} = \omega_{3i}\mathrm{r}_3\mathrm{e}^{\mathrm{i}\theta_{3i}} = \omega_3 r_3(cos\,\theta_{3i} + j\,sin\,\theta_{3i})$$

$$V_{B1/A1} = 0.044 * 48.6(cos\,6.81 + j\,sin\,6.81)$$

$$V_{B1/A1} = 1.848 + j1.075$$

$$V_{B1/A1} = 2.14\ m/s$$

$$V_{B2/A2} = 0.073\ * 48.6 * (\cos(11.81) + jsin(11.81)$$

$$V_{B2/A2} = (2.58 - j2.435)$$

$$V_{B2/A2} = 3.55\ m/s$$

$$V_{B3/A3} = 0.106 * 48.6 * (\cos(16.51) + jsin(16.51)$$

$$V_{B3/A3} = (-3.58 - j\ 3.7)$$

$$V_{B3/A3} = 5.15\ m/s$$

$$V_B = \omega_{4i} r_4 e^{i\theta_{4i}} = \omega_4 r_4(cos\,\theta_4 + j\,sin\,\theta_4)$$

$$V_{B1} = 0.0379 * 88.8 * (\cos(-1.68) + jsin(-1.68)$$

$$V_{B1} = (-0.366 - j3.345)$$

$$V_{B1} = 3.36\ m/s$$

$$V_{B2} = 0.042 * 88.8 * (\cos(13.32) + j\sin(13.32))$$

$$V_{B2} = (2.72 + j2.55) = 3.73\ m/s$$

$$V_{B3} = -0.0699 * 88.8 * (cos(28.32) + sin(28.32))$$

$$V_{B3} = (6.2 + j0.283)$$

$$V_{B3} = 6.21\ m/s$$

Acceleration Analysis of Planar Four-bar Mechanisms

$$\frac{d^2}{dt^2}(\mathrm{r}_3\mathrm{e}^{\mathrm{j}\theta_{3i}} - \mathrm{r}_4\mathrm{e}^{\mathrm{j}\theta_{4i}} = \mathrm{r}_1\mathrm{e}^{\mathrm{j}\theta_i} - \mathrm{r}_2\mathrm{e}^{\mathrm{j}\theta_2 i}) = 0$$

$$\alpha_{3i} = \frac{\begin{pmatrix} -\mathrm{r}_2\alpha_2 sin(\theta_{4i}-\theta_{2i}) + \mathrm{r}_2{\omega_{2i}}^2 cos(\theta_{4i}-\theta_{2i}) \\ +\mathrm{r}_3{\omega_{3i}}^2 cos(\theta_{3i}-\theta_{4i}) - \mathrm{r}_4{\omega_{4i}}^2 \end{pmatrix}}{\mathrm{r}_3 \sin(\theta_{3i}-\theta_{4i})}$$

Assume $\alpha_2 = 0.02 \;\; rad/s^2$

$$\alpha_{31} = \frac{\begin{pmatrix} -98.82 * 0.02 * sin(-1.68+52.3) + 98.82 * 0.05^2 cos(-1.68+52.3) \\ +48.6 * (0.044)^2 cos(6.81+1.68) - 88.8 * (0.0379)^2 \end{pmatrix}}{48.6 * \sin(6.81+1.68)}$$

$$\alpha_{31} = -0.163 \, rad/s^2$$

$$\alpha_{32} = \frac{\begin{pmatrix} -98.82 * 0.02 * sin(13.32-17.3) + 98.82 * (0.05)^2 cos(13.32-17.3) \\ +48.6 * (0.073)^2 cos(11.81-13.32) - 88.8 * (0.042)^2 \end{pmatrix}}{48.6 * \sin(11.81-13.32)}$$

$$\alpha_{32} = 0.037 \, rad/s^2$$

$$\alpha_{33} = \frac{\begin{pmatrix} -98.82 * 0.02 * sin(28.32+2.3) + 98.82 * (0.05)^2 cos(28.32+2.3) \\ +48.6 * (0.106)^2 cos(16.51-28.32) - 88.8 * (-0.0699)^2 \end{pmatrix}}{48.6 * \sin(16.51-28.32)}$$

$$\alpha_{33} = 0.0464 \, rad/s^2$$

Similarly, the following formula for α_4 can be derived by multiplying Eqn.(4.36) by $\mathrm{e}^{-\mathrm{i}\theta_3}$

$$\alpha_{4i} = \frac{\begin{matrix} \mathrm{r}_2\alpha_{2i} sin(\theta_{2i}-\theta_{3i}) - \mathrm{r}_2{\omega_{2i}}^2 cos(\theta_{2i}-\theta_{3i}) \\ + \mathrm{r}_4{\omega_{4i}}^2 cos(\theta_{4i}-\theta_{3i}) - \mathrm{r}_3{\omega_{3i}}^2 \end{matrix}}{\mathrm{r}_4 \sin(\theta_{4i}-\theta_{3i})}$$

$$\alpha_{41} = \frac{\begin{matrix} 98.82 * 0.02 * sin(-52.3+6.81) - 98.82 * (0.05)^2 cos(-52.3+6.81) \\ + 88.8 * (0.0379)^2 cos(-1.68+6.81) - 48.6(0.044)^2 \end{matrix}}{88.8 * \sin(-1.68+6.81)}$$

$$\alpha_{41} = 0.025 \, rad/s^2$$

$$\alpha_{42} = \frac{\begin{matrix} 98.82 * 0.02 * sin(-17.3+11.81) - 98.82 * (0.05)^2 cos(-17.3+11.81) \\ + 88.8 * (0.042)^2 cos(13.32+11.81) - 48.6(0.073)^2 \end{matrix}}{88.8 * \sin(13.32+11.81)}$$

$$\alpha_{42} = -4.65 \;\; rad/s^2 \backslash$$

$$\alpha_{43} = \frac{98.82 * 0.02 * sin(-2.3 + 16.51) - 98.82 * (0.05)^2 cos(-2.3 + 16.51) + 88.8 * (-0.0699)^2 cos(28.32 + 16.51) - 48.6(0.106)^2}{88.8 * \sin(28.32 + 16.51)}$$

$$\alpha_{43} = 0.0259 \ rad/s^2$$

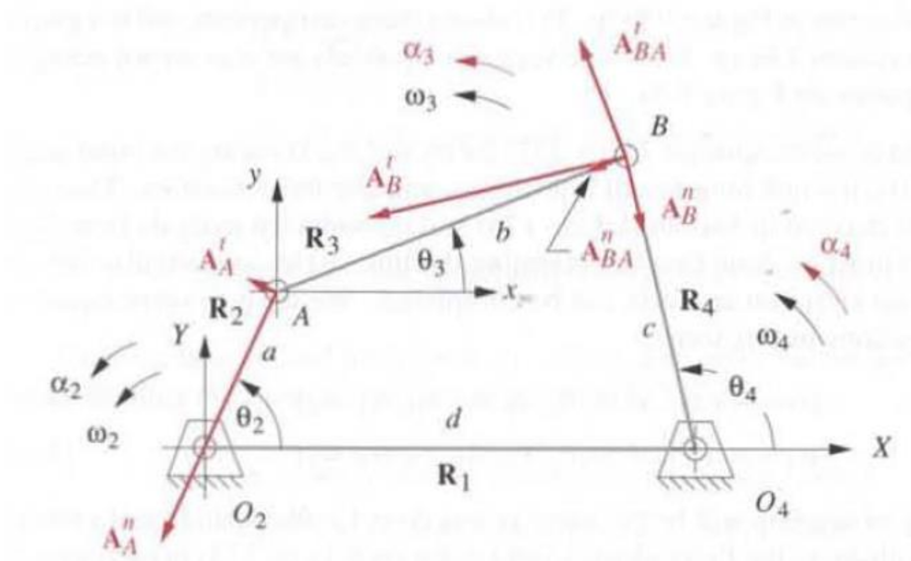


Figure 25: Position vector loop showing acceleration vector [25]

The relative acceleration or acceleration difference equation

$$A_A + A_{B/A} - A_B = 0$$

$$A_A = (A_A^t + A_A^n) = \alpha_{2i} r_2 e^{j\theta_{2i}} - \omega_{2i}{}^2 r_2 e^{j\theta_2} = r_2(\alpha_{2i} - \omega_{2i}{}^2)(\cos\theta_{2i} + j\sin\theta_{2i})$$

$$A_{A21} = 98.82 * (0.02 \ - (0.05)^2)(cos(-52.3) + j\,sin(-52.3))$$

$$A_{A1} = (-0.774 - j1.54) = vector\ solution$$

$$A_{A1} = \sqrt{A_{A1n}{}^2 + A_{A1t}{}^2} = 1.73\, rad/s^2$$

$$A_{A22} = 98.82 * (0.02 \ - (0.05)^2)(cos(-17.3) + j\,sin(-17.3))$$

$$A_{A2} = (0.0367 + j1.73) = 1.73\, rad/s^2$$

$$A_{A23} = 98.82 * (0.02 \ - (0.05)^2)(cos(-2.3) + j\,sin(-2.3))$$

$$A_{A3} = (-1.152 - j1.29) = 1.73\, rad/s^2$$

$$A_{B/A} = \left(A_{B/A}^t + A_{B/A}^n\right) = \alpha_{3i} r_3 e^{j\theta_{3i}} - \omega_{3i}{}^2 r_3 e^{j\theta_{3i}}$$

$$= r_3(\alpha_{3i} - \omega_{3i}{}^2)(cos\,\theta_{3i} + j\,sin\,\theta_{3i})$$

$$A_{B31/A31} = 48.6 * (0.163 - (0.044)^2)(cos(6.81) + j\,sin(6.81))$$

$$A_{B31/A31} = (6.766 + j3.94) = 7.83\, rad/s^2$$

$$A_{B32/A32} = 48.6 * (0.037 - (0.073)^2)(cos(11.81) + j\, sin(11.81))$$

$$A_{B32/A32} = (1.12 - j1.056) = 1.53\, rad/s^2$$

$$A_{B33/A33} = 48.6 * (0.0464\ - (0.106)^2)(cos(16.51) + j\, sin(16.51))$$

$$A_{B3/A3} = (-1.2 - j1.23) = 1.72\, rad/s^2$$

$$A_B = (A_B^t + A_B^n) = \alpha_4 r_4 e^{j\theta_{4i}} + \omega_4{}^2 r_4 e^{j\theta_4} = r_4(\alpha_{4i} - \omega_{4i}{}^2)(cos\,\theta_{4i} + j\, sin\,\theta_{4i})$$

$$A_{B41} = 88.8 * (0.025 - 0.0379\ ^2)(cos(-1.68) + j\, sin(-1.68))$$

$$A_{B41} = (-0.228 - j2.08) = 2.1\, rad/s^2$$

$$A_{B42} = 88.8 * (-4.65 - 0.042^2)(cos\, 13.32 + j\, sin\, 13.32)$$

$$A_{B42} = (-301.12 - j400.15) = 5{,}00\, rad/s^2$$

$$A_{B43} = 88.8 * (0.0259 - (-0.0699)^2)(\cos 28.32 + j\, sin\, 28.32)$$

$$A_{B43} = (-1.864 - j - 0.085) = 1.87\, rad/s^2$$

6.13 Velocity and acceleration analysis of coupler point

Having found the angular velocity of link 3, we can compute the linear velocity of point P on the link as shown below. Starting from positioning vector

$$\mathrm{R}_p = \mathrm{R}_P \mathrm{e}^{\mathrm{j}\theta_6} = \mathrm{r}_2 \mathrm{e}^{\mathrm{j}\theta_{2i}} + \mathrm{r}_5 \mathrm{e}^{\mathrm{j}(\theta_{3i}+\alpha)} = \mathrm{W}_1 \mathrm{e}^{\mathrm{j}\theta_{2i}} + \mathrm{Z}_1 \mathrm{e}^{\mathrm{j}(\theta_{5i})}$$

The Velocity of point p as follows;

$$\vec{V}_{Rp} = i\mathrm{W}_1 \omega_{2i} \mathrm{e}^{\mathrm{j}\theta_{2i}} + iZ_1 \omega_{3i} \mathrm{e}^{\mathrm{j}(\theta_{5i})}$$

The real and imaginary components are then

Real part

$$\vec{V}_{Rpx} = -\mathrm{W}_1 \omega_{2i} \sin\theta_{2i} - Z_1 \omega_{3i} \sin\theta_{5i}$$

$$\vec{V}_{Rpx1} = -98.82 * 0.05 * \sin(-52.3) - 70.5 * 0.044 * \sin 16.48 = 6.58\ dis/s$$

$$\vec{V}_{Rpx2} = -98.82 * 0.05 * \sin(-17.3) - 70.5 * 0.073 * \sin 21.5 = -7.367\ dis/s$$

$$\vec{V}_{Rpx3} = -98.82 * 0.05 * \sin(-2.3) - 70.5 * 0.106 * \sin 31.5 = 3.06\ dis/s$$

Imaginary

$$\vec{V}_{Rpy} = \mathrm{W}_1 \omega_{2i} \cos\theta_{2i} + Z_1 \omega_{3i} \cos\theta_{5i}$$

$$\vec{V}_{Rpy} = 98.82 * 0.05 * \cos(-52.3) + 70.5 * 0.044 * \cos 16.48 = -4.43\ dis/s$$

$$\vec{V}_{Rpy} = 98.82 * 0.05 * \cos(-17.3) + 70.5 * 0.073 * \cos 21.5 = -4.43\ dis/s$$

$$\vec{V}_{Rpy} = 98.82 * 0.05 * \cos(-2.3) + 70.5 * 0.106 * \cos 31.5 = 4.15\ dis/s$$

The magnitude and direction of $\vec{V}_{Rp}$ are.

$$V_{Rp} = \sqrt{{V_{Rpx}}^2 + \left(iV_{Rpy}\right)^2}$$

$$\theta_{Rp} = \tan^{-1}\frac{(V_{Rpy})}{(V_{Rpx})}$$

$$V_{Rp1} = \sqrt{{V_{Rpx1}}^2 + \left(iV_{Rpy1}\right)^2} = \sqrt{6.58^2 + (-4.43)^2} = 7.93\ dis/s$$

$$\theta_{Rp1} = \tan^{-1}\frac{(V_{Rpy1})}{(V_{Rpx1})} = \tan^{-1}\frac{(6.58)}{(-4.43)} = -56.05^0$$

$$V_{Rp2} = \sqrt{{V_{Rpx2}}^2 + (iV_{Rpy2})^2} = \sqrt{(-7.367)^2 + (-4.43)^2} = 8.596\ dis/s$$

$$\theta_{Rp2} = \tan^{-1}\frac{(V_{Rpy2})}{(V_{Rpx2})} = \tan^{-1}\frac{(-7.367)}{(-4.43)} = 58.98^0$$

$$V_{Rp3} = \sqrt{{V_{Rpx3}}^2 + (iV_{Rpy3})^2} = \sqrt{3.06^2 + 4.15^2} = 5.156\ dis/s$$

$$\theta_{Rp3} = \tan^{-1}\frac{(V_{Rpy3})}{(V_{Rpx3})} = \tan^{-1}\frac{(3.06)}{(4.15)} = 36.4^0$$

To find acceleration at point p, differentiating velocity equation with respect to time.

$$\vec{a}_{Rp} = (-{\omega_{2i}}^2 - i\alpha_{2i})\mathrm{W}_1\mathrm{e}^{\mathrm{j}\theta_{2i}} + (-{\omega_{3i}}^2 + i\alpha_{3i})\mathrm{Z}_1\mathrm{e}^{\mathrm{j}\theta_{5i}}$$

The real and imaginary components are then

Real

$$\vec{a}_{Rpx} = -\mathrm{W}_1({\omega_{2i}}^2\cos\theta_{2i} + \alpha_{2i}\sin\theta_{2i}) - \mathrm{Z}_1({\omega_{3i}}^2\cos\theta_{5i} + \alpha_{3i}\sin\theta_{5i})$$

$$\vec{a}_{Rpx1} = -98.82 * (0.05^2\cos(-52.3) + 0.02\sin(-52.3)) - 70.5(0.044^2\cos 16.48 + 0.163\sin 16.48) = -2.207\ dis/s^2$$

$$\vec{a}_{Rpx2} = -98.82 * (0.05^2\cos(-17.3) + 0.02\sin(-17.3)) - 70.5(0.073^2\cos 21.5 + 0.037\sin 21.5) = -0.954\ \ dis/s^2$$

$$\vec{a}_{Rpx3} = -98.82 * (0.05^2\cos(-2.3) + 0.02\sin(-2.3)) - 70.5(0.106^2\cos 31.5 + 0.0464\sin 31.5) = -2.55\ dis/s^2$$

Imaginary

$$\vec{a}_{Rpy} = -\mathrm{W}_1({\omega_{2i}}^2\sin\theta_{2i} - \alpha_{2i}\cos\theta_{2i}) - \mathrm{Z}_1({\omega_{3i}}^2\sin\theta_{5i} - \alpha_{3i}\cos\theta_{5i})$$

$$\vec{a}_{Rpy1} = -98.82(0.05^2\sin(-52.3) - 0.02 * \cos(-52.3)) - 70.5(0.044^2\sin 16.48 - 0.163 * \cos 16.48) = 12.31\ dis/s^2$$

$$\vec{a}_{Rpy2} = -98.82(0.05^2\sin(-17.3) - 0.02 * \cos(-17.3)) - 70.5(0.073^2\sin 21.5 - 0.037 * \cos 21.5) = 4.25\ dis/s^2$$

$$\vec{a}_{Rpy3} = -98.82(0.05^2 \sin(-2.3) - 0.02 * \cos(-2.3)) - 70.5(0.106^2 \sin 31.5 - 0.0464 * \cos 31.5) = 4.45\ dis/s^2$$

The magnitude and direction of $\vec{a}_{Rp}$ are.

$$a_{Rp} = \sqrt{{a_{Rpx}}^2 + (ia_{Rpy})^2}$$

$$a_{Rp1} = \sqrt{{a_{Rpx1}}^2 + (ia_{Rpy1})^2} = \sqrt{(-2.207)^2 + (12.31)^2} = 12.51\quad dis/s^2$$

$$\theta_{Rp1} = \tan^{-1}\frac{(a_{Rpy1})}{(a_{Rpx1})} = \tan^{-1}\frac{(12.31)}{(-2.207)} = -79.84^0$$

$$a_{Rp2} = \sqrt{{a_{Rpx2}}^2 + (ia_{Rpy2})^2} = \sqrt{(-0.954)^2 + (4.25)^2} = 4.355\quad dis/s^2$$

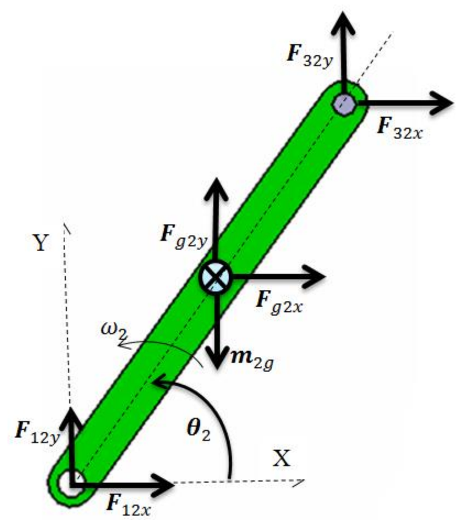
$$\theta_{Rp2} = \tan^{-1}\frac{(a_{Rpy2})}{(a_{Rpx2})} = \tan^{-1}\frac{(4.25)}{(-0.954)} = -77.35^0$$

$$a_{Rp3} = \sqrt{{a_{Rpx3}}^2 + (ia_{Rpy3})^2} = \sqrt{(-2.55)^2 + (4.45)^2} = 3.31\quad dis/s^2$$

$$\theta_{Rp3} = \tan^{-1}\frac{(a_{Rpy2})}{(a_{Rpx2})} = \tan^{-1}\frac{(4.45)}{(-2.55)} = -60.2^0$$

6.2 Dynamic Force Analysis

Link 2



Length of Link $2W_1 = r_2 = 98.82$ cm

$$W_{1x} = r_{2x} = 60.41$$
$$W_{1y} = r_{2y} = -78.2$$

$m_2 = 8.38\ kg$

Acceleration of link 2 at center of gravity

$$A_{C.G2} = r_{2\ CG}(\alpha_{2i} - \omega_{2i}{}^2)(\cos\theta_{2i} + j\sin\theta_{2i})$$
$$A_{\ C.G} = 49.42 * (0.02 - 0.05^2)(\cos(-52.3) + j\sin(-52.3))$$
$$A_{\ C.G} = 0.53 - J0.682 = 0.862\frac{cm}{s^2}@-52.3^0$$

Mass moment of inertia about its CG= $r_2 = 49.42\ cm$

$$I_{g2} = \frac{m_2 r_2{}^2}{g12} = \frac{8.38 * 98.82^2}{12} = 682\ \ kg - cm^2$$

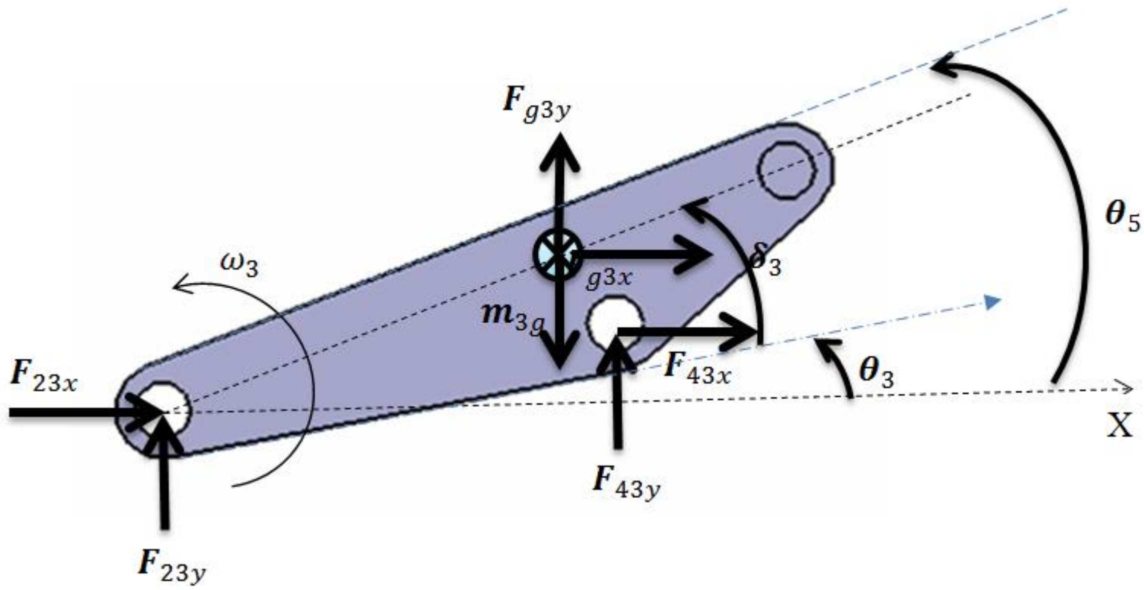
Moment about the center of gravity of the link 2

$$T_{g2} = I_{g2} * \alpha_2 = 6.95\ \ kg - cm^2 * 0.02\frac{rad}{s^2} = 0.139 kg.cm/s^2$$

$$F_{g2x} = m_{2g} * A_{CG2x} = (8.38 * 0.53) = 4.44\ kg.cm/s^2$$

$$F_{g2y} = m_{2g} * A_{CG2y} = 8.38 * (-0.682) = -5.72\ kg.cm/s^2$$

Link 3



Length of Link 2 $Z_1 = r_3 = 70.5$ cm

$$Z_{1x} = r_{Px} = 67.6$$
$$Z_{1y} = r_{Py} = 20$$

$m_3 = 7.89\ kg$

Acceleration of link 3 at center of gravity (C.G)

$$\bar{x} = \mathrm{r}_{Px\ CG} = \frac{\mathrm{r}_P\cos(\beta)+\mathrm{r}_3}{3} = \frac{70.5*\cos(9.969)+48.6}{3} = 39.35\ cm$$

$$\bar{y} = \mathrm{r}_{Py\ CG} = \frac{\mathrm{r}_P\cos(\beta)}{3} = \frac{70.5*\sin 9.969}{3} = 4.06\ cm$$

$$\mathrm{r}_{P\ CG} = 39.55cm\ @\ 5.89^0$$

$$\vec{a}_{pCGx} = -\mathrm{r}_2({\omega_{21}}^2\cos\theta_{21} + \alpha_{21}\sin\theta_{21}) - \mathrm{r}_{P\ CG}({\omega_{31}}^2\cos\theta_{51} + \alpha_{3i}\sin\theta_{51})$$

$$\vec{a}_{Rpx} = -98.82 * (0.05^2\cos(-52.3) + 0.02\sin(-52.3)) - 39.55\ (0.044^2\cos 16.48 + 0.163\sin 16.48)$$

$$\vec{a}_{pC.Gx} = -0.5\ cm/s^2$$

$$\vec{a}_{Rpy} = -\mathrm{r}_2({\omega_{21}}^2\sin\theta_{21} - \alpha_{21}\cos\theta_{21}) - \mathrm{r}_{pCG}({\omega_{31}}^2\sin\theta_{51} - \alpha_{31}\cos\theta_{51})$$

$$\vec{a}_{Rpy} = -98.82(0.05^2\sin(-52.3) - 0.02\cos(-52.3)) - 39.55(0.044^2\sin(16.48) - 0.163\cos(16.48))$$

$$\vec{a}_{Rpy} = 7.56\ \ cm/s^2$$

$$\vec{a}_{C.G3} = 7.581\frac{cm}{s^2}\ @-3.78^0$$

Mass moment of inertia about its CG= $r_p = 39.55\ cm$

$$I_{g3} = 4715\ \ kg - cm^2$$

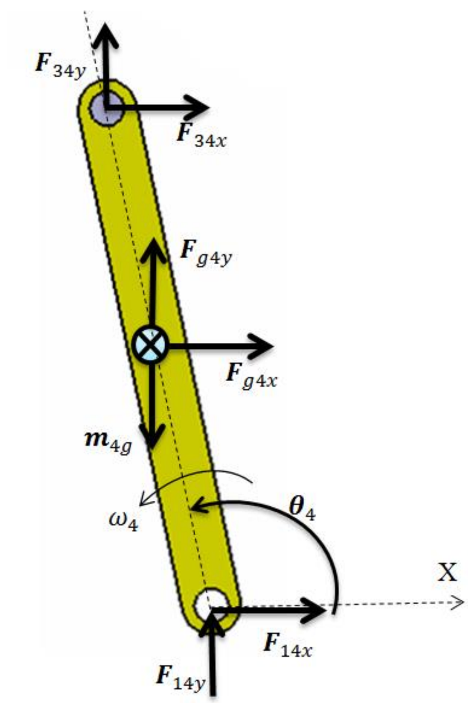
Moment about the center of gravity of the link 3

$$T_{g3} = I_{g3} * \alpha_3 = 19.74\ kg - cm^2 * 0.163\ rad/s^2 = 3.22kg - cm^2/s^2$$

$$F_{g3x} = m_{3g} * A_{CG3x} = 7.89\ kg * (-0.5\ )cm/s^2\ = -3.945\ kg\ cm\ /s^2$$

$$F_{g3y} = m_{3g} * A_{CG3y} = 7.89\ kg * 7.56\ cm/s^2\ = 59.65\ kg\ cm\ /s^2$$

Link 4



Length of Link $2U_1 = r_4 = 88.8$ cm@ -1.68^0

$$r_{4x} = 23.95$$

$$r_{4y} = -3.8$$

Weight= $7.57\ kg$

Acceleration of link 4 at center of gravity

$$A_{C.G4} = r_{4\,CG}(\alpha_{41} - \omega_{41}{}^2)(cos\,\theta_{41} + j\,sin\,\theta_{41})$$

$$A_{\,C.G4} = 44.4 * (0.025 - 0.0379\,^2)(\cos(-1.68) + j\sin(-1.68))$$

$$A_{\,C.G4} = 1.045 - J0.031 = 1.045\frac{cm}{s^2}@{-1.68^0}$$

Mass moment of inertia about its CG= $r_4 = 44.4\ cm$

$$I_{g4} = \frac{m_4 r_4{}^2}{12} = \frac{7.57 * 88.8^2}{12} = 4974\ \ kg - cm^2$$

Moment about the center of gravity of the link 4

$$T_{g4} = I_{g4} * \alpha_{41} = 19.74\ kg.cm^2 * 0.163\frac{rad}{s^2} = 3.22kg.cm^2/s^2$$

$$F_{g4x} = m_4 * A_{C.G4x} = 7.57\ kg * 1.045\ cm/s^2\ = 7.91\ kgcm/s^2$$

$$F_{g4y} = m_4 * A_{C.G4y} = 7.57\ kg * (-0.031cm/s^2)\ = -0.235\ kgcm/s^2$$

$M = r_2 sin\theta_2 - r_{g2} cos\theta_2 = 98.82 * \cos(-52.3) - 49.41\cos(-52.3) = 30.2\ cm$

$N = -r_2\cos\theta_2 + r_{g2} cos\theta_2 = -98.82 * \cos(-52.3) + 49.41\cos(-52.3) = -30.2\ cm$

$L = r_3 \sin\theta_3 - r_{g3}\cos(\theta_3 + \delta_3) = 70.5\sin(6.81^0) - 39.55\cos(6.81 + 5.89^0) = 30.58\ cm$

$s = -r_3\cos\theta_3 + r_{g3}\cos(\theta_3 + \delta_3) = -70.5\cos(6.81) + 39.55\cos\left(6.81 + 5.89^0\right) = -31.42\ cm$

$Q = -r_4 \sin\theta_4 + r_{g4}\cos\theta_4 = -88.8\sin(-1.68^0) + 44.4\cos(-1.68) = 47\ cm$

$T = r_4\cos\theta_4 + r_{g4}\cos\theta_4 = 88.8\cos(-1.68) + 44.4\cos(-1.68) = 133.14\ cm$

$r_{g2} sin\theta_2 = 49.41 sin(-52.3) = -39.1\ cm$

$-r_{g2} cos\theta_2 = -49.41\cos(-52.3) = -30.21\ cm$

$r_{g3}\sin(\theta_3 + \delta_3) = 39.55 * \sin(6.81 + 5.89^0) = 8.5\ cm$

$-r_{g3}\sin(\theta_3 + \delta_3) = -39.55 * \sin(6.81 + 5.89^0) = -8.69\ cm$

$r_{g4} sin\theta_4 = 44.4 * \sin(-1.68) = -1.3\ cm$

$-r_{g4} sin\theta_4 = -44.4 * \sin(-168) = 1.3\ cm$

$T_1 = 100kg.cm/s^2$ Expected output torque

$$\begin{bmatrix} 4.44 \\ -5.72 \\ 0.139 \\ -3.945 \\ 59.65 \\ 3.22 \\ 7.91 \\ -0.235 \\ 103 \end{bmatrix} = \left\{ \begin{matrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -39.1 & -30.21 & 30.2 & -30.2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 8.5 & -8.69 & 30.58 & -31.42 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 47 & 133.14 & -1.3 & -1.3 & 0 \end{matrix} \right\} \begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{34y} \\ F_{14x} \\ F_{14y} \\ T_s \end{bmatrix}$$

$$\boldsymbol{b} = \boldsymbol{Ax}$$

$$\boldsymbol{A^{-1}} = \left\{\begin{matrix} 1 & 0 & 0 & 0.8102 & 0.19397 & 0.0223 & -0.185 & 0.0086 & 0.0066 \\ 0 & 1 & 0 & -0.122 & 1.124 & 0.0143 & -1.109 & 0.0152 & 0.0117 \\ 0 & 0 & 0 & 0.8103 & 0.1939 & 0.0223 & -0.185 & 0.0086 & 0.0066 \\ 0 & 0 & 0 & -0.1215. & 1.12428 & 0.0143 & -1.109 & 0.015 & 0.0117 \\ 0 & 0 & 0 & -0.1897 & 0.1939 & 0.0223 & -0.185 & 0.0086 & 0.0066 \\ 0 & 0 & 0 & 0.06811 & -0.0696 & -0.00801 & 0.076 & 0.0065 & 0.005 \\ 0 & 0 & 0 & 0.1897 & -0.1939 & -0.02232 & 1.185 & -0.0086 & -0.006 \\ 0 & 0 & 0 & -0.068 & 0.06968 & 0.0080 & -0.076 & 0.993 & -0.005 \\ -39.1 & -30.21 & -1 & -3.538 & -35.69 & -0.630 & 35.15 & -0.536 & -0.412 \end{matrix}\right\}$$

$$\boldsymbol{x = A^{-1}x}$$

$$\begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{34y} \\ F_{14x} \\ F_{14y} \\ T_s \end{bmatrix} = \left\{\begin{matrix} 1 & 0 & 0 & 0.8102 & 0.193 & 0.0223 & -0.185 & 0.0086 & 0.006 \\ 0 & 1 & 0 & -0.122 & 1.124 & 0.0143 & -1.109 & 0.0152 & 0.011 \\ 0 & 0 & 0 & 0.8103 & 0.193 & 0.0223 & -0.185 & 0.0086 & 0.006 \\ 0 & 0 & 0 & -0.12 & 1.12 & 0.0143 & -1.109 & 0.015 & 0.0117 \\ 0 & 0 & 0 & -0.189 & 0.1939 & 0.0223 & -0.185 & 0.0086 & 0.0066 \\ 0 & 0 & 0 & 0.068 & -0.06 & -0.008 & 0.076 & 0.0065 & 0.005 \\ 0 & 0 & 0 & 0.1897 & -0.19 & -0.022 & 1.185 & -0.00 & -0.006 \\ 0 & 0 & 0 & -0.068 & 0.069 & 0.0080 & -0.076 & 0.993 & -0.005 \\ -39.1 & -30.21 & -1 & -3.538 & -35.69 & -0.630 & 35.15 & -0.536 & -0.412 \end{matrix}\right\} \begin{bmatrix} 4.44 \\ -5.72 \\ 0.139 \\ -3.945 \\ 59.65 \\ 3.22 \\ 7.91 \\ -0.235 \\ 103 \end{bmatrix}$$

$$\begin{bmatrix} F_{12x} \\ F_{12y} \\ F_{23x} \\ F_{23y} \\ F_{34x} \\ F_{34y} \\ F_{14x} \\ F_{14y} \\ T_s \end{bmatrix} = \begin{bmatrix} 12.10 \\ 54.29 \\ 7.66 \\ 60.01 \\ 11.60 \\ -3.33 \\ -3.698 \\ 3.095 \\ -1882 \end{bmatrix}$$

$$F_{12} = \sqrt{{F_{12x}}^2 + {F_{12y}}^2} = 55.62\ N\ @77.4^0$$

$$F_{23} = \sqrt{{F_{23x}}^2 + {F_{23y}}^2} = 60.5\ N@82.7^0$$

$$F_{34} = \sqrt{{F_{34x}}^2 + {F_{34y}}^2} = 12.1\ N@-16.02^0$$

$$F_{14} = \sqrt{{F_{14x}}^2 + {F_{14y}}^2} = 4.82\ N@-39.9^0$$

CHAPTER SEVEN

RESULTS AND DISCUSSIONS

7.1 Results

The results found analytical and CAD starting from estimated for the three prescribed path points configurations of planer four bar linkage, the result are approximately the same.

Assume value to generate point “p’’ p_x & p_y Horizontal and vertical projection of a coupler point with respect to coupler link.

p_x = [0 40 50 80] X - axis distance relative to origin 0_2

p_y = [0 120 300 270] Y- axis distance relative to origin 0_2

Table 2**:** four bar linkage configuration of generated analytical and MATLAB solution in (cm)

linkages	Linkage length	
	Analytical solution	MATLB solution
Fixed link (bar) $G_1 = r_1$	139.69	135.9278
Input link ($W_1 = r_2$)	98.82	96.6465
Coupler link ($V_1 = r_3$)	48.6	56.9016
Output link ($U_1 = r_4$)	88.8	86.8353
coupler point ($Z_1 = r_p$)	70.5	78.4478
Link (S_1)	24.25	26.5453

Table 3**:** Output rotational angle of linkages

	Input of Prescribed path points		Output linkage rotational angle			
For i	$\boldsymbol{p_x}$	$\boldsymbol{p_y}$	θ_{2i}	θ_{3i}	θ_{4i}	θ_{5i}(coupler point)
1	40	120	-52.3^0	6.81^0	-1.68^0	16.48^0
2	50	300	-17.3^0	11.81^0	13.32^0	21.5^0
3	80	270	-2.3^0	16.51^0	28.32^0	31.5^0

Table 4**:** Output angular velocity and acceleration of linkages

$\omega_2 = 0.05\, rad/s$,$\alpha_2 = 0.02\, rad/s^2$,

	Output linkage angular velocity (rad/s)			Output linkage angular acceleration (rad/s^2)		
For i	$\omega_{3i} = \omega_{5i}$ (coupler point)		ω_{4i}	α_{3i}	$\alpha_{4i} = \alpha_{5i}$	
1	0.044		0.0379	0.163	0.025	
2	0.073		0.042	0.037	−4.65	
3	0.106		−0.0699	0.0464	0.0259	
	Output linkage linear velocity $(c.m/s)$			Output linkage linear acceleration $(c.m/s^2)$		
	$v_{3i} = V_{Bi/Ai}$	$v_{4i} = V_{Bi}$	v_{5i}(coupler point)	$a_{3i} = A_{Bi/Ai}$	$a_{4i} = A_{Bi}$	a_{5i} (coupler point)
1	2.14	3.36	7.93	7.83	2.1	12.51
2	3.55	3.73	8.596	1.53	-	4.355
3	5.15	6.21	5.156	1.72	1.87	3.31

For

Table 5: Joint forces acting on links (MATLAB solution)

For $\theta_2 = -52$

	Link 2	link 3	Link 4	Link 1
Joint force	$F_{12} =$ 1.6698e+02 + 2.6902e+01 i	$F_{23} =$ 89.1775 +14.4128i	$F_{34} = 1.0691e+02 - 1.4376e+01i$	$F_{14} = 1.7009e+02 - 2.3391e+01i$
Force angle	$theta\_F_{12} = 55.1681 +$	$theta\_F_{23} = 10.6805 + 14.7750i$	$theta\_F_{34} = -28.4777 + 17.2606i$	$theta\_F_{14} = 1.1544e+02 + 5.0517e-01i$

	2.1100i			

For $\theta_2 = -17.3$

	Link 2	link 3	Link 4	Link 1
Joint force	$F_{12} =$ 651.3817	$F_{23} =$ 570.4392	$F_{34} =$ 494.0204	$F_{14} =$ 397.5791
Force angle	$theta\_F_{12} =$ 101.3288	$theta\_F_{23} =$ 98.8030	$theta_{F_{34}} =$ 106.4203	$theta_{F_{14}} = -65.7822$

For $\theta_2 = -2.3$

	Link 2	link 3	Link 4	Link 1
Joint force	$F_{12} = 3.7321 - 1.6919i$	$F_{23} = 3.0139 - 1.7245i$	$F_{34} = 2.4012 - 2.5369\text{i}$	$F_{14} = 1.3769 - 2.5541\text{i}$
Force angle	$theta\_F_{12} = 87.9010 - 21.8406i$	$theta\_F_{23} = -89.0367 - 25.0178i$	$theta_{F_{34}} = -82.1424 - 10.7902i$	$theta_{F_{14}} = -83.5237 - 1.0964i$

CATIA V5R21 Result

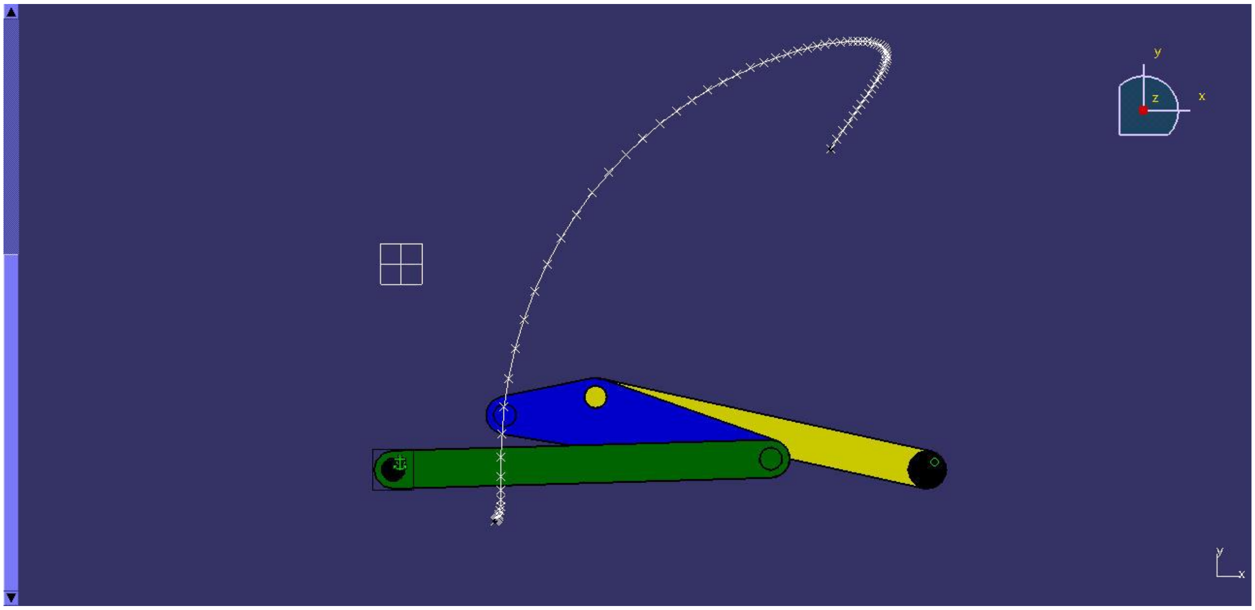


Figure 26: generated path point result one

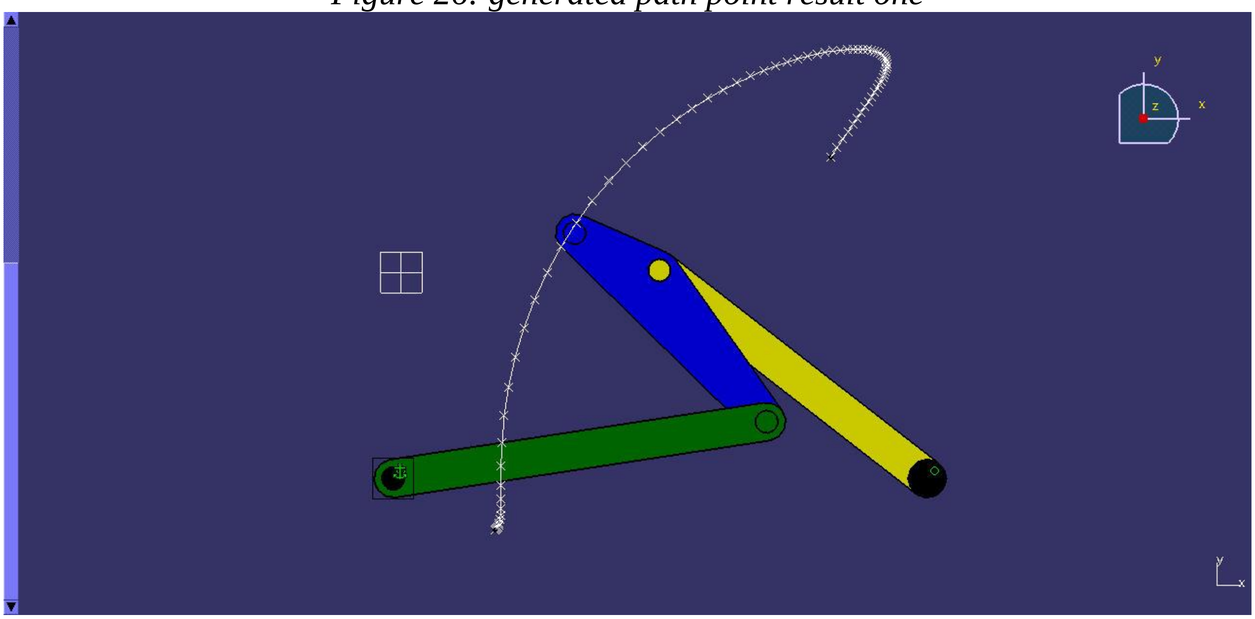


Figure 27: generated path point result tow

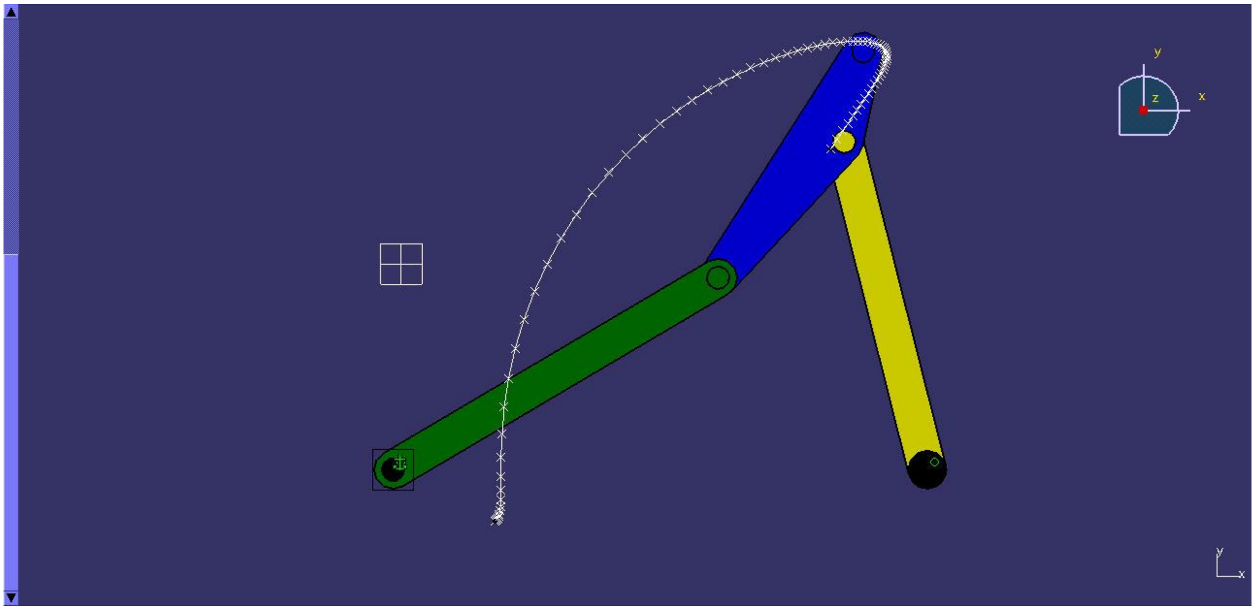


Figure 28: generated path point result three

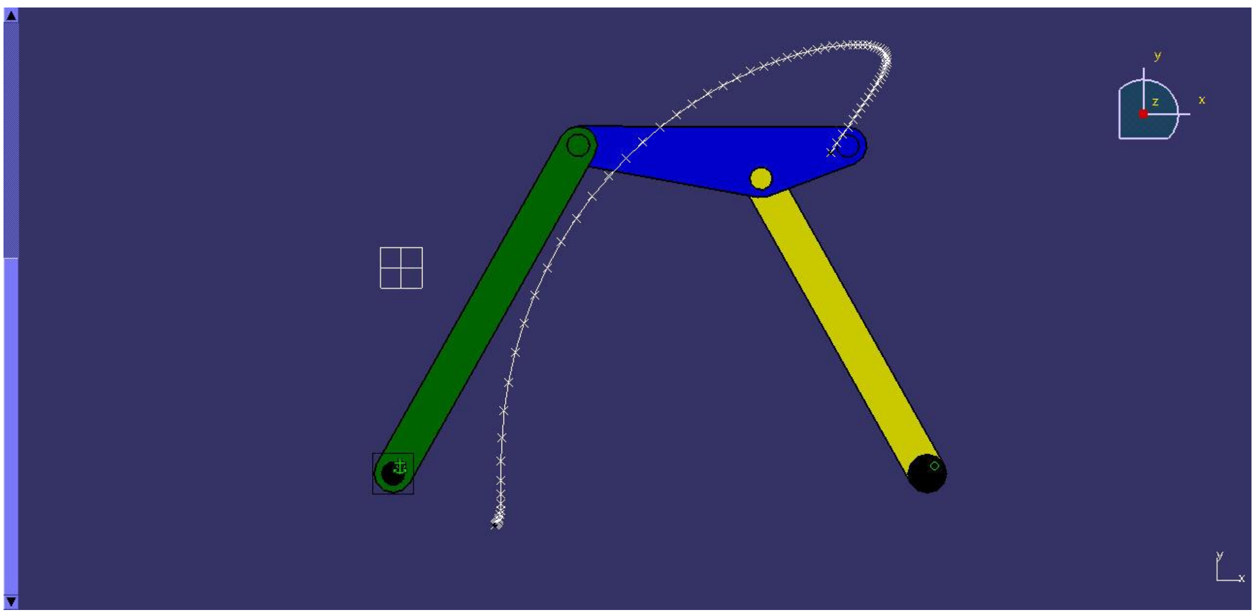


Figure 29 : generated path point result four

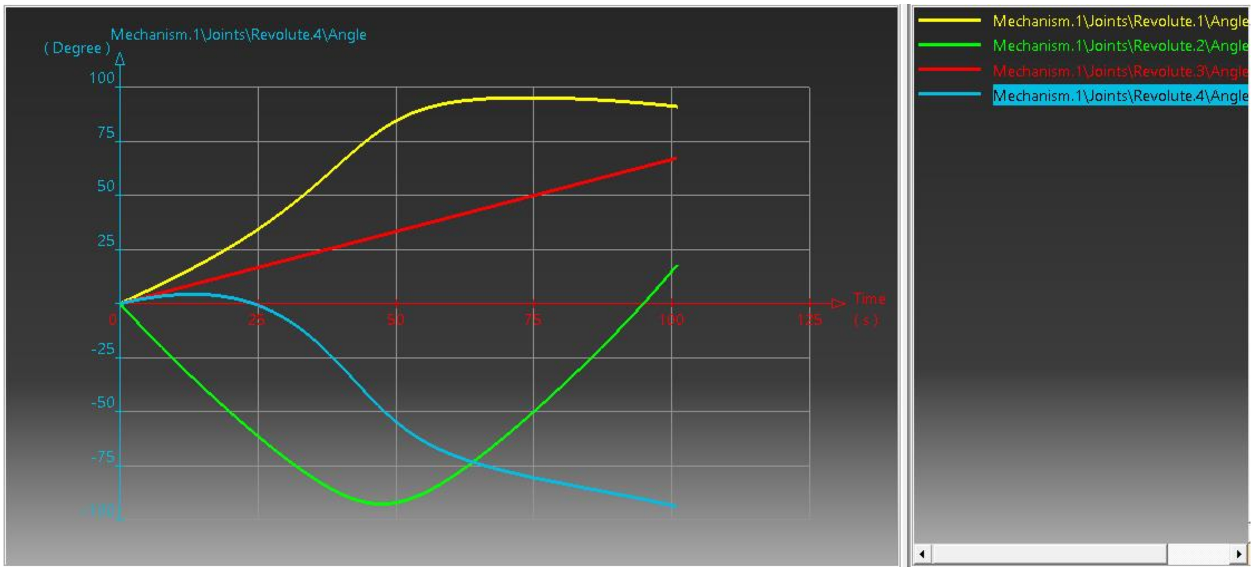


Figure 30: Input and Output rotational angle of linkages

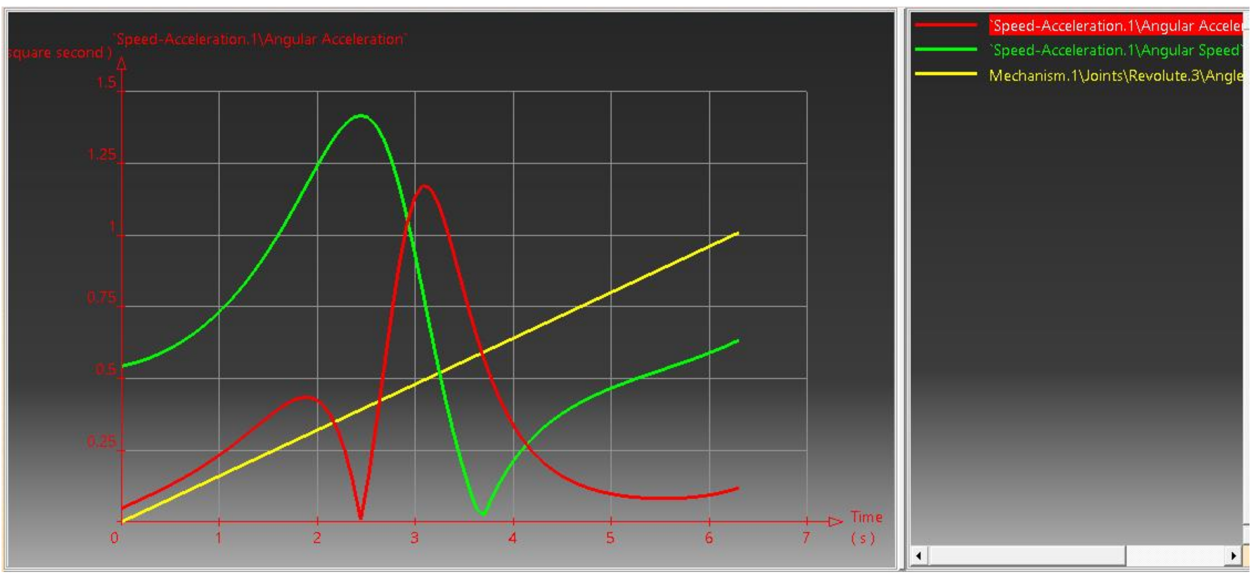


Figure 31: Angular speed and angular acceleration of coupler linkage point

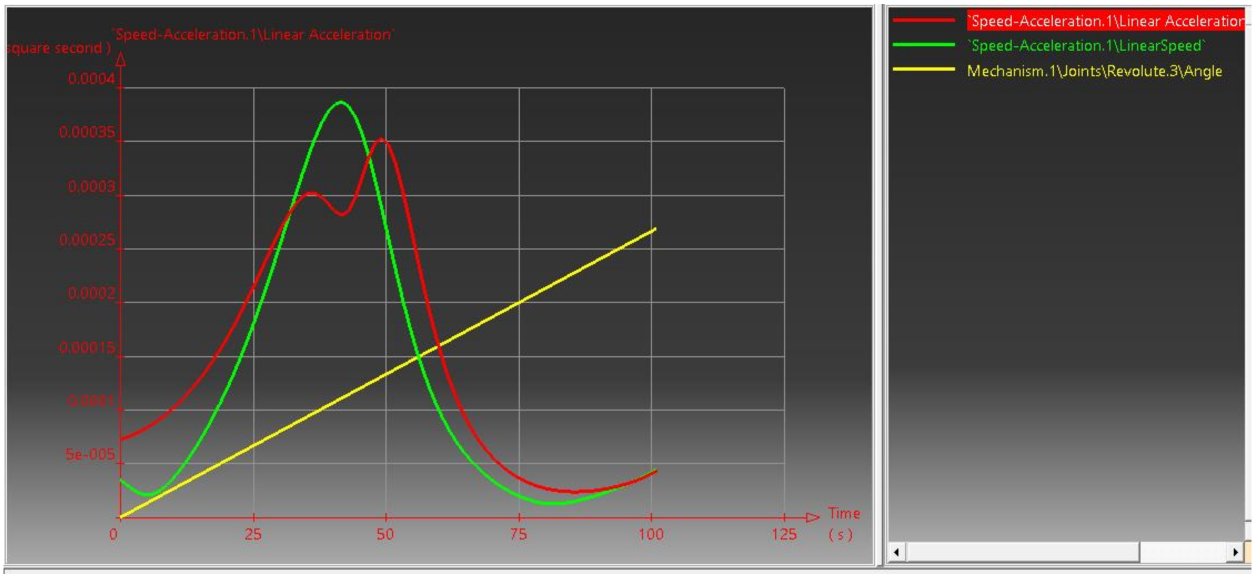


Figure 32 linear speed and linear acceleration of coupler linkage point

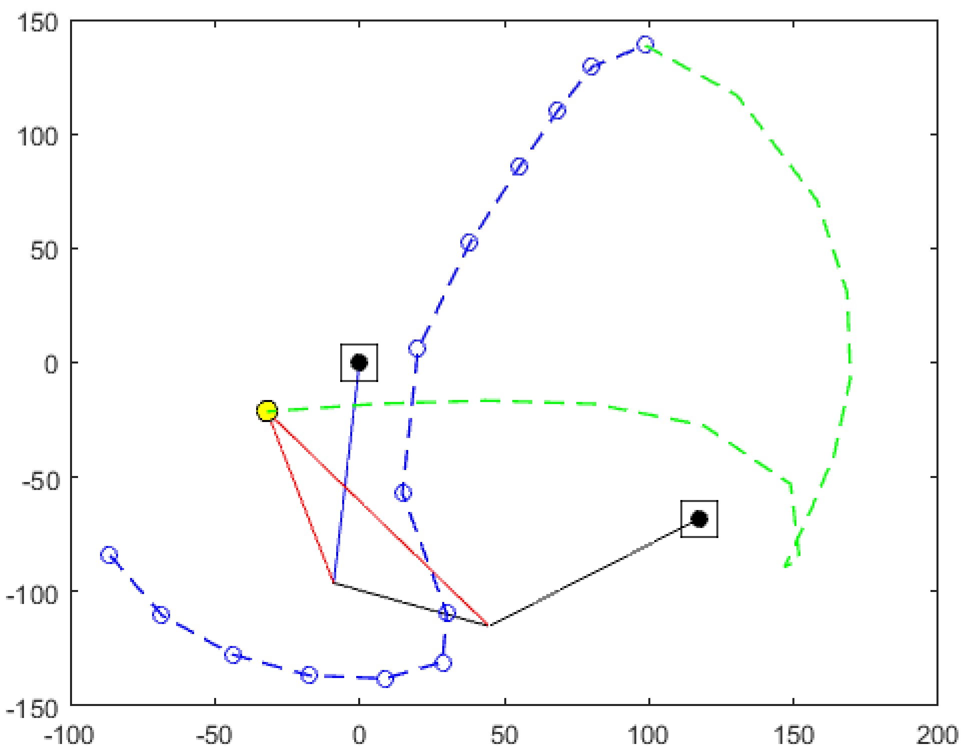


Figure 33: planaer four bar link generated path (MATLAB® R2016b)

CONCULISION

In this work, mechanisms and in particular planar four bar linkages have been studied and their components and classifications are introduced that are capable of producing a different output path. The objective of this work is kinematic and dynamic analysis of planar for bar linkage for path generation and to remind or support a designer with analytical & CAD tools and knowledge to design a four-bar mechanism to produce a different path of coupler curve that would meet the design criteria. Position analyses of all the links in a four-bar mechanism have done using the loop closure equation in complex polar form method. Analytical example has done at three positions and an application model built upon MATLAB code developed to evaluate the performance of the various conditions. position, velocity, acceleration and dynamic force in each pin joint reaction due to self-weight of linkages and using the calculated or generated linkage dimension easily in analytical and MATLAB have simulated in CATIA to determine the kinematic motion and generated path of planer four bar linkage.

REFERENCE

[1] Tong, Y., *Four-bar linkage synthesis for combination of motion and path-point generation*, M. Sc. Thesis, School of Engineering, University of Dayton, Ohio, USA, May. 2013.

[2] Galal Ali Hassaan, *synthesis of planar mechanisms, part iii: four-bar mechanisms for three coupler-positions generation*, Global journal of advanced research, Vol-2, Issue-4 PP. 726-734 ISSN: 2394, 2015.

[3] Wen-Yi Lin, *A GA–DE hybrid evolutionary algorithm for path synthesis of four-bar linkage*, Mechanism and Machine Theory 45, pp. 1096–1107. Elsevier Ltd., 2010

[4] H. Zhou, Kwun-Lon Ting, *Adjustable slider–crank linkages for multiple path generation*, Mechanism and Machine Theory 37, pp. 499–509, 2002.

[5] H. Zhou a, Edmund H.M. Cheung, *Adjustable four-bar linkages for multi-phase motion generation*, Mechanism and Machine Theory 39, pp. 261–279, 2004.

[6] Yu Hongying, Tang Dewei, Wang Zhixing, *Study on a new computer path synthesis method of a four-bar linkage*, Mechanism and Machine Theory 42, pp.383–392, 2007.

[7] S.K. A charyya, M. Mandal, *Performance of EAs for four-bar linkage synthesis*, Mechanism and Machine Theory 44, pp. 1784–1794, 2009.

[8] Monish P. Wasnik1, M. K. Sonpimple, S. K. Undirwade, *Optimal synthesis of a Path Generator Linkage using Non-Conventional Approach*, IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE), PP 38-44, 2014.

[9] Ren-Chung Soong and Sun-Li Wu, *Design of Variable Coupler Curve Four-bar Mechanisms*, Journal of the Chinese Society of Mechanical Engineers, Vol.30, No.3, pp.249-257, 2009.

[10] Hong Zhou, *Dimensional synthesis of adjustable path generation linkages using the optimal slider adjustment*, Mechanism and Machine Theory 44, pp. 1866–1876, 2009.

[11] Hong-Sen Yan, Ren-Chung Soong, *Kinematic and dynamic design of four-bar linkages by links counterweighing with variable input speed*, Mechanism and Machine Theory 36, pp.1051-1071, 2001.

[12] N. Nariman-Zadeh, M. Felezi, A. Jamali, M. Ganji, *Pareto optimal synthesis of four-bar mechanisms for path generation*, Mechanism and Machine Theory 44, pp. 180-191, 2009.

[13] Mekonnen Gebreslasie and Alem Bazezew, *Synthesis, Analysis and Simulation of a Four-Bar Mechanism Using Matlab Programming*, Department of Mechanical Engineering Addis Ababa University, Journal of EAEA, Vol. 18,2001.

[14] Gordon R. Pennock Ali Israr, BKinematic *analysis and synthesis of an adjustable six-bar linkage*, Mechanism and Machine Theory 44, pp. 306–323, 2009.

[15] H. Zhou, Edmund H.M. Cheung, *Analysis and optimal synthesis of adjustable linkages for path generation*, Mechatronics 12, pp. 949–961, 2002.

[16] Himanshu Chaudhary, S. Kumar Saha, *balancing of four-bar linkages using maximum recursive dynamic algorithm*, Mechanism and Machine Theory 42, pp. 216-232, (2007)

[17] Mehmet ismet Can Dede, Duygu Comen, *kinematic synthesis of path generation of planer mechanism*, Scientific journal of IFToMM, No 3(48),2012.

[18] Radovan R. Bulatovic ,Stevan R. Djordjevic, *Optimal Synthesis of a Four-Bar Linkage by Method of Controlled Deviation*, Theoret, Appl. Mech., Vol.31, No.3-4,pp. 265-280, Belgrade 2004.

[19] Han Jianyou, Qian Weixiang, Zhao Huishe, *Study on synthesis method of λ-formed 4-bar linkages approximating a straight line*, Mechanism and Machine Theory 44, pp.57–65, 2009.

[20] J. L. Meriam, L. G. Kraige, *Engineering Mechanics Dynamics*, Vol. 2, Seven edition, John Wiley & Sons Inc., 2012.

[21] A.N. Erdman, G.N. Sandor, *Advanced Mechanism Design: Analysis and Synthesis*, vol. 2, Prentice-Hall Inc.,1984.

[22] J.S. Rao, R.V. Dukkipati, *Mechanism and Machine theory*, New Age International (P) Ltd.1992.

[23] David H. Myszka, *Machines and Mechanisms: Applied Kinematic Analysis*, fourth edition, Prentice-Hall Inc., 2012.

[24] Sadhu Singh, *Theory of Machines: Kinematics and Dynamics*, third edition, Pearson Education India.

[25] Robert L. Norton, *Design of Machinery: An Introduction to The Synthesis and Analysis of Mechanisms and Machines*, Third Edition, McGraw-Hill Inc., 2004.

[26] Oleg Vinogradov, *Fundamentals of Kinematics and Dynamic of Machines and Mechanisms*, CRC Press LLC, 2000.

[27] George H. Martin, *Kinematics and Dynamic of Machines*. Second edition, McGraw-Hili inc., 1982.

[28] Richard S. hartenberg and Jacques Denavit, *Kinematic Synthesis of Linkage*, McGraw-Hill Inc., 1964.

[29] Wandling, George Randall, *Synthesis of mechanisms for function, path, and motion generation using invariant characterization, storage and search methods*, Retrospective Theses and Dissertations. Paper 12383, 2000.

[30] Natesan, Arun K., *Kinematic analysis and synthesis of four-bar mechanisms for straight line coupler curves*. Thesis. Rochester Institute of Technology. Accessed from, (1994).

[31] Mohammad Y. Saadeh ‘‘Plot Any Four-Bar Linkage’ ’May, 10, 2010, University of Nevada Las Vegas,

[32] Radovan R. Bulatovic & Stevan R. Djordjevic, *Optimal Synthesis of a Four-Bar Linkage by Method of Controlled Deviation*, University of Kragujevac, Kraljevo, Serbia and Montenegro. Theoret. Appl. Mech., Vol.31, No.3-4, pp. 265-280, Belgrade 2004.

APPENDEX

Part A

```
clc;
Px0=[0 40 50 80];     % input any path point you want to generate
Py0=[0 120 300 270];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
       bet2=35;      alph2=5;    phi2=15;
       bet3=50;      alph3=10;   phi3=30;

   thp21=atan2((Py0(2)-Py0(1)),(Px0(2)-Px0(1)));
   thp31=atan2((Py0(3)-Py0(1)),(Px0(3)-Px0(1)));
        p21=sqrt((Py0(2)-Py0(1))^2+(Px0(2)-Px0(1))^2);
        p31=sqrt((Py0(3)-Py0(1))^2+(Px0(3)-Px0(1))^2);
                delta21 = 180+thp21*(180/pi);
                delta31 = 180+thp31*(180/pi);
Awz=[(cos(bet2)-1) -sin(bet2) (cos(alph2)-1) -sin(alph2);...
    (cos(bet3)-1) -sin(bet3) (cos(alph3)-1) -sin(alph2);...
    sin(bet2) (cos(bet2)-1) sin(alph2) (cos(alph2)-1);...
    sin(bet3) (cos(bet3)-1) sin(alph3) (cos(alph3)-1)];

P=[p21*cos(delta21);p31*cos(delta31);p21*sin(delta21);p31*sin(delta31)];
disp(P)
%R=[R2x;R2y;Rpx;Rpy];
W1=Awz\P;
W1x=W1(1,1);
W1y=W1(2,1);
Z1x=W1(3,1);
Z1y=W1(4,1);
W1=sqrt(W1(1,1)^2+W1(2,1)^2);% = W1
Z1=sqrt((Z1y)^2+(W1y)^2);% = Z1
r2=W1;
rp=Z1;
      theta21=atan2((W1y), W1x);
Bus=[(cos( phi2)-1) -sin( phi2) (cos(alph2)-1) -sin(alph2);...
    (cos(phi3)-1) -sin(phi3) (cos(alph3)-1) -sin(alph2);...
    sin( phi2) (cos( phi2)-1) sin(alph2) (cos(alph2)-1);...
    sin(phi3) (cos(phi3)-1) sin(alph3) (cos(alph3)-1)];
U=Bus\P;
U1x=U(1,1);
U1y=U(2,1);
S1x=U(3,1);
S1y=U(4,1);
U1=sqrt(U1x^2+U1y^2);% =
    theta41=atan2(U1y,U1x);
theta3=atan2((W1y-U1y),(W1x-U1x));
theta5=atan2(Z1y,Z1x);
beta = theta5-theta3;
S1=sqrt(S1x^2+S1y^2);
     psi1=atan2(S1y,S1x);
```

```
V1=sqrt(Z1^2+S1^2- 2*Z1*S1*cos(theta5-psi1));

G1=sqrt(W1^2+V1^2+U1^2+2*W1*V1*cos(theta21-theta3)...
    -2*W1*U1*cos(theta21-theta41)-2*V1*U1*cos(theta3-theta41));
%--------------------------------------------------------------------
r1 = G1;
r4 = U1;
r3 = V1 ;
% ASSUMED VALUE TO BE GENARATED BY POINT " P " IN Px0 AND Py0 DIRECTIION
Px=rp*cos(beta);%---------------------horizontal projection of coupler point wrt coupler
Py=rp*sin(beta);%                     vertical projection of coupler point wrt coupler

X = [r1  r2  r3  r4  Px  Py ];
% ----------------------------------- Cu: x coordinate for coupler point wrt crank-coupler point
%------------------------------------ Cv: y coordinate for coupler point wrt crank-coupler point

cycles = 1.5;%------------------------- number of crank rotations
Incriment = 35;%------------------- divide a rotation into this number
% let
P = X(1:4);
check = P;
[L, locL] = max(check);
check(locL) = [];
[S, locS] = min(check);
check(locS) = [];
if S==X(4) && sum(check)>(L+S)
    TITLE = 'This is a Double-Crank Mechanism';
elseif (S==X(1)||S==X(3)) && sum(check)>(L+S)
    TITLE = 'This is a Rocker-Crank Mechanism';
elseif S==X(2) && sum(check)>(L+S)
    TITLE = 'This is a Double-Rocker Mechanism';
    flag  = 1;
elseif sum(check)==(L+S)
    TITLE = 'This is a Change Point Mechanism';
elseif sum(check)<(L+S)
    TITLE = 'This is a Double-Rocker Mechanism';
end
theta21 = linspace(0,2*pi,Incriment);% ------------------Input angle theta1
dig = 10;% -------------------------------------------divide links into this number
R1 = X(1);
          r1 = linspace(0,R1,dig);
R2 = X(2);
          r2 = linspace(0,R2,dig);
R3 = X(3);
          r3 = linspace(0,R3,dig);
R4 = X(4);
          r4 = linspace(0,R4,dig);
Pu = X(5);
          cu = linspace(0,Pu,dig);
Pv = X(6);
          cv = linspace(0,Pv,dig);
th1=-30*(pi/180);
d = sqrt(R2^2 + R1^2 - 2*R2*R1*cos(theta21-th1));% -----------diagonal distance between
```

```
%                                                     crank-coupler point and rocker-frame point
th5 = acos((R4^2+d.^2-R3^2)./(2*R4*d));% -------------angle between rocker and diagonal link (d)
IMAG = imag(th5);
LOCATION = IMAG==0;
LOCATION1 = find(IMAG==0);
LOC = LOCATION;
n = length(LOCATION);
n1 = length(LOCATION1);
Check = 0;
direction = 1;
for i=1:n-1
    if LOC(i+1)~=LOC(i)
        if Check==0
            direction = LOC(i);
        end
        Check = Check+1;
    end
end
Rotate = 0;
if isempty(LOCATION1)
    error('unvalid linkage');
elseif direction==0 && Check==2
    LOC1 = find(LOCATION==1);
    th2 = [theta21(LOC1) theta21(fliplr(LOC1))];
elseif n1==n
    th2 = theta21;
elseif direction==1 && Check==2
    Rotate = 1;
    loc1 = LOC(1:end-1);
    loc2 = LOC(2:end);
    [Value, deadpoint] = find((loc2-loc1)~=0);
    deadp = deadpoint + [0 1];
    LOC2 = [deadp(2):n 1:deadp(1)];
    th2 = [theta21(LOC2) theta21(fliplr(LOC2))];
elseif Check==4
    Rotate = 1;
    loc1 = LOC(1:end-1);
    loc2 = LOC(2:end);
    [Value, deadpoint] = find((loc2-loc1)~=0);
deadp1 = deadpoint(1:2) + [1 0];
    deadp2 = deadpoint(3:4) + [1 0];
    if DIREC == 1
        LOC3 = deadp1(1):deadp1(2);
    else
        LOC3 = deadp2(1):deadp2(2);
    end
    th2 = [theta21(LOC3) theta21(fliplr(LOC3))];
end
if Rotate == 1
    d = sqrt(R2^2 + R1^2 - 2*R2*R1*cos(th2-th1));
    th5 = acos((R4^2+d.^2-R3^2)./(2*R4*d));% --angle between rocker and diagonal link (d)
    th5 = [th5(1:end/2) -th5(end/2+1:end)];
end
```

```
Xa = R2*cos(th2);% ----------------------------x coordinate for the crank-coupler point
Ya = R2*sin(th2);% ----------------------------y coordinate for the crank-coupler point
a = R1 - R2*cos(th2-th1);% ------------------------horizontal distance between rocker-frame point
and
%                                               projection of crank-coupler point
b = R2*sin(th2-th1);% --------------------------------------vertical projection of crank-coupler
point
th6 = atan2(b,a);% ----------------------------angle between frame and diagonal link (d)
th4 = pi+th1 - th5 - th6;% ------------------------angle the rocker makes with horizon
Xb = R4*cos(th4) + R1*cos(th1);% -----------------------horizontal distance between frame-crank
point and
%                                               projection of coupler-rocker point
Yb = R4*sin(th4)+R1*sin(th1);% ----------------------------vertical projection of coupler-rocker
point
th3 = atan2((Yb-Ya),(Xb-Xa));% ----------------angle the coupler makes with the horizon
Px = Xa + Pu*cos(th3) - Pv*sin(th3);% ---------horizontal projection of coupler
%                                               point wrt coupler
Py = Ya + Pu*sin(th3) + Pv*cos(th3);% ---------vertical projection of coupler wrt coupler
increments = length(th2);
for i=1:increments
    bar1x(i,:) = r2*cos(th2(i));
    bar1y(i,:) = r2*sin(th2(i));
    bar2x(i,:) = linspace(Xa(i),Xb(i),dig);
    bar2y(i,:) = linspace(Ya(i),Yb(i),dig);
    bar3x(i,:) = R1*cos(th1) + r4*cos(th4(i));
    bar3y(i,:) = R1*sin(th1)+ r4*sin(th4(i));
    Couplx1(i,:) = linspace(Xa(i),Px(i),dig);
    Couply1(i,:) = linspace(Ya(i),Py(i),dig);
    Couplx2(i,:) = linspace(Px(i),Xb(i),dig);
    Couply2(i,:) = linspace(Py(i),Yb(i),dig);
end
for k=1:cycles
    for i = 1:increments
        plot(bar1x(i,:),bar1y(i,:),'b',bar2x(i,:),bar2y(i,:),'k',...
            bar3x(i,:),bar3y(i,:),'k',Couplx1(i,:),Couply1(i,:),'r',...
            Couplx2(i,:),Couply2(i,:),'r')
        hold on
        plot(0,0,'sk',R1*cos(th1),R1*sin(th1),'sk','MarkerSize',18)
        plot(0,0,'ok',R1*cos(th1),R1*sin(th1),'ok','MarkerFaceColor','k')
        plot(Couplx1(i,end),Couply1(i,end),'ok','MarkerSize',8,...
            'MarkerFaceColor','y')
        if  Rotate == 1 && i<=increments/2
            plot(Couplx1(1:i,end),Couply1(1:i,end),'c*','linewidth',2)
        elseif Rotate == 1
            plot(Couplx1(1:increments/2,end),Couply1(1:increments/2,end),'b--o','linewidth',1)
            plot(Couplx1(increments/2:i,end),Couply1(increments/2:i,end),'--g','linewidth',1)
        end
        clc
        fprintf('th2 = %5.2f,   th3 = %5.2f, d = %7.2f\n',th2(i),th3(i),d(i))
        fprintf('th4 = %5.2f,   th5 = %5.2f\n',th4(i),th5(4))
        fprintf('Px = %5.2f,  Py = %5.2f\n',Px(i),Py(i))
        hold off
        M(k) = getframe(gcf);
```

```
    end
end
disp('calculated output linkage length=')
disp('fixed link (r1)=')
 disp(R1)
disp('input link (r2)=')
 disp(R2)
disp('coupler link(r3)=')
 disp(R3)
disp('out put link (r4)=')
 disp(R4)
disp('path point position(rp) and angle beta=')
                 disp(rp)
disp('angle beta=')
                 disp(beta)
 %TAKING INPUTS FROM PRIVEIOUSE LINKAGE SOLUTION FOR THE FOUR-BAR MECHANISM
% %  mass of Links (kg)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
mr2 = 8.38;% mass of link 2
mr3 = 7.89;%;% mass of link 3
mr4 =7.57;%% mass of link 4
% Link moments of inertia (kg-m^2)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
kr2 = R2;% -----------------------radius of gyration of link Ao,A
kr3 = R3;% -----------------------link A,B
kr4 = R4;% -----------------------link B,Bo
% Position Vector for Center of Gravity Magnitudes (m)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
rg2 = R2; %-----------------  link Ao,A from the fixed pivot Ao
rg3 = 39.55;%-------------------- link A,B from the pivot B
rg4 = R4;% -------------------link B,Bo from the fixed pivot Bo
%deviation angle
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
deltar2 =0;% ------------------deviation angle of Link Ao,B from the Position Vector. of pivot A
deltar3 = 5.89;% --------------Link A,B from the p.v. of pivot B(wrt A)
deltar4 =0;% ------------------Link B,Bo from the p.v. of pivot B(wrt Bo)
thetar2 = -17.3;%-----------------angle between Ao,A and Ao,Bo
omegar2 = 0.1;% ----------------Assumed angular velocity of link Ao,A
alphar2 = 0.05;%  ----------------Assumed angular acceleration of the link Ao,A
Tl = 100;%---------------------initial load torque
%CONVERTING DEGREES TO RADIANS AND CHECKING FOR FEASIBILITY
%thetar2 = pi*thetar2/180;
K = ((R2*R2) - (R3*R3) + (R4*R4) + (R1*R1))/2;
P = K - (R2*(R1-R4)*cos(thetar2)) - (R4*R1) ;
Q = -2*R2*R4*sin(thetar2);
R = K - (R2*(R1+R4)*cos(thetar2)) - (R4*R1);
flag=0;

if ((Q*Q - 4*P*R)<0)
    disp('wrong values of the link lengths');
    flag=1;
end
```

```
%CALCULATION OF OTHER ANGLES
while(flag==0)
    thetar41 = 2*atan( ((-1*Q) + sqrt(Q*Q - 4*P*R))/(2*P));
    thetar42 = 2*atan( ((-1*Q) - sqrt(Q*Q - 4*P*R))/(2*P));
if(thetar41<=0)
    thetar41 = 2*atan( ((-1*Q) + sqrt(Q*Q - 4*P*R))/(2*P)) + pi;
end
if(thetar42<=0)
    thetar42 = 2*atan( ((-1*Q) - sqrt(Q*Q - 4*P*R))/(2*P)) + pi;
end
thetar31 = asin( ((R4*sin(thetar41)) - (R2*sin(thetar2)))/R3) ;
thetar32 = asin( ((R4*sin(thetar42)) - (R2*sin(thetar2)))/R3) ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                    CALCULATION OF ANGULAR VELOCITIES                           %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
omegar31 = (-1*R2*omegar2*sin(thetar41 - thetar2))/(R3*sin(thetar41 - thetar31));
omegar32 = (-1*R2*omegar2*sin(thetar42 - thetar2))/(R3*sin(thetar42 - thetar32));
omegar41 = (-1*R2*omegar2*sin(thetar31 - thetar2))/(R4*sin(thetar41 - thetar31));
omegar42 = (-1*R2*omegar2*sin(thetar32 - thetar2))/(R4*sin(thetar42 - thetar32));
%-------------------------------------------------------------------------------------
disp('angulr velocities =')
disp(omegar31)
disp(omegar41)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%                    CALCULATION OF ANGULAR ACCELERATIONS                        %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
alphar31 = ((-1*R2*alphar2*sin(thetar41 - thetar2))...
           + (R2*omegar2*omegar2*cos(thetar41 - thetar2))...
           + (R3*omegar31*omegar31*cos(thetar41 - thetar31))...
           - (R4*omegar41*omegar41))/(R3*sin(thetar41 - thetar31));
alphar32 = ((-1*R2*alphar2*sin(thetar42 - thetar2))...
           + (R2*omegar2*omegar2*cos(thetar42 - thetar2))...
           + (R3*omegar32*omegar32*cos(thetar42 - thetar32))...
           - (R4*omegar42*omegar42))/(R3*sin(thetar42 - thetar32));
alphar41 = ((-1*R2*alphar2*sin(thetar31 - thetar2))....
           + (R2*omegar2*omegar2*cos(thetar31 - thetar2))...
           + (R3*omegar31*omegar31)...
           - (R4*omegar41*omegar41*cos(thetar41 - thetar31)))/(R4*sin(thetar41 - thetar31));
alphar42 = ((-1*R2*alphar2*sin(thetar32 - thetar2)) + (R2*omegar2*omegar2*cos(thetar32 -
thetar2))...
    + (R3*omegar32*omegar32) - (R4*omegar42*omegar42*cos(thetar42 - thetar32)))/(R4*sin(thetar42
- thetar32));
%-----------------------------------------------------------------------------------------------
----------------
disp('angulr acceleration =')
disp(alphar31)
disp(alphar41)
          %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
          %           Ax = b                     % WHERE X=[
F12x,F12y,F23x,F23y,F34x,F34y,F14x,F14y,Ts ]
          %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%       b=[ Fg2x,Fg2y- m2g,Tg2,Fg3x,Fg3y -
m3g,Tg3,Fg4x,Fg4y - m4g,Tg4+T1 ]
```

```
%  CALCULATION OF THE ELEMENTS OF THE 'b' MATRIX
%--------------------------------------------------------------------------------
Fg2x = -1*mr2*rg2*(alphar2*cos(thetar2 + deltar2 - (3.1415926/2))...
      + omegar2*omegar2*cos(thetar2 + deltar2));
Fg2y = mr2*9.81 - mr2*rg2*(alphar2*sin(thetar2 + deltar2 - (3.1415926/2))...
      + omegar2*omegar2*sin(thetar2 + deltar2));
Tg2 = mr2*kr2*kr2*alphar2;
Fg3x = -1*mr3*rg3*(alphar31*cos(thetar31 + deltar3 - (3.1415926/2))...
      + omegar31*omegar31*cos(thetar31 + deltar3));
Fg3y = mr3*9.81 - mr3*rg3*(alphar31*sin(thetar31 + deltar3 - (3.1415926/2))...
      + omegar31*omegar31*sin(thetar31 + deltar3));
Tg3 = mr3*kr3*kr3*alphar31;
Fg4x = -1*mr4*rg4*(alphar41*cos(thetar41 + deltar4 - (3.1415926/2))...
      + omegar41*omegar41*cos(thetar41 + deltar4));
Fg4y = mr4*9.81 - mr4*rg4*(alphar41*sin(thetar41 + deltar4 - (3.1415926/2))...
      + omegar41*omegar41*sin(thetar41 + deltar4));
Tg4 = mr4*kr4*kr4*alphar41 - Tl;
%CALCULATIONS OF THE ELEMENTS OF THE 'A' MATRIX
%---------------------------------------------------------------------------------
A31 = rg2*sin(thetar2 + deltar2);
A32 = -1*rg2*cos(thetar2 + deltar2);
A33 = R2*sin(thetar2) - rg2*cos(thetar2 + deltar2);
A34 = rg2*cos(thetar2 + deltar2) - R2*cos(thetar2);
A39 = 1;
A63 = rg3*sin(thetar31 + deltar3);
A64 = -1*rg2*cos(thetar31 + deltar3);
A65 = R3*sin(thetar31) - rg3*cos(thetar31 + deltar3);
A66 = rg3*cos(thetar31 + deltar3) - R3*cos(thetar31);
A95 = rg4*cos(thetar41 + deltar4) - R4*sin(thetar41);
A96 = R4*cos(thetar41) - rg4*cos(thetar41 + deltar4);
A97 = rg4*sin(thetar41 + deltar4);
A98 = -1*rg4*cos(thetar41 + deltar4);
B = [Fg2x Fg2y Tg2 Fg3x Fg3y Tg3 Fg4x Fg4y Tg4];
A = [1 0 -1 0 0 0 0 0 0;...
     0 1 0 -1 0 0 0 0 0;...
     A31 A32 A33 A34 0 0 0 0 1;...
     0 0 1 0 -1 0 0 0 0;...
     0 0 0 1 0 -1 0 0 0;...
0 0 A63 A64 A65 A66 0 0 0;...
     0 0 0 0 1 0 1 0 0;...
     0 0 0 0 0 1 0 1 0;...
     0 0 0 0 A95 A96 A97 A98 0];
     %CALCULATION OF OUTPUT THE SOLUTION
X = (inv(A))*B';
theta112 = (atan(X(2,1)/X(1,1)))*180/pi;
  if(X(1,1)<0)
      theta112 = (atan(X(2,1)/X(1,1)) + pi)*180/pi;
  end
theta123 = (atan(X(4,1)/X(3,1)))*180/pi;
if(X(3,1)<0)
    theta123 = (atan(X(4,1)/X(3,1)) + pi)*180/pi;
end
theta134 = (atan(X(6,1)/X(5,1)))*180/pi;
```

```
if(X(5,1)<0)
    theta134 = (atan(X(6,1)/X(5,1)) + pi)*180/pi;
end
theta114 = (atan(X(8,1)/X(7,1)))*180/pi;
if(X(7,1)<0)
    theta114 = (atan(X(8,1)/X(7,1)) + pi)*180/pi;
end
     %DISPLAY OF RESULTS
disp('A=');
  disp(A);
disp('X = [F12x F12y F23x F23y F34x F34y F14x F14y Ts] ');
  disp(X);
disp('F12 = ');
  disp(sqrt(X(1,1)^2 + X(2,1)^2));
disp('theta_F12 = ');
  disp(theta112);
disp('F23 = ');
  disp(sqrt(X(3,1)^2 + X(4,1)^2));
disp('theta_F23 = ');
  disp(theta123);
disp('F34 = ');
  disp(sqrt(X(5,1)^2 + X(6,1)^2));
disp('theta_F34 = ');
  disp(theta134);
disp('F14 = ');
  disp(sqrt(X(7,1)^2 + X(8,1)^2));
disp('theta_F14 = ');
  disp(theta114);
flag=flag+1;
end
```

```
 -176.1190
 -128.1635
   38.4980
  -87.6020

th2 =   4.62,   th3 =  0.78, d =  129.75
th4 =   3.01,   th5 =  0.70
Px = -86.38,  Py = -83.84
th2 =   4.80,   th3 =  1.12, d =  112.41
th4 =   2.88,   th5 =  0.70
Px = -68.26,  Py = -110.28
th2 =   4.99,   th3 =  1.39, d =   94.63
th4 =   2.78,   th5 =  0.70
Px = -43.99,  Py = -127.50
th2 =   5.17,   th3 =  1.63, d =   76.91
th4 =   2.69,   th5 =  0.70
Px = -17.24,  Py = -136.67
th2 =   5.36,   th3 =  1.82, d =   60.18
th4 =   2.58,   th5 =  0.70
Px =   8.74,  Py = -137.98
th2 =   5.54,   th3 =  1.93, d =   46.38
th4 =   2.44,   th5 =  0.70
```

```
Px = 28.61,  Py = -130.85
th2 =  5.73,   th3 =  1.78, d =   39.44
th4 =  2.15,   th5 =  0.70
Px = 30.16,  Py = -109.60
th2 =  5.91,   th3 =  1.22, d =   43.06
th4 =  1.66,   th5 =  0.70
Px = 14.80,  Py = -56.86
th2 =  6.10,   th3 =  0.63, d =   55.11
th4 =  1.30,   th5 =  0.70
Px = 20.23,  Py =  5.97
th2 =  6.28,   th3 =  0.21, d =   71.16
th4 =  1.16,   th5 =  0.70
Px = 38.13,  Py = 52.25
th2 =  0.00,   th3 =  0.21, d =   71.16
th4 =  1.16,   th5 =  0.70
Px = 38.13,  Py = 52.25
th2 =  0.18,   th3 = -0.10, d =   88.68
th4 =  1.17,   th5 =  0.70
Px = 55.09,  Py = 85.30
th2 =  0.37,   th3 = -0.36, d =  106.51
th4 =  1.27,   th5 =  0.70
Px = 68.63,  Py = 110.36
th2 =  0.55,   th3 = -0.61, d =  124.04
th4 =  1.44,   th5 =  0.70
Px = 80.37,  Py = 129.30
th2 =  0.74,   th3 = -0.99, d =  140.90
th4 =  1.74,   th5 =  0.70
Px = 98.62,  Py = 138.69
th2 =  0.74,   th3 = -1.48, d =  140.90
th4 =  2.07,   th5 =  0.70
Px = 130.22,  Py = 117.04
th2 =  0.55,   th3 = -1.95, d =  124.04
th4 =  2.28,   th5 =  0.70
Px = 158.02,  Py = 70.91
th2 =  0.37,   th3 = -2.26, d =  106.51
th4 =  2.40,   th5 =  0.70
Px = 168.46,  Py = 30.72
th2 =  0.18,   th3 = -2.52, d =   88.68
th4 =  2.49,   th5 =  0.70
Px = 169.57,  Py = -6.60
th2 =  0.00,   th3 = -2.75, d =   71.16
th4 =  2.58,   th5 =  0.70
Px = 163.90,  Py = -40.39
th2 =  6.28,   th3 = -2.75, d =   71.16
th4 =  2.58,   th5 =  0.70
Px = 163.90,  Py = -40.39
th2 =  6.10,   th3 = -2.92, d =   55.11
th4 =  2.69,   th5 =  0.70
Px = 154.30,  Py = -69.12
th2 =  5.91,   th3 = -2.97, d =   43.06
th4 =  2.87,   th5 =  0.70
Px = 146.82,  Py = -89.13
th2 =  5.73,   th3 = -2.68, d =   39.44
```

```
th4 =  3.24,   th5 =  0.70
Px = 152.06,  Py = -86.50
th2 =  5.54,   th3 = -2.05, d =   46.38
th4 =  3.72,   th5 =  0.70
Px = 148.94,  Py = -53.08
th2 =  5.36,   th3 = -1.52, d =   60.18
th4 =  4.00,   th5 =  0.70
Px = 118.82,  Py = -27.28
th2 =  5.17,   th3 = -1.14, d =   76.91
th4 =  4.08,   th5 =  0.70
Px = 81.14,  Py = -17.92
th2 =  4.99,   th3 = -0.86, d =   94.63
th4 =  4.04,   th5 =  0.70
Px = 43.68,  Py = -16.43
th2 =  4.80,   th3 = -0.61, d =  112.41
th4 =  3.92,   th5 =  0.70
Px =  6.70,  Py = -17.82
th2 =  4.62,   th3 = -0.34, d =  129.75
th4 =  3.72,   th5 =  0.70
Px = -31.94,  Py = -21.24
calculated output linkage length=
fixed link (r1)=
  135.9278

input link (r2)=
   96.6465

coupler link(r3)=
   56.9016

out put link (r4)=
   86.8353

path point position(rp) and angle beta=
   78.4478

angle beta=
    2.2061

angulr velocities =
    0.2250

   -0.0574

angulr acceleration =
    0.1648

   -0.0666

X = [F12x F12y F23x F23y F34x F34y F14x F14y Ts]
   1.0e+04 *

   -0.0128
```

```
    0.0639
   -0.0087
    0.0564
   -0.0140
    0.0474
    0.0163
   -0.0363
    2.5845

F12 =
  651.3817

theta_F12 =
  101.3288

F23 =
  570.4392

theta_F23 =
   98.8030

F34 =
  494.0204

theta_F34 =
  106.4203

F14 =
  397.5791

theta_F14 =
  -65.7822
```

Part B

THREE-POSITION MOTION GENERATION BY ANALYTICAL SYNTHESIS

Figure 24 shows n four bar linkage in one general position with a coupler point located at its first precision position P_1 . Second and third precision positions (points P_2 and P_3) are also shown. These are to be achieved by the rotation of the input rocker. link 2. through as yet unspecified angles φ_2 and φ_3. Note also that the angles of the coupler link 3 at each of the precision positions are defined by the angles of the position vectors $Z_{1,}$ Z_2 and Z_3. The linkage shown in the figure is schematic. Its dimensions are unknown at the outset and are to be found by this synthesis technique. Thus, for example. The length of the Position vector Z_1 as shown is not indicative of the final length of thai edge of link 3 nor are the lengths or angles of any of the links shown predictive of the final result.

For convenience. we will place the global coordinate system X, Y at the first precision point P_1. We define the Other two desired precision positions in the plane with respect to this global system as shown in Figure 24. The position difference vectors P_{21}. drawn from P_1 to P_2. and P_{31}. drawn from P_1 to P_3. have angles δ_2 and δ_3, respectively.

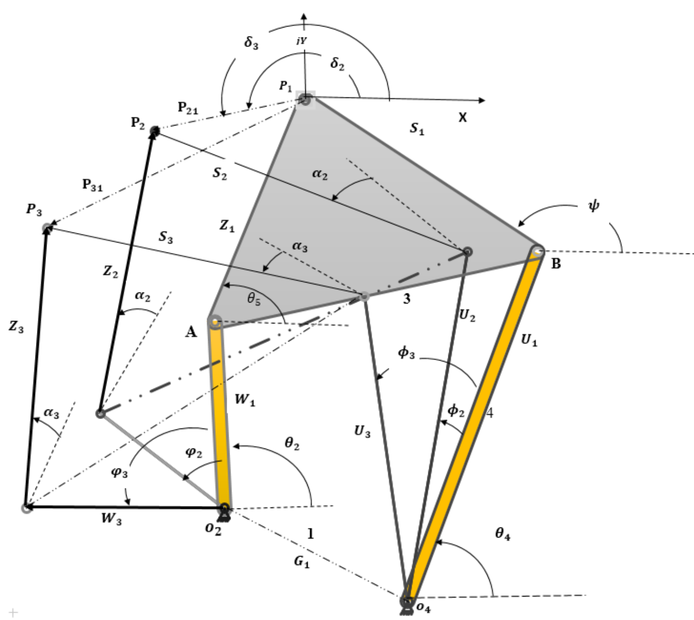


Figure 34:Three-positioning analytical synthesis

The position difference vectors P_{21} and P_{31} define the displacements of the output motion of point P from point 1 to 2 and from 1 to 3, respeclivcly. The dyad $W_1\, Z_1$ defines the left half oflhe linkage. The dyad $U_1\, S_1$, defines the right half of the linkage. Vectors Z_1 and S_1 , are both embedded(pin connected) in the rigid coupler (link 3), and both will undergo the same rotations. through angle α_2 from position 1 to position 2 and through angle α_3 from position 1 to position 3. The pin-to-pin length and angle of link 3 (vector V_1) is defined in terms of vectors Z_1 and S_1.

$$V_1 = Z_1 - S_1$$

The ground link is defined by equation

$$G_1 = W_1 + V_1 - U_1$$

A we did in the two-position case. we will first solve for the left side of the linkage (vectors W_1 and Z_1) and later use the same procedure to solve for the right side (vectors U_1 and S_1). To solve for W_1 and Z_1 we need to now write two vector loop equations. one around the loop which includes positions P_1 and P_2 and lhe second one around the loop which includes positions P_1 and P_3.We will go clockwise around the first loop for motion from position 1 to 2.starting with W_2, and then write the second loop equation for motion from position 1 to 3 starting with W_3.

$$W_2 + Z_2 - P_{21} - Z_1 - W_1 = 0$$
$$W_3 + Z_3 - P_{31} - Z_1 - W_1 = 0 \qquad 1$$

Substituting the complex number equivalents for the vectors.

$$we^{j(\theta_2+\varphi_2)} + ze^{j(\theta_5+\alpha_2)} - p_{21}e^{j\delta_2} - ze^{j\theta_5} - we^{j\theta_2} = 0$$
$$we^{j(\theta_2+\varphi_3)} + ze^{j(\theta_5+\alpha_3)} - p_{31}e^{j\delta_3} - ze^{j\theta_5} - we^{j\theta_2} = 0 \qquad 2$$

Rewriting the sums of angles in the exponents as products of terms.

$$we^{j\theta_2}e^{j\varphi_2} + ze^{j\theta_5}e^{j\alpha_2} - p_{21}e^{j\delta_2} - ze^{j\theta_5} - we^{j\theta_2} = 0$$
$$w(e^{j\theta_2}e^{j\varphi_2} - e^{j\theta_2}) + z(e^{j\theta_5}e^{j\alpha_2} - e^{j\theta_5}) = p_{21}e^{j\delta_2}$$
$$we^{j\theta_2}e^{j\varphi_3} + ze^{j\phi}e^{j\alpha_3} - p_{31}e^{j\delta_3} - ze^{j\theta_5} - we^{j\theta_2} = 0 \qquad 3a$$
$$w(e^{j\theta_2}e^{j\varphi_3} - we^{j\theta_2}) + z(e^{j\theta_5}e^{j\alpha_3} - ze^{j\theta_5}) = p_{31}e^{j\delta_3}$$

Rearranging:

$$we^{j\theta_2}(e^{j\varphi_2} - 1) + ze^{j\theta_5}(e^{j\alpha_2} - 1) = p_{21}e^{j\delta_2}$$

$$we^{j\theta_2}(e^{j\varphi_3}-1)+ze^{j\theta_5}(e^{j\alpha_3}-1) = p_{31}e^{j\delta_3} \qquad 3b$$

The magnitude w of vectors W_1, W_2, and W_3 is the same in all three positions because it represents the same line in a rigid link. The same can be said about vectors Z_1, Z_2, and Z_3 whose common magnitude is z.

The scalar equations can be revealed by substituting Euler's identity and separate the real and imaginary.

$$e^{\pm j\theta} = \cos\theta + j\sin\theta$$

Real part:

$$w\cos\theta_2\,(\cos\varphi_2-1) - w\sin\theta_2\sin\varphi_2 + z\cos\phi\,(\cos\alpha_2-1)$$
$$-z\sin\theta_5\sin\alpha_2 = p_{21}\cos\delta_2 \qquad 4a$$
$$w\cos\theta_2\,(\cos\varphi_3-1) - w\sin\theta_2\sin\varphi_3 + z\cos\phi\,(\cos\alpha_3-1)$$
$$-z\sin\theta_5\sin\alpha_3 = p_{31}\cos\delta_3 \qquad 4b$$

Imaginary part (with complex operator j divided out):

$$w\sin\theta_2\,(\cos\varphi_2-1) - w\cos\theta_2\sin\varphi_2 + z\sin\phi\,(\cos\alpha_2-1)$$
$$-z\cos\theta_5\sin\alpha_2 = p_{21}\sin\delta_2 \qquad 4c$$
$$w\sin\theta_2\,(\cos\varphi_3-1) - w\cos\theta_2\sin\varphi_3 + z\sin\phi\,(\cos\alpha_3-1)$$
$$-z\cos\theta_5\sin\alpha_3 = p_{31}\sin\delta_3 \qquad 4d$$

There are twelve variables in these four equations 4 $w.\theta_2, \varphi_2, \varphi_3, z, \theta_5, \alpha_2, \alpha_3, p_{21}, p_{31}$, $\delta_2. and\ \delta_3$ We can solve for only four. Six of them are defined in the problem statement, namely $\alpha_2, \alpha_3, p_{21}, p_{31}, \delta_2. and\ \delta_3$. Of the remaining six. $w.\theta_2, \varphi_2, \varphi_3, z, \theta_5$ we must choose two as free choices (assumed values) in order to solve for the other four. One strategy is to assume values for the two angles. φ_2 and φ_3, on the premise that we may want to specify the angular excursions of link 2 to suit some driving constraint. (This choice also has the benefit of leading to a set of linear equations for simultaneous solution.)

This leaves the magnitudes and angles of vectors W and Z to be found

$(w.\theta_2, \varphi_2, \varphi_3, , z, \theta_5)$.

x and y components of the two unknown vectors W and Z. Can be writen in polar coordinates.

$$W_{1_x} = w\cos\theta_2 : \quad Z_{1_x} = z\cos\theta_5 :$$

$$W_{1_y} = w\sin\theta_2 : \quad Z_{1_y} = z\sin\theta_5 : \qquad 5$$

Substituting equations 5 in to 4 we obtain:

$$W_{1_x}(\cos\varphi_2 - 1) - W_{1_y}\sin\varphi_2 + Z_{1_x}(\cos\alpha_2 - 1) - Z_{1_y}\sin\alpha_2 = p_{21}\cos\delta_2 \qquad 6\text{ a}$$

$$W_{1_x}(\cos\varphi_3 - 1) - W_{1_y}\sin\varphi_3 + Z_{1_x}(\cos\alpha_3 - 1) - Z_{1_y}\sin\alpha_3 = p_{31}\cos\delta_3 \qquad 6\text{ b}$$

$$W_{1_y}(\cos\varphi_2 - 1) - W_{1_x}\sin\varphi_2 + Z_{1_y}(\cos\alpha_2 - 1) - Z_{1_x}\sin\alpha_2 = p_{21}\cos\delta_2 \qquad 6\text{ c}$$

$$W_{1_y}(\cos\varphi_3 - 1) - W_{1_x}\sin\varphi_3 + Z_{1_y}(\cos\alpha_3 - 1) - Z_{1_x}\sin\alpha_3 = p_{31}\sin\delta_3 \qquad 6\text{ d}$$

These are four equations in the four unknowns W_{1_x}, $W_{1_y} Z_{1_x}$and Z_{1_y} By setting the coefficients which contain the assumed and specified terms equal to some constams. we can simplify the notation and obtain the following general fomula of Matrix equation..

$$\begin{bmatrix} \cos\varphi_2 - 1 & -\sin\varphi_2 & \cos\alpha_2 - 1 & -\sin\alpha_2 \\ \cos\varphi_3 - 1 & -\sin\varphi_3 & \cos\alpha_3 - 1 & -\sin\alpha_3 \\ \sin\varphi_2 & \cos\varphi_2 - 1 & \sin\alpha_2 & \cos\alpha_2 - 1 \\ \sin\varphi_3 & \cos\varphi_3 - 1 & \sin\alpha_3 & \cos\alpha_3 - 1 \end{bmatrix} \times \begin{bmatrix} W_{1_x} \\ W_{1_Y} \\ Z_{1_x} \\ Z_{1_Y} \end{bmatrix} = \begin{bmatrix} p_{21}\cos\delta_2 \\ p_{31}\sin\delta_3 \\ p_{21}\cos\delta_2 \\ p_{31}\sin\delta_3 \end{bmatrix} \qquad 7$$

$$\begin{bmatrix} W_{1_x} \\ W_{1_Y} \\ Z_{1_x} \\ Z_{1_Y} \end{bmatrix} = \begin{bmatrix} \cos\varphi_2 - 1 & -\sin\varphi_2 & \cos\alpha_2 - 1 & -\sin\alpha_2 \\ \cos\varphi_3 - 1 & -\sin\varphi_3 & \cos\alpha_3 - 1 & -\sin\alpha_3 \\ \sin\varphi_2 & \cos\varphi_2 - 1 & \sin\alpha_2 & \cos\alpha_2 - 1 \\ \sin\varphi_3 & \cos\varphi_3 - 1 & \sin\alpha_3 & \cos\alpha_3 - 1 \end{bmatrix}^{-1} \times \begin{bmatrix} p_{21}\cos\delta_2 \\ p_{31}\sin\delta_3 \\ p_{21}\cos\delta_2 \\ p_{31}\sin\delta_3 \end{bmatrix}$$

Equations 5 and 7 solve the three-position synthesis problem for the left-hand side of the linkage using any pair of assumed values for $\varphi_2\ and\ \varphi_3$.

For the righi-hand side of the linkage to find vectors U and S. Figure 24 also shows the three positions of the US dyad. and the angles $\sigma, \phi_2, \phi_3, \psi, \alpha_2, and\ \alpha_3$. which define those vector rotations for all three positio;

The vector loop equations are:

$$U_2 + S - P_{21} - S_1 - U_1 = 0$$

$$U_3 + S_3 - P_{31} - S_1 - U_1 = 0 \quad 8$$

Substituting the complex number equivalents for the vectors.

$$ue^{j(\sigma+\phi_2)} + se^{j(\psi+\alpha_2)} - p_{21}e^{j\delta_2} - se^{j\psi} - ue^{j\sigma} = 0$$

$$ue^{j(\sigma+\phi_3)} + se^{j(\psi+\alpha_3)} - p_{31}e^{j\delta_3} - se^{j\psi} - ue^{j\sigma} = 0 \quad 9$$

Rewriting the sums of angles in the exponents as products of terms.

$$ue^{j\sigma}e^{j\phi_2} + se^{j\psi}e^{j\alpha_2} - p_{21}e^{j\delta_2} - se^{j\psi} - ue^{j\sigma} = 0$$

$$u(e^{j\sigma}e^{j\phi_2} - e^{j\sigma}) + s(e^{j\psi}e^{j\alpha_2} - e^{j\psi}) = p_{21}e^{j\delta_2}$$

$$ue^{j\sigma}e^{j\phi_3} + se^{j\psi}e^{j\alpha_3} - p_{31}e^{j\delta_3} - se^{j\psi} - ue^{j\sigma} = 0 \quad 10a$$

$$ue(^{j\sigma}e^{j\phi_3} - e^{j\sigma}) + s(e^{j\psi}e^{j\alpha_3} - e^{j\psi}) = p_{31}e^{j\delta_3}$$

Rearranging:

$$ue^{j\sigma}(e^{j\phi_2} - 1) + se^{j\psi}(e^{j\alpha_2} - 1) = p_{21}e^{j\delta_2}$$

$$ue^{j\sigma}(e^{j\phi_3} - 1) + se^{j\psi}(e^{j\alpha_3} - 1) = p_{31}e^{j\delta_3} \quad 10b$$

Separate the real and imaginary.

Real part:

$$u\cos\sigma\,(\cos\phi_2 - 1) - u\sin\sigma\sin\phi_2 + s\cos\psi\,(\cos\alpha_2 - 1)$$
$$-s\sin\psi\sin\alpha_2 = p_{21}\cos\delta_2 \quad 11a$$

$$u\cos\sigma\,(\cos\phi_3 - 1) - u\sin\sigma\sin\phi_3 + s\cos\psi\,(\cos\alpha_3 - 1)$$
$$-s\sin\psi\sin\alpha_3 = p_{31}\cos\delta_3 \quad 11b$$

Imaginary part (with complex operator j divided out):

$$u\sin\sigma\,(\cos\phi_2 - 1) - u\cos\sigma\sin\phi_2 + s\sin\psi\,(\cos\alpha_2 - 1)$$
$$-s\cos\psi\sin\alpha_2 = p_{21}\sin\delta_2 \quad 11c$$

$$u\sin\sigma\,(\cos\phi_3 - 1) - u\cos\sigma\sin\phi_3 + s\sin\psi\,(\cos\alpha_3 - 1)$$
$$-s\cos\psi\sin\alpha_3 = p_{31}\sin\delta_3 \quad 11d$$

x and y components of the two unknown vectors U and S.

$U_{1_x} = u \cos \sigma$: $S_{1_x} = s \cos \psi$:

$U_{1_y} = u \sin \sigma$: $S_{1_y} = s \sin \psi$:   12

Substituting equations 5.23 into 5.22 we obtain:

$$U_{1_x}(\cos \phi_2 - 1) - U_{1_y} \sin \phi_2 + S_{1_x}(\cos \alpha_2 - 1) - S_{1_y} \sin \alpha_2 = p_{21} \cos \delta_2 \qquad 13\text{ e}$$

$$U_{1_x}(\cos \phi_3 - 1) - U_{1_y} \sin \phi_3 + S_{1_x}(\cos \alpha_3 - 1) - S_{1_y} \sin \alpha_3 = p_{31} \cos \delta_3 \qquad 13\text{ f}$$

$$U_{1_y}(\cos \phi_2 - 1) - U_{1_x} \sin \phi_2 + S_{1_y}(\cos \alpha_2 - 1) - S_{1_x} \sin \alpha_2 = p_{21} \cos \delta_2 \qquad 13\text{ g}$$

$$U_{1_y}(\cos \phi_3 - 1) - U_{1_x} \sin \phi_3 + S_{1_y}(\cos \alpha_3 - 1) - S_{1_x} \sin \alpha_3 = p_{31} \sin \delta_3 \qquad 13\text{ h}$$

These are four equations in the four unknowns U_{1_x}, $U_{1_y} S_{1_x}$and S_{1_y} we can simplify the notation and obtain the following solutions.

$$\begin{bmatrix} \cos \phi_2 - 1 & -\sin \phi_2 & \cos \alpha_2 - 1 & -\sin \alpha_2 \\ \cos \phi_3 - 1 & -\sin \phi_3 & \cos \alpha_3 - 1 & -\sin \alpha_3 \\ \sin \phi_2 & \cos \phi_2 - 1 & \sin \alpha_2 & \cos \alpha_2 - 1 \\ \sin \phi_3 & \cos \phi_3 - 1 & \sin \alpha_3 & \cos \alpha_3 - 1 \end{bmatrix} \times \begin{bmatrix} U_{1_x} \\ U_{1_Y} \\ S_{1_x} \\ S_{1_Y} \end{bmatrix} = \begin{bmatrix} p_2 * \cos \delta_2 \\ p_3 * \cos \delta_3 \\ p_2 * \sin \delta_2 \\ p_3 * \sin \delta_3 \end{bmatrix} \qquad 14$$

Equations 14 can be solve using the approach of equations 7.