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Wisdom at the source of the Blue Nile

BAHIR DAR UNIVERSITY

BAHIR DAR INSTITUTE OF TECHNOLOGY

SCHOOL OF RESEARCH AND POSTGRADUATE STUDIES

FACULTY OF CIVIL AND WATER RESOURCE ENGINEERING

Development of Conceptual Mathematical Model to Estimate Compressive Strength of In-Place Cured Cement Concrete

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Bahir Dar, Ethiopia
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Development of Conceptual Mathematical Model to Estimate Compressive Strength of In-Place Cured Cement Concrete

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A thesis submitted to the School of Research and Graduate Studies of Bahir Dar Institute of Technology, BDU in partial fulfillment of the requirements for the degree of Master of Science in the Construction Technology and Management in the Faculty of Civil and Water Resource Engineering.

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Bahir Dar, Ethiopia
February, 2019

DECLARATION

I, Habtegebreal Adane, a student of Bahir Dar Institute of Technology honestly declare that this thesis comprises my own original work. I have acknowledged and refereed all materials used in this work. This document was not submitted to any university or institute.

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This thesis has been submitted for examination with my approval as a university advisor.

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To my Father, Mother, Sisters, Brother, and Aki

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ABSTRACT

In standard practice, the most widely used test for cement concrete is compressive strength test using standard specimens which are kept and tested in a laboratory environment. However, the result of compression test done in a controlled laboratory environment can only show the adequacy of mix proportion to achieve the required strength. Unlike the laboratory specimens, the strength of in-place concrete is affected by many factors including compaction, curing, and ambient environmental conditions. Compressive strength test alone is a normal practice due to the absence of a standard method or model to relate the compressive strength result of the standard cured specimen with the strength of in-place cured concrete structures.

In this research, mathematical models are developed to estimate compressive strength of in-place concrete putting five determining factors into consideration, i.e. curing method, curing duration, type and size of the building structure, and the result of standard compressive strength test. In order to determine the values of coefficients for equations and to study the effect of curing, 0.4, 0.5 and 0.6 water to cement ratios are considered. For each water to cement ratio, curing methods of wet burlap covering, plastic covering, water spraying and air curing were assessed. And also, for each curing method, 3, 7, and 14 days of curing duration were used.

The formulae developed in this research can be used in order to take into account strength affecting factors. The use of these formulae could enhance the confidence of both contractors and consultants about their concrete work and it also can be used to compensate for the strength loss of in-place concrete due to curing, type and size of the structure while carrying out mix design. However, the formulae developed in this research work are only for the age of concrete on the 28th day. Therefore, this research must be continued to fully develop to make it applicable for all ages; especially, for those at early ages.

Keywords: concrete, curing, in-place strength, curing affected zone, mathematical model, compressive strength.

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LIST OF ABBREVIATIONS

ACI	American Concrete Institute
AD	Air Dry
ASTM	American Society for Testing Materials
BS	British Standard
CAZ	Curing Affected Zone
CA	Coarse Aggregate
CUZ	Curing Unaffected Zone
FA	Fine Aggregate
NA	Neutral Axis
OD	Oven Dry
OPC	Ordinary Portland cement
PPC	Pozzolanic Portland cement
SSD	Saturated Surface Dry
W/C	Water to Cement Ratio
MUDHC	Ministry of Urban Development, Housing, and Construction

1. INTRODUCTION

1.1. Background

Virtually, in all countries where structural cement concrete is used, concrete quality control is based on compression testing on molded specimens sampled from the concrete which is being poured and tested at 28 days after casting (Watkins, et al., 1996). Compressive strength test is not intended for determining in-place strength of the concrete because it makes no allowance for the effect of compaction, curing method, curing duration and the ambient environment (ACI 228.1R, 2003). The result of compression test on specimens which are kept and cured under controlled laboratory condition only shows the potential strength which the mix proportion can achieve.

Watkins et al. (1996) reported that it has long been recognized that compression test under laboratory conditions suffers a major weakness. The principal shortcoming of this test method of quality control is that it provides no direct measurement of the in-place strength of concrete, which is more directly relevant to the serviceability of the structure.

Neville (2013) also pointed out that, due to compaction, segregation, curing and other factors, the strength of concrete in actual structures are actually inferior compared with the strength of standard cured specimens. Of these factors, curing is one of the major factors which influence the strength of in-place concrete especially slabs, pavements, and small section structures.

In order to deal with the aforementioned shortcomings, ASTM provides four methods of estimating the strength of in-place concrete using concrete specimens, i.e., ASTM C 31 Making and Curing Concrete Test Specimens in the Field (ASTM C 31/C 31M, 2008), ASTM C 873 Cylinders Cast in Place in Cylindrical Molds (ASTM C 873/C 873M, 2004), ASTM C 1074 Concrete Strength by the Maturity Method (ASTM C 1074, 2004) and ASTM C 42/C 42M Obtaining and Testing Drilled Cores and Sawed Beams of Concrete (ASTM C 42/C 42M, 2004). In this study, the limitation of those methods are discussed and the new method is attempted to develop to estimate the strength of in-place concrete as a function of curing method, curing duration, type of structure, size of the structure and compressive strength of standard cured specimens.

1.2. Statement of the Problem

Even though concrete specimens are made for quality control from the same mix which is being poured to cast structures, they are cured in a condition where both moisture and temperature are maintained till the day of testing. Such different condition invites/brings a problem of dissimilarity with the strength of in-place concrete. As a result, the test results only show the adequacy of mix proportion to attain the required target mean strength. In this regard, other determining factors which affect the strength of in-place concrete such as compaction, curing method and duration, and type of structure and size of structures are not normally taken into account. It is the in-place strength which matters a lot because the overall serviceability of the structure depends on its strength. In addition to checking the adequacy of mix proportion, other factors which affect the in-place strength shall be considered.

Approving or rejecting concrete works only by using the result of compression test on standard cured specimen has its own limitations due to the fact that compression test is just measurement of one parameter from many other determining factors. However, consideration of compressive strength test is a normal practice due to the absence of standard method or model to relate the compressive strength result of the standard cured specimen with the strength of in-place cured concrete structures.

In developed nations, it is common to estimate in-place strength using different methods such as Pullout test (ASTM C 900), Rebound Number (ASTM C 805), Penetration Resistance (ASTM C 803), Break-off Number (ASTM C 1150), and Ultrasonic Pulse Velocity (ASTM C 597). But from observation in Bahir Dar and Addis Ababa, the practices of such tests are very rare because these tests need well-trained technicians and sophisticated equipment. As a result, it is very important to develop a new and very simple method of estimating the strength of in-place cured concrete.

1.3. Research Questions

- What are the limitations of existing methods to estimate the in-place compressive strength of concrete?
- Can the compressive strength of concrete specimens which is cured under standard condition alone tell us the in-place strength?
- What are the other factors which affect the strength of in-place concrete and how should they be included in the determination of in-place strength?
- How to develop an alternative and fairly exact method of estimating in-place strength by considering different factors into account?

1.4. The Objective of the Study

General objective

- The main objective of this research is to develop mathematical equations by which compressive strength of in-place concrete can be estimated taking in to account curing method, curing duration, type of structural element, size of the structural element, and water to cement ratio in reference to standard cured concrete specimens into account.

Specific objective

- To assess the limitation of existing methods to estimate the in-place strength of concrete
- To develop theoretical equations for different structural members which could help to estimate the compressive strength of in-place cured concrete structures
- To experimentally quantify the influence of curing method and duration for the determination of values of coefficients by which theoretically developed equations can be supported.

1.5. The Scope of the Study

Due to time, resource and financial constraints, this research has the following scope:

- Even though the environmental conditions which affect the strength of in-place concrete differ from place to place, this research is conducted in the City of Bahir Dar, Ethiopia.
- The study was not conducted for all seasons of the year. As a result, it was conducted only from May to June 2018.
- Even though cement types like PPC and OPC are available in Bahir Dar, this research is conducted using cement type of Ordinary Portland cement (OPC 42.5N) because OPC cement is widely used (Walelign, 2014; Molla, 2017).
- The effect of compaction and other issues which affect the strength of in-place strength was not assessed.

1.6. Limitation of the Study

- The experiments were done by taking the 25mm depth of curing affected zone but the depth of curing affected zone may vary depending on many factors.
- The strength variation due to the vertical height of concrete structures was not assessed. Concrete structures which are vertically long like columns and shear walls have different strengths at the top and bottom due to the fact that pressure at the bottom causes better compaction (Mostafa & Jules, 1998).
- Concrete may be cast and cured at any season of the year but this study was conducted only from May to June 2018. Not only the season difference but also even within the same season, the ambient condition will vary slightly year to year.

1.7. The significance of the Study

The developed formulae will be used for different applications. Some of them are stated below.

- The compressive strength of actual field structures can be estimated after knowing the compressive strength result of standard laboratory cured concrete specimen. Construction often requires operations such as formwork removal and termination of curing be carried out as early as possible to enable these operations at the earliest possible time and to proceed to the next work safely, it requires the use of the reliable method to estimate the in-place strength (ACI 228.1R, 2003). In other words, estimating the strength of in-place concrete increase the confidence of the client, contractor and supervisor regarding the safety of the building.

  As more data comes in the future, those formulae will be updated, developed and confirmed. This longtime development helps to make the formulae standard and compliance criteria.
- While preparing a mix design, there is no consideration of the curing method, curing duration, type and size of the structure. Since these equations consider those factors, they can be used as an input for mix design. In other words, using the formulae, it is possible to estimate the expected difference in strength between the standard cured specimen and in-place strength. This difference in strength may be added as an additional margin while calculating the required average strength (target mean strength) as described in ACI 318.
- If one can estimate or predict the strength of in-place concrete, it is possible to reduce safety factors during design and can reduce the size of the section or reduce the amount of reinforcement. In other words, the cost may reduce significantly in doing so.

2. Literature Review

2.1. Introduction

In freshly placed concrete, moisture and temperature play an important role in the performance of hardened concrete. Water, which is very important for hydration of cement may be lost from freshly placed concrete due to the relative humidity difference between the ambient environment and interior part of concrete, speed of the wind, and also due to evaporation that may be caused by the temperature of both fresh concrete and the ambient environment. Self-desiccation also contributes to lack of water by which water is consumed for cement hydration.

Unless this lost water is either substituted by additional water or retained from escaping the freshly placed concrete, the hydration of cement will not be completed and the concrete will end up to be partially hydrated.

Curing has a significant influence on the properties of hardened concrete, such as strength, permeability, abrasion resistance, and volume stability (ACI 308R, 2001). Even though curing is an important step in concrete production, we do not have any method of checking and monitoring either curing is sufficient or not. The Ethiopian specifications including Standard Technical Specification for Building Works, MUDHC do not contain any methods to confirm curing (MUDHC, 2014).

In Ethiopia, curing methods of spraying water over the surface of concrete early in the morning and late in the evening, covering with plastic and wet burlap are the most applicable methods of curing (Walelign, 2014; Molla, 2017). Those different methods of curing with a combination of different curing duration have a different effect on the strength of in-place concrete.

2.2. Definition of Curing

Different standards and literature give approximately the same definition of curing. Some of them are:

ASTM C 125

> "Curing, action taken to maintain moisture and temperature conditions in a freshly-placed cementitious mixture to allow hydraulic cement hydration and (if applicable) pozzolanic reactions to occur so that the potential properties of the mixture may develop"

Neville, 2013

> "Curing is the name given to procedures used for promoting the hydration of cement and consists of control of temperature and of the moisture movement from and into the concrete."

Kosmatka, et al., 2003

> "Curing is the maintenance of satisfactory moisture content and temperature in concrete for a period of time immediately following placing and finishing so that the desired properties may develop."

Senbetta, 1981

> "By definition, curing of concrete is the maintenance of proper moisture and temperature conditions of newly placed concrete for a sufficient period to assure satisfactory hydration of the cementitious materials and proper hardening of the concrete."

Generally, curing of concrete is an action taken to maintain moisture condition and temperature of freshly placed concrete for a sufficient period to sustain hydration reaction between cement and water.

2.3. Curing Methods

The methods of curing applied to concrete vary depending on the ambient condition, the orientation of structural member which include size and shape of concrete, the need for construction access during curing, availability of curing materials, aesthetic appearance, and cost (Taylor, 2014; Edwards, 2006; Kosmatka, et al., 2003).

Generally, curing can be classified into three categories. Those are:

1. **Wet Curing:** In this curing method, the moisture loss can be substituted by continuous and frequent application of water over the exposed surface of the concrete. In addition to supplying water, it can be used as a cooling mechanism in hot weather concreting. This method is also known as conventional curing (Rao, et al., 2010). The application of water may be employed in either of the following methods or combination of them:
    a) **Sprinkling:** This method is carried out by spraying water through a system of the nozzle over the surface of the concrete. It is a good method of curing for an environment with low relative humidity and high temperature but it is costly. If sprinkling is done at intervals, the concrete must be prevented from drying between applications of water; otherwise, alternate cycles of wetting and drying can cause surface cracking (Kosmatka, et al., 2003; McCarter & Watson, 1997).
    b) **Ponding:** Garber (2006), explains ponding as keeping the surface covered with water which involves building a short dam on all sides of the floor. For slabs and pavements, short retaining dikes can be made around the perimeter of the surface to pond water.
    c) **Wet Coverage:** This method relies on materials that absorb large quantities of water. The absorbent material is spread over the new floor and soaked with water. The materials must be wet again from time to time (Garber, 2006). Saturated cover materials, such as burlap, sand and other moisture retaining fabrics should be free from any substance which may cause discoloration of the concrete surface.
2. **Membrane Curing:** This method relies on the prevention of loss of water from the surface of the concrete. This can be done by covering the surface of the concrete with an impervious membrane like plastic sheets or applying membrane curing compounds. This could be called a water-barrier method (Neville, 2013).
3. **Accelerated Curing:** This method is used to accelerate strength gain by applying heat and water to the concrete (Kosmatka, et al., 2003). This method is typically used for pre-cast concrete products and concrete specimens (Rao, et al., 2010).

2.4. Selection of Curing Method

Curing methods applied to different structures are mainly based on suitability and effectiveness of the curing method with the type of structures. Beside their suitability, other factors are also considered such as cost, availability, size and type of structure and so on.

Flat concrete surfaces which are laid horizontally, such as floors and concrete pavement can be cured by ponding, wet burlap, and spraying. Structural elements like columns and walls may be more efficiently protected by leaving their forms in place, or by wrapping them in plastic and burlap (Taylor, 2014).

The most effective method of curing is to keep the exposed concrete surfaces continuously moist by ponding or spraying with water (Al-Gahtani, 2010). However, in hot and aired areas, ponding and spraying methods are not economical. Austin & Robins, (1992) and Samir & Mokdad, (1988) state that wet burlap curing was the most effective and air curing was the least effective between 7 and 28 days in a hot climate.

Data from Walelign, (2014) in Addis Ababa and from Molla, (2017) in Bahir Dar, Ethiopia shows that water spraying, covering with wet burlap and covering with plastic are the most applicable methods. The percentage of mostly applied curing methods are tabulated below in Table 1.

Table 1 - Mostly applied curing method in Addis Ababa (a) (Walelign, 2014) and in Bahir Dar (b) (Molla, 2017).

Curing Method	**Column (%)**	**Beam (%)**	**Slab (%)**
Water Sprinkling	38	69	77
Covering with Wet cloth	40	18	9
Covering with Plastic Sheet	22	13	14

(a)

Curing Method	**Column (%)**	**Beam (%)**	**Slab (%)**
Water Sprinkling	77.5	95	100
Covering with Wet cloth	17.5	5	0
Covering with Plastic Sheet	5	0	0

(b)

2.5. Duration of Curing

2.5.1. When to Start Curing?

Concrete gains much of its strength rapidly at early ages so that the greatest benefit of curing is to secure the need of moisture around the surface of concrete (Samir & Mokdad, 1988). As a result, curing measures must be started as soon as the concrete is at risk of drying and when such drying will damage the concrete or inhibit the development of required properties (ACI 308R, 2001). But if concrete is allowed to dry for a sufficient period, it never regains the strength of continuously cured concrete, even after long period of subsequent moist curing and the fine hair cracks will not be healed. Samir & Mokdad (1988) shows that the effect of curing delay on compressive strength of specimen is significant in a hot climate. Figure 1 and Figure 2 shows the effect of curing delayed by 3, 7, 14, 28, and 91 days on the compressive strength of specimen on different age of testing.

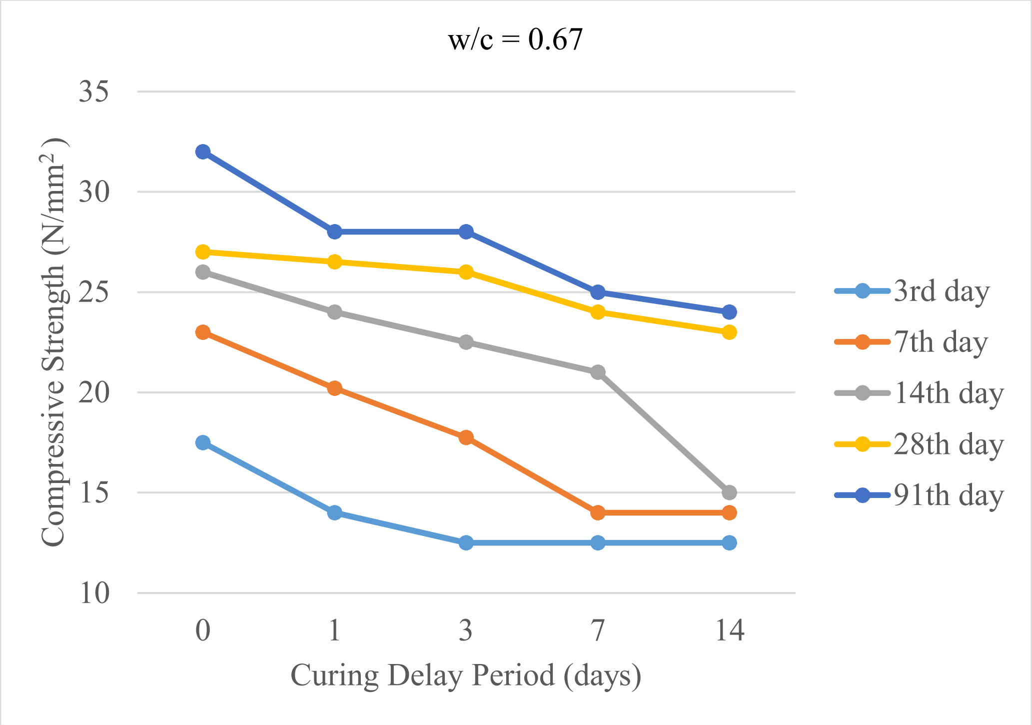

Figure 1 - Effect of curing delay on strength for w/c = 0.67 (Samir & Mokdad, 1988)

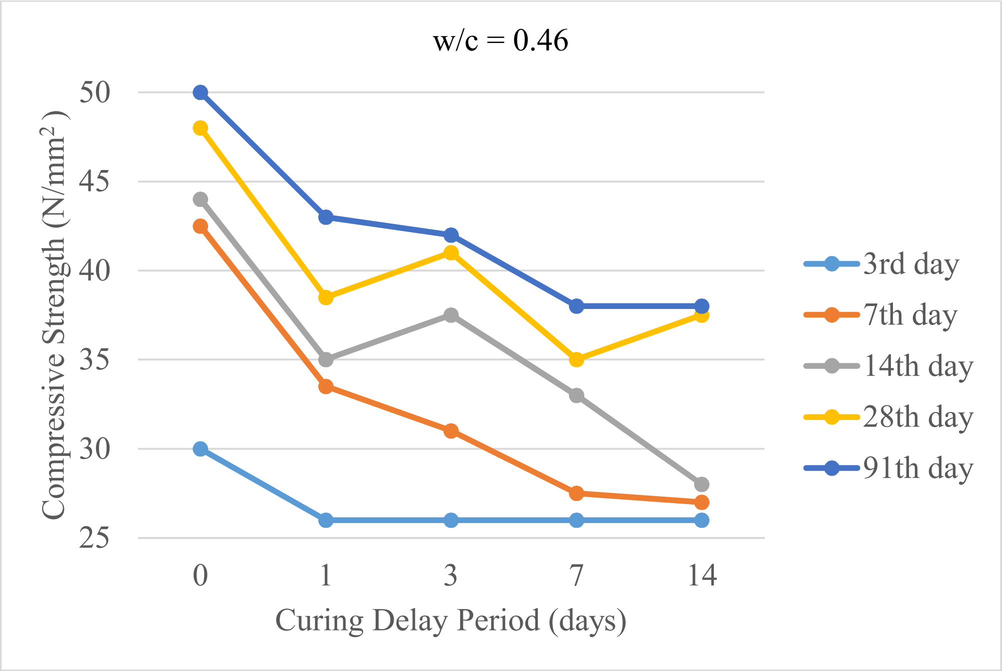

Figure 2- Effect of curing delay on strength w/c = 0.46 (Samir & Mokdad, 1988)

2.5.2. Length of Curing

The duration of curing depends on a number of factors: the type of cementing materials used, mixture proportions, required strength, size and shape of the concrete member; ambient weather, and future exposure conditions (Kosmatka, et al., 2003). ACI 308R also state that the required duration of curing depends on the composition and proportions of the concrete mixture, the values to be achieved for desired concrete properties, the rate at which desired properties are developing while curing measures are in place, and the rates at which those properties will develop after curing measures are terminated.

Figure 3 demonstrates that the rate of continued strength development decreases significantly after curing procedures are terminated (ACI 308R, 2001). This postcuring rate of continued development should be considered in approving the termination of curing anytime before full attainment of specified concrete properties. Similar research was done by Baris & Hulusi (2003) also shows that concrete specimens which are continuously cured for long period time experience relatively high strength than that of cured for short period in both OPC and PPC concrete.

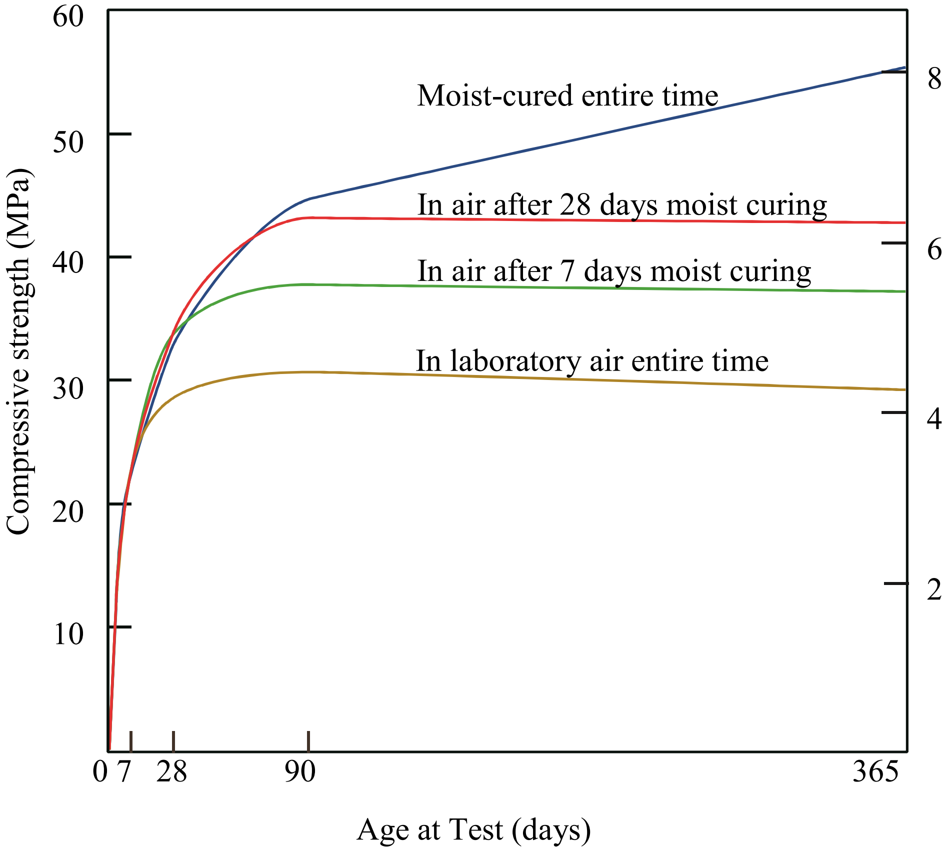

Figure 3 - Effect of curing time on strength (Gonnerman & Shuman, 1928)

Concrete mixture with low w/c ratio needs curing immediately after finishing due to the fact that high heat of hydration around the surface leads to evaporation of water and the duration of curing is often for short period of time because it became impervious enough within few days. Concrete with lean mixture needs a long period of curing because moisture can escape easily due to its large pore structure. Kosmatka, et al., (2003) also states that the curing period may be 3 weeks or longer for lean concrete mixtures used in massive structures such as dams; conversely, it may be only a few days for rich mixes, especially if Type III or HE cement is used.

Walelign (2014) state that increasing the duration of moist curing improves the mechanical and durability properties of concrete. The challenge is to determine the minimum duration of curing that is necessary to achieve the required level of performance for the specific application, taking into account all relevant parameters.

Similarly, ACI 308R also pointed out that duration of curing required to achieve the desired levels of strength, durability, or both, depends on the chemical composition and fineness of the cementitious materials, water to cement ratio, mixture proportions, aggregate characteristics, chemical and mineral admixtures, the temperature of the concrete, and the effectiveness of the curing method. This complex set of factors makes it difficult to confidently state the minimum curing time required to achieve the desired level of performance with the particular mixture. For concrete with and without pozzolan and chemical admixtures, a 7-day minimum duration of curing will often be sufficient to attain approximately 70% of the specified compressive strength. It is not necessarily true, however, durability characteristics, such as abrasion resistance or surface absorption, will reach satisfactory levels in the same minimum time.

A better approach to determine the duration of curing may be to rely on performance testing, either in-place strength assessed using thermal methods or preferably some other approach, such as abrasion resistance or permeability (ACI 308R, 2001). When testing is not performed to verify in-place strength, ACI 308R state that concrete should be maintained above 10°C and kept moist for minimum curing periods as shown in Table 2.

Table 2 - Recommended minimum duration of curing (ACI 308R, 2001)

Cement type	**Minimum curing period**
ASTM C 150 Type I	7 days
ASTM C 150 Type II	10 days
ASTM C 150 Type III or when accelerators are used to achieve results demonstrated by test to be comparable to those achieved using ASTM C 150 Type III cement	3 days
ASTM C 150 Type IV or Type V cement	14 days

2.6. Curing Affected zone

Curing measures applied on concrete only affects the surface of concrete up to a certain depth. As demonstrated in figure 4, the interior part of concrete is not affected by curing (ACI 308R, 2001).

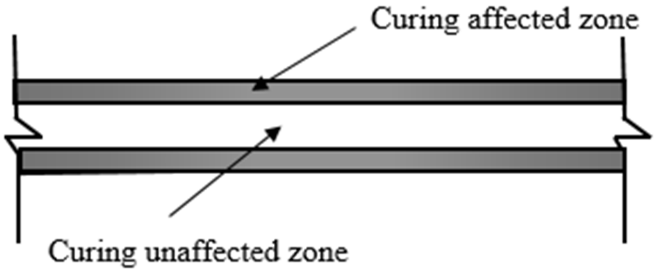

Figure 4 – Curing affected zone and curing unaffected zone

Many researchers call this zone in different names; some of them are:

- Curing affected zone (Cather, 1994; ACI 308R, 2001)
- Cover concrete (Parrott, 1992)
- Outer-zone concrete (Neville, 2013)
- Surface zone (Soroka & Baum, 1994)
- Skin of concrete (Kreijger, 1984)

Although many names are given for a portion of concrete which is affected by curing, the name "Curing affected zone" is used in this study due to the fact that many researchers and standards like ACI use this name and it is like the conventional name.

Concrete properties in the curing-affected zone will be strongly influenced by curing effectiveness, while properties further from the surface will be less susceptible to moisture loss (ACI 308R, 2001).

2.6.1. The Depth of Curing Affected Zone

The thickness of curing affected zone varies depending on mixture proportion, curing method, curing duration, ambient environment and so on. Many researchers estimate the thickness of curing affected zone as follows:

- “This zone extends from the surface to a depth varying from approximately 5 to 20mm (1/4 to 3/4 in.)” (Cather, 1994).
- “It should be added that concrete remote from the surface, that is at depth, is hardly subjected to moisture movement, which affects only an outer zone, typically 30mm deep, but occasionally up to a depth of 50mm. In reinforced concrete, this depth represents all or most of the depth of cover” (Neville, 2013).
- “The effect of poor curing seemed to affect approximately the top 30mm of the slabs, and the affected zone decreased with the quality of the curing” (Senbetta, 1981).
- “The curing affected zone (CAZ) extended to approximately 20mm below the surface of the concrete that was exposed to the winter and summer climate, and 40-50mm for the concrete exposed to the Muscat climate (hot, arid climate with long and very hot summers and warm winters), with notable variation in properties due to climate and curing.” (AI-Kindy, 1998).
- “Concrete, in fact, has three skins, the cement skin (about 0.1mm thick), the mortar skin (about 5mm) and the concrete skin (about 30mm).” (Kreijger, 1984).

2.6.2. Moisture Movement in Curing Affected Zone

Water movement and moisture distribution within the continuously evolving pore network is of fundamental importance in many aspects of the science of cement-based materials and are directly responsible for the development of intrinsic material properties such as strength, permeability, and shrinkage (McCarter, et al., 2001). Zhang, et al. (2016), also states that the moisture content in concrete pores is critically important for most of the degradation processes suffered by concrete. Furthermore, moisture content in pores directly affects strength, thermal properties and the rate of cement hydration.

Zhang, et al., (2016) study moisture movement in early-age concrete to improve the knowledge of moisture movement and simulation of concrete at an early age. First, the moisture movement in three concretes with water to cement ratios of 0·62, 0·43 and 0·30, representing low (30MPa), middle (50MPa) and high (80MPa) strength concrete in practice, respectively, were experimentally investigated by measuring the internal relative humidity at 20mm, 70mm and 120mm from the surface of concrete. Second, based on the experimental findings, mathematical modeling of moisture movement, in terms of internal relative humidity distribution and its variation with time was developed (Zhang, et al., 2016).

The relative humidity field, showing the comparison between model and test results, is displayed in Figure 5, Figure 6, and Figure 7, where the progress of internal relative humidity of the three individual measured points in the test sample and the humidity profile at selected ages are displayed, respectively.

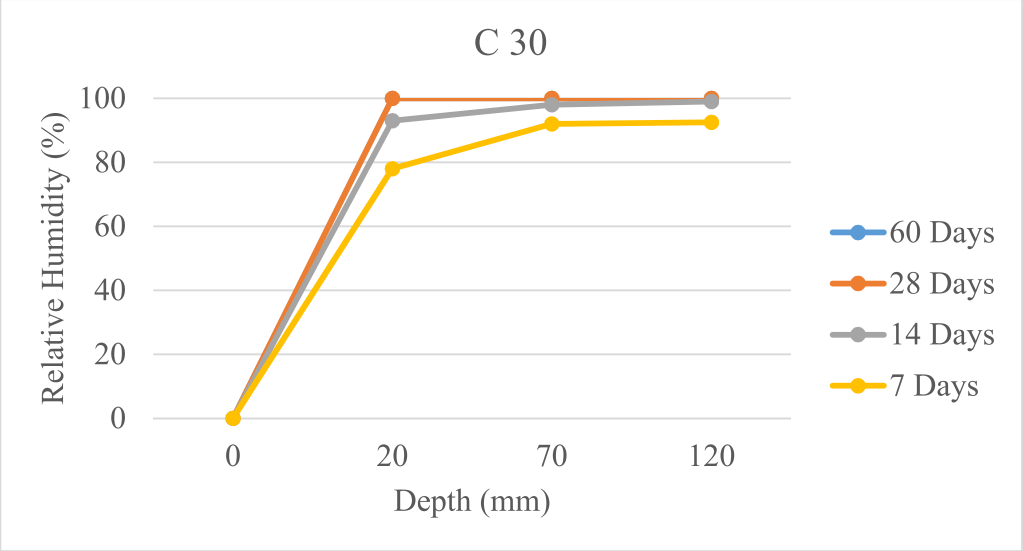

Figure 5 - Internal relative humidity for C 30 concrete grade (Zhang, et al., 2016).

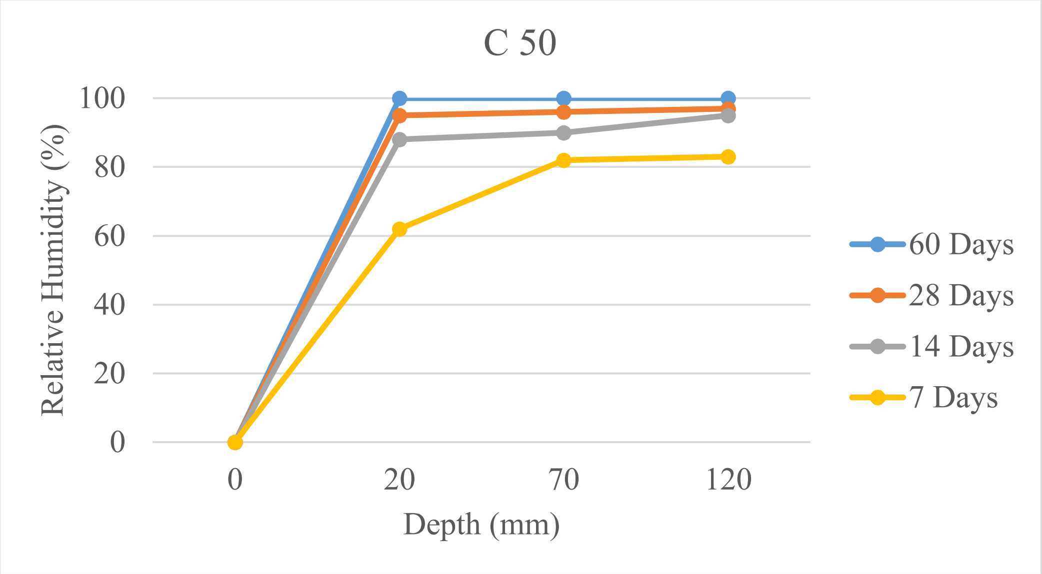

Figure 6 - Internal relative humidity for C 50 grade concrete (Zhang, et al., 2016).

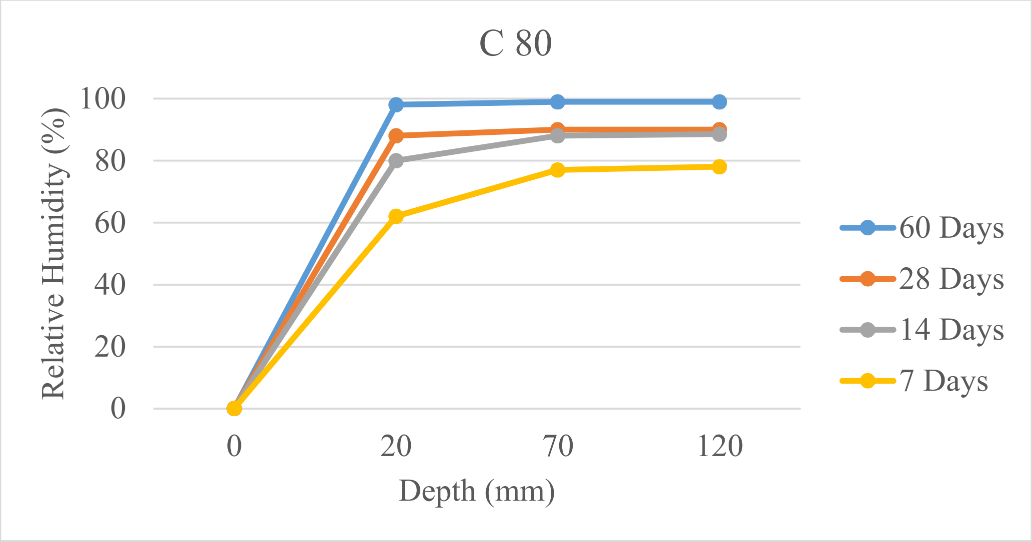

Figure 7 - Internal relative humidity for C 80 grade concrete (Zhang, et al., 2016).

From the above charts, one can realize that internal relative humidity increases significantly to approximately 50mm from the surface of the concrete. This is the zone which is also very much affected by curing. And starting from 50mm to the heart of concrete, relative humidity seems constant.

The other important point which the above graphs show is that, as water to cement ratio reduce, the relative humidity drops significantly due to the fact that the effect of self-desiccation on consumption of moisture is very significant in low water to cement ratio compared with high water to cement ratio.

2.7. Curing and Compressive Strength

2.7.1. Effect of Curing on Compressive Strength

The effect of curing on compressive strength of concrete will vary depending on the water to cement ratio. Concrete mixtures with high cement contents and low water to cement ratios (less than 0.40) may require special curing needs. As cement hydrates (chemically combining with water) the internal relative humidity decreases causing the paste to self-desiccate (dry out). The paste can self-desiccate to a level where hydration stops (Kosmatka, et al., 2003). This may influence desired concrete properties, especially compressive strength. Within a few days concrete becomes impermeable so, that it is practically impossible to inject water into the heart of concrete for hydration reaction. But the exterior face of concrete can be cured continuously. As a result, the exterior face of concrete become stronger than the interior part of the concrete.

Cook (1989) prepares a 760 x 760mm column with high strength concrete (77Mpa) to investigate the strength difference between the inner and exterior part of concrete with low water to cement ratio. The result shows that high initial temperatures generated by hydration significantly reduce the strength of the interior region of concrete. Figure 8 indicates that the strength of the core obtained from the middle of the concrete column is consistently less than the strength of the core obtained from the exterior face.

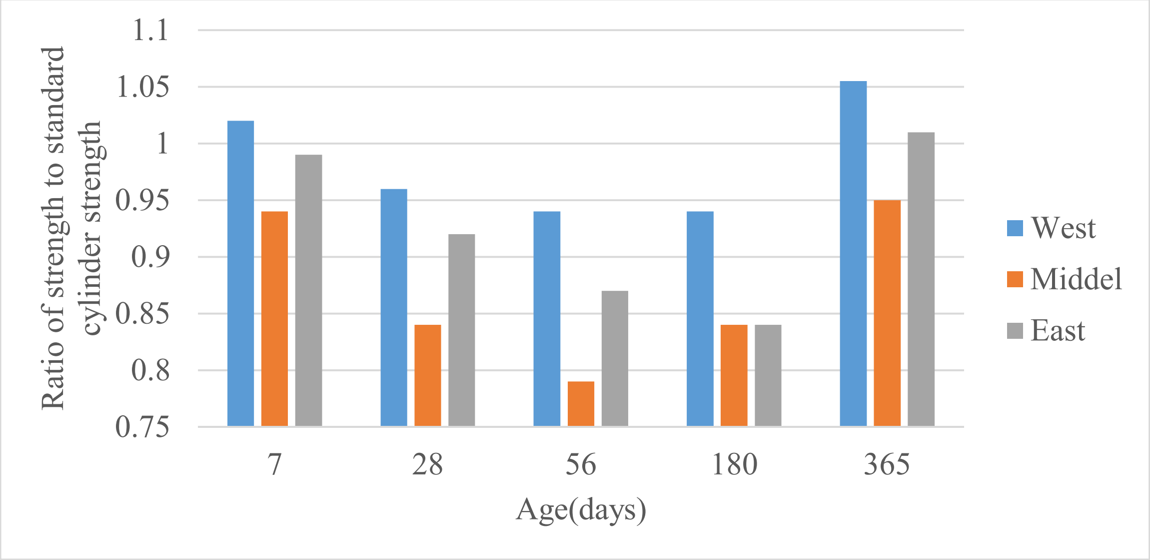

Figure 8 – Comparison of interior and exterior concrete strength (Cook, 1989)

A related issue discussed in ACI 308R which reports that self-desiccation can be remedied near the concrete surface by externally providing curing water to sustain hydration. At such low values of w/c, however, the permeability of the paste is normally so low that externally applied curing water will not penetrate far beyond the surface layer.

To the opposite, the concrete with high water to cement ratio (lean mix) contain more water than is required for hydration of the cement; however, excessive loss of water from the surface of concrete by evaporation can delay or prevent adequate hydration (Kosmatka, et al., 2003). The concrete within the heart concrete section is virtually self-curing (McCarter & Watson, 1997).

The other important factors which affect the influence of curing on compressive strength of in-place concrete is the size, and orientation of structures. As concrete become larger and larger, the percentage of curing affected zone from the total concrete section became smaller and smaller because the thickness of curing affected zone is independent of the size of the structure. In another hand, as concrete sections become small, the percentage of curing affected zone will be significant. Therefore, as more volume curing gets to affect, the less compressive strength will be.

Generally, the effect of curing on the compressive strength of large concrete sections is not significant. As Neville, (2013) point out, concrete in the interior of a structural member is generally unaffected by curing, so that curing is of little importance with respect to structural strength except in the case of very thin members. Small section concrete members and very thin concrete structures like slab and pavement in which most of their surface is exposed to the environment are more affected by curing than massive structures.

2.7.2. Comparison of In-Place and Standard Cube Strength

The final in-place quality of concrete depends on proportioning, placing, consolidation, and curing practices. Checking the quality of fresh concrete does not ensure quality in-place concrete (ACI 214.4R, 2003). It is obvious to expect large compressive strength result from standard cured concrete specimen compared with in-place compressive strength because the strength of in-place concrete is affected by curing time, curing duration, consolidation, exposure to fluctuated wind and temperature and micro-cracking due to loading.

Bungey J. and Millard S. (2006) pointed out that if measured in-place strength values are expressed as equivalent cube strengths, it will usually be found that they are less than the strengths of cubes made of concrete from the same mix which are compacted and cured in a standard way. In-place compaction and curing will vary widely. Typical comparisons between in-place concrete strength and standard cured concrete specimen are tabulated in Table 3.

Table 3 - Comparison of in-place and standard cube strengths (Bungey and Millard 2006)

Member type	**Typical 28 days in-place equivalent cube strength as % of standard cube strength**	
	Average	**Likely range**
Column	65	55-75
Wall	65	45-95
Beam	75	60-100
Slab	50	40-60

2.7.3. The Influence of Specimen Size on Effect of Curing

Strength is considered the primary property of concrete that governs system performance, particularly in the context of structural design. However, it has also been recognized since the 1980s that the strength and curing relationship typically presented (using standardly sized specimen) may not be entirely valid in structural sized elements, because the relative volume of material affected by poor curing reduces with increasing specimen size (Meyer, 1987). Most data available on the relationship between strength and curing is based on laboratory sized specimens, typically 50 to 150 mm cubes (Taylor, 2014).

Cather (1994) also reports that much of the published information concentrates on the effects of curing on strength. However, as the specimens are normally 100 mm or 150 mm cubes or 150 mm dia. cylinders, the influence of the CAZ is much greater than for most real structures. For example, if we were to assume a CAZ of 25 mm and then relate this to variously sized cubic specimens, the data given in Table 4 would result.

Table 4 - Volume of cube affected by curing: % (Cather, 1994)

Size of Cube in mm	**The Volume of Cube Affected by Curing: %**
50	100
100	87.5
150	70.0
200	57.8

A similar concept was developed by Soroka & Baum (1994) which states that concrete quality and the effectiveness of curing conditions are usually determined from the compressive strength of test specimens. Compressive strength, however, is a bulk property whereas curing affects mainly the quality of concrete outer layers. The thickness of the affected outer layers may be assumed to be independent of specimen size for a given curing condition. Hence, it is to be expected that the effect of the curing conditions on the measured compressive strength of concrete would decrease with an increase in the size of the test specimen.

In order to study the influence of specimen size on the effect of curing conditions on cube strength, Soroka & Baum (1994) determines the compressive strength of concrete at the ages of 28 and 90 days, on 70, 120, 200, and 250mm cubes. The concrete cubes were subjected to four curing conditions:

- **Condition A**: Exposed unprotected, immediately after casting to the age of testing, to 30˚C/40% relative humidity (RH).
- **Condition B**: The same as in A but exposed to 20˚C/65% RH.
- **Condition C**: Covered in molds for 24 hours with polyethylene sheeting at 20˚C/65% RH and then placed in water at 20˚C for six days followed by storage at 20˚C/40% RH to the age of testing.
- **Condition D:** Covered in molds as in C but later stored continuously in water at 20˚C to the age of testing.

The result presented in Figure 9 and Figure 10 shows that the effect of curing was greater in small specimens and decreased as the size increased. For example, the 28-day strength of concrete cubes subjected to standard curing (i.e., in water to the age of seven days followed by storage at 20˚C/65% RH), was about twice the strength of the corresponding uncured cubes when 70mm cubes were tested, and only 10% greater when 250mm cubes were tested.

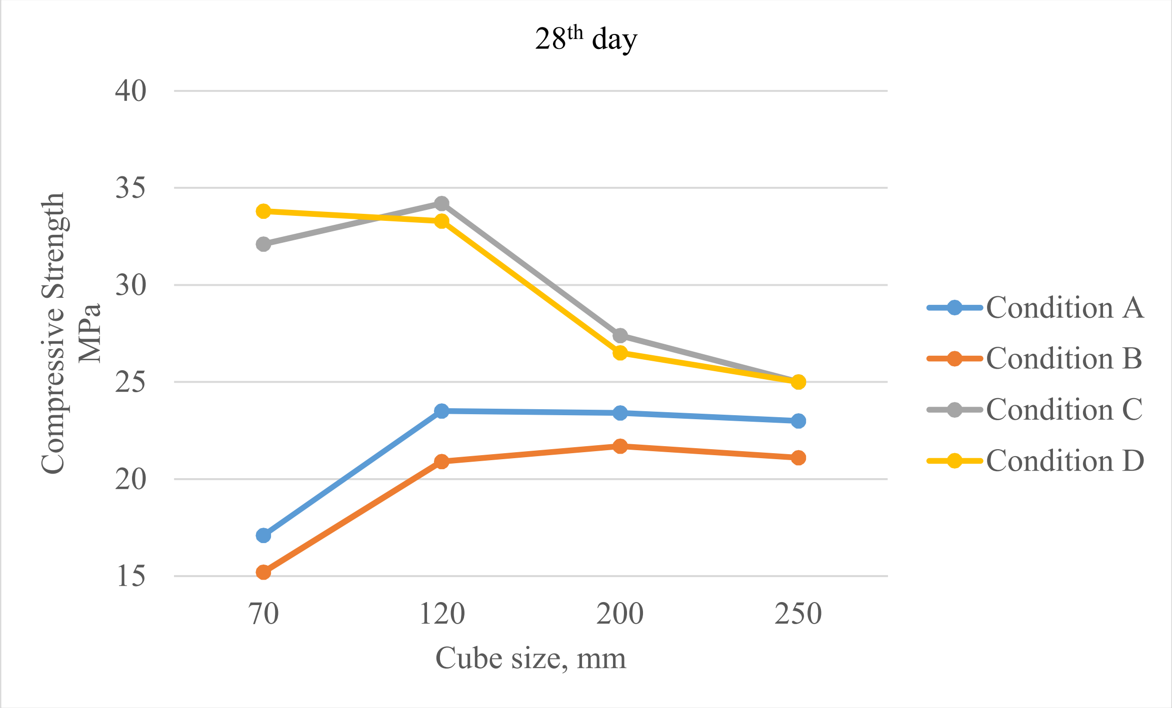

Figure 9 - Compressive strength as a function of cube size (Soroka & Baum, 1994)

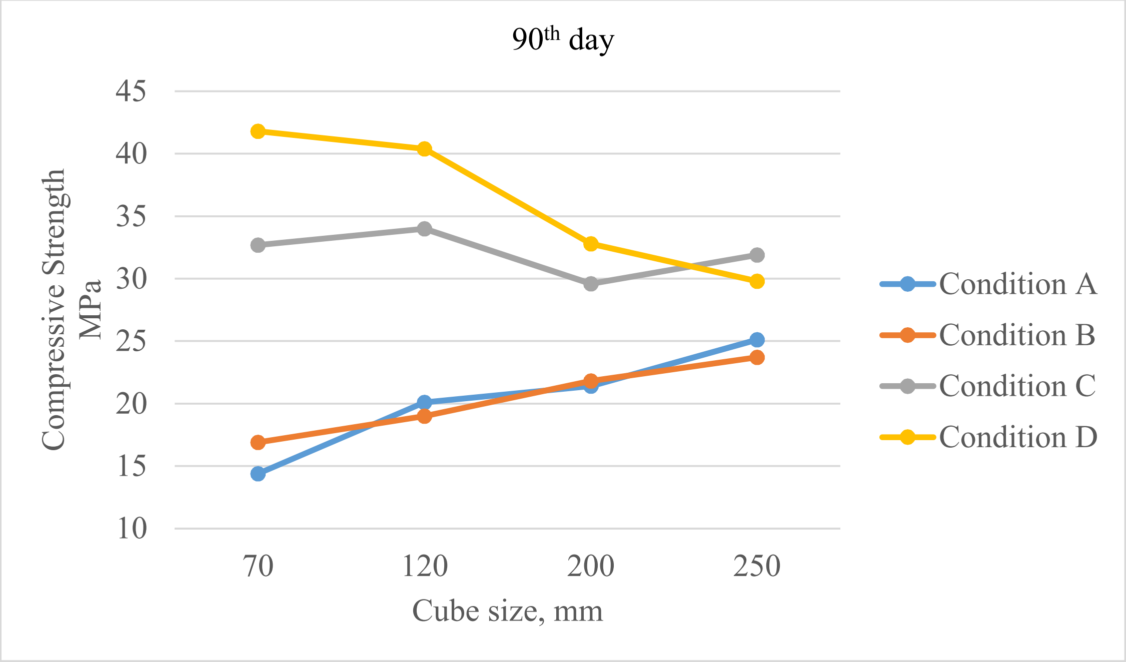

Figure 10 - Compressive strength as a function of cube size (Soroka & Baum, 1994)

This effect of size is attributable to the slowed hydration and the internal cracking that take place on drying in the outer layers of the specimens and thereby adversely affect their strength. Assuming the thickness of the affected layers to be independent of specimen size, their relative volume increases as the size decreases. Hence, the strength of the smaller specimens is more adversely affected than the strength of the larger specimens (Soroka & Baum, 1994). Taylor (2014) conclude similarly by saying that the magnitude of the effect of curing on strength of a sample will be significantly affected by the size of the sample, with larger effects to be observed in smaller sized samples.

2.8. Limitation of Existing Methods of Estimating Strength of In-place Concrete

2.8.1. Cast in Place in Cylinders (ASTM C 873/C 873M)

Cast in place cylinder is a technique for obtaining a cylindrical concrete specimen from newly casted slab without drilling a core (ACI 228.1R, 2003). The test is conducted by fastening concrete cylinder mold and a tubular support member within the concrete formwork prior to placement of the concrete as shown in Figure 11. (ASTM C 873/C 873M, 2004).

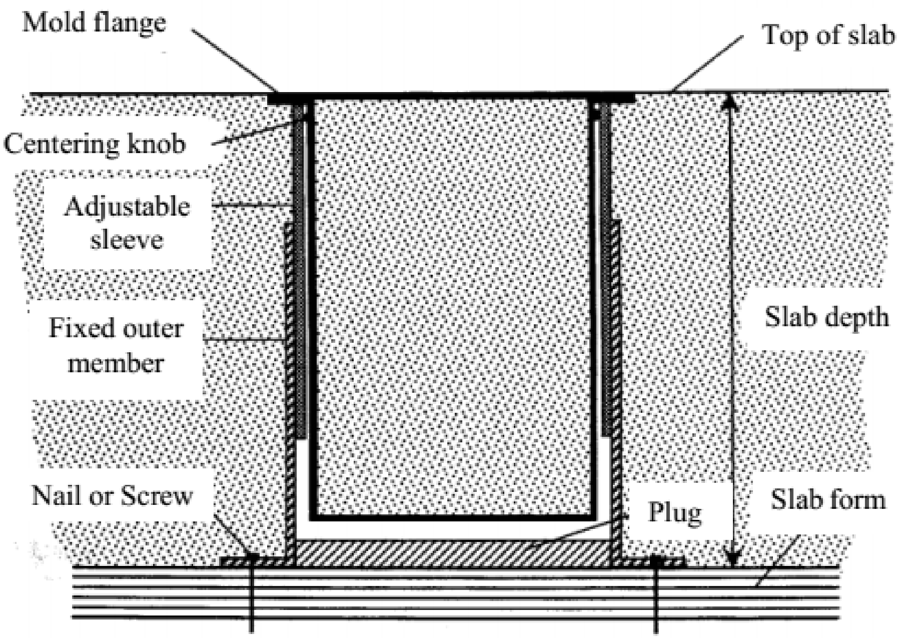

Figure 11 - Cast in-place cylinder mold assembly (ASTM C 873/C 873M, 2004)

The mold is filled when the slab is cast, and the concrete in the mold is allowed to cure with the slab. The objective of this test is to obtain the test specimen that has been subjected to the same thermal and moisture history as the concrete in structure (ACI 228.1R, 2003).

ASTM C 873/C 873M also states that cast in place cylinder strength relates to the strength of concrete in the structure due to the similarity of curing conditions since the cylinder is cured within the slab.

The strength of cast-in-place cylinders may be used for various purposes, such as estimating the load-bearing capacity of slabs, determining the time of form and shore removal, and determining the effectiveness of curing and protection. (ASTM C 873/C 873M, 2004)

The main limitation of this test method is that, since slabs are flexural members, the compressive loading direction is horizontal but in the case of compression testing, the testing direction is vertical and also tests are done in casting direction which gives the relatively higher result than lateral direction. Not only the direction of load is a problem but also the layer of curing affected zone is parallel to the actual slab compression load but in case of testing it is perpendicular as shown in Figure 12.

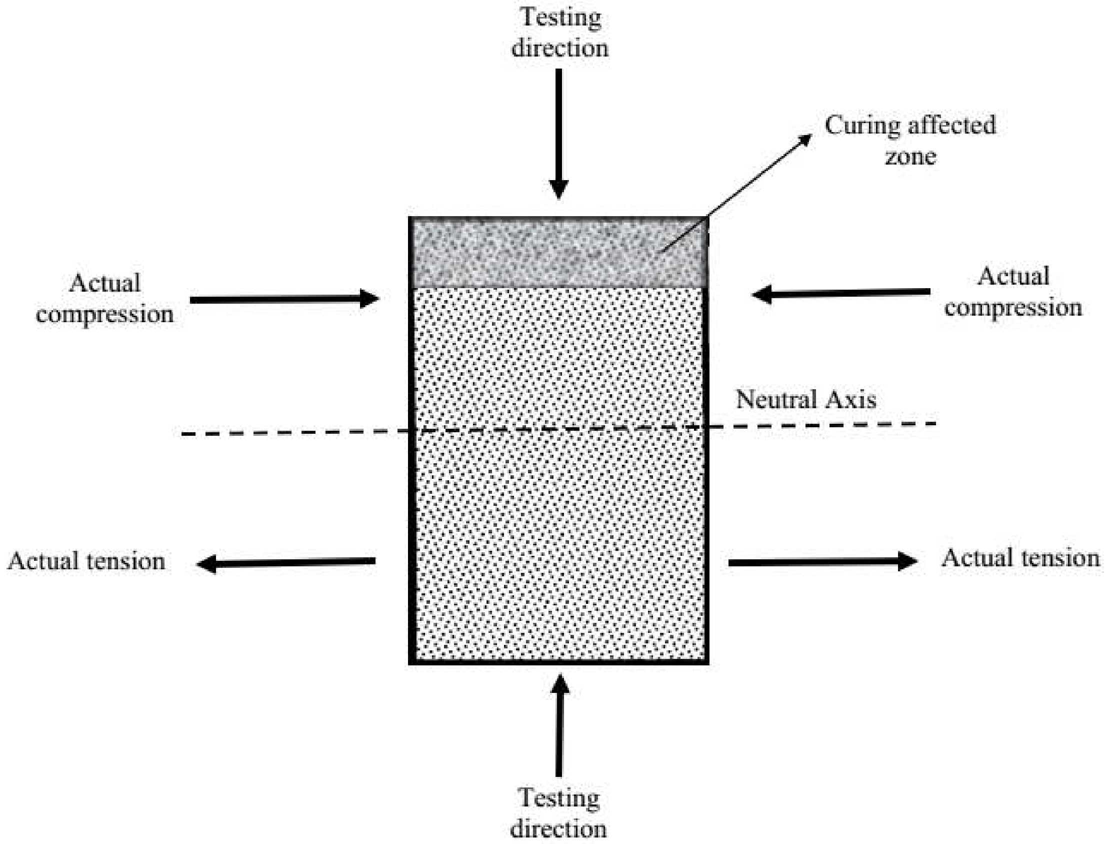

Figure 12 – Testing direction of cast-in-place cylinder and actual slab loads

2.8.2. Maturity Method (ASTM C 1074)

The maturity method is a technique to estimate in-place strength by accounting for the effects of temperature and time on strength development (ACI 228.1R, 2003). First, the strength-maturity relationship is developed by laboratory tests on the concrete mixture to be used as shown in Figure 13. Then the temperature history of the field concrete, for which strength is to be estimated, is recorded from the time of concrete placement to the time when the strength estimation is desired. The recorded temperature history is used to calculate the maturity index of the field concrete. Finally, using the calculated maturity index and the strength-maturity relationship, the strength of the field concrete is estimated (Celik & Nicholas, 2006).

ASTM C 1074 pointed out that there are two alternative functions for computing the maturity index from the measured temperature history of the concrete. One of the commonly used maturity equation which is used to compute the temperature-time factor is as follows:

$$M(t) = \sum (T_a - T_o)\Delta t \qquad \text{2-1}$$

where:

$M(t)$ = The temperature-time factor at age t, degree-days or degree-hours,

Δt = A time interval, days or hours,

T_a= The average concrete temperature during time interval, Δt, °C, and

T_o= Datum temperature, °C.

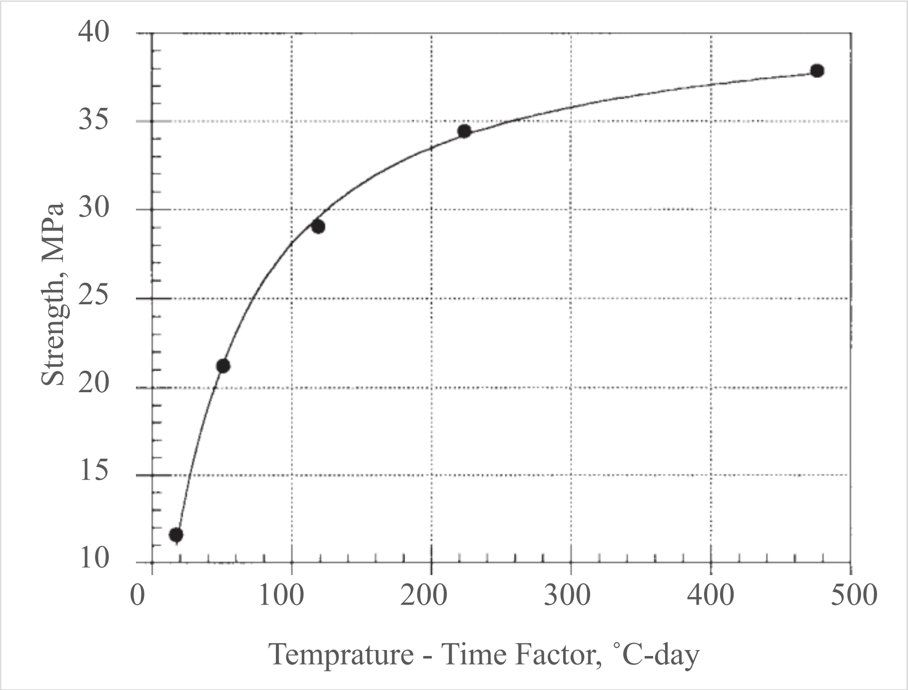

Figure 13- Example of a relationship between compressive strength and temperature time factor (ASTM C 1074, 2004)

This practice can be used to estimate the in-place strength of concrete to allow the start of critical construction activities such as (ASTM C 1074, 2004):

- Removal of formwork;
- Post-tensioning of tendons;
- Termination of cold weather protection; and
- Opening of the roadways to traffic.

This practice can also be used to estimate the strength of laboratory specimens cured under non-standard temperature conditions.

The major limitations of the maturity method are (ASTM C 1074, 2004):

- The concrete must be maintained in a condition that permits cement hydration;
- The method does not take into account the effects of early-age concrete temperature on the long-term strength; and
- The method needs to be supplemented by other indications of the potential strength of the concrete mixture.

2.8.3. Curing Concrete Test Specimens in the Field (ASTM C 31/C 31M)

This test method involves placing concrete specimens in or on the structure as near to the point of deposit of the concrete represented as possible and by providing the specimen with the same temperature and moisture environment as the structural work (ASTM C 31/C 31M, 2008).

If the specimens are made and field cured, the resulting strength test data when the specimens are tested are able to be used for the following purposes (ASTM C 31/C 31M, 2008):

- Determination of whether a structure is capable of being put in service
- Comparison with test results of standard cured specimens or with test results from various in-place test methods,
- Adequacy of curing and protection of concrete in the structure, or form or shoring removal time requirements

The major limitation of this method is that it assumes that if both concrete specimen and in-place concrete structures get the same temperature and moisture environment, the result will be approximately the same. As discussed in section 2.7.3, a significant volume of specimen is affected by curing compared with in-place concrete structures. Cather, (1994) assume a 25mm thickness of curing affected zone and calculate the volume of cube affected by curing as tabulated in Table 5.

Table 5 - Volume of cube affected by curing (Cather, 1994)

Size of Cube in mm	**The Volume of Cube Affected by Curing: %**
50	100
100	87.5
150	70.0
200	57.8

Based on Cather (1994), the percentage of curing affected volume of column (3m height) and slab (1m strip width) are tabulated in Table 6, and Table 7.

Table 6 – Volume of column affected by curing

Square Column: mm	**The Thickness of CAZ: mm**	**The Volume of Column Affected by Curing:**
400	25	25%
600	25	17%
800	25	14%

Table 7 – Volume of solid slab affected by curing

Depth: mm	**NA from the Top Surface: mm**	**The Thickness of CAZ: mm**	**The Volume of Solid Slab Affected by Curing:**
100	50	25	50%
150	75	25	33%
200	100	25	25%

From those tables, one can realize that a significant volume of concrete specimen is affected by curing compared with in-place concrete structures. Concrete specimens and in-place concrete structures have relatively different curing affected volume. Which means a concrete with large curing affected volume will have relatively lower compressive strength compared to the one with small curing affected volume. Therefore, the compressive strength result of concrete specimens made and cured in the field to represent the in-place concrete underestimate the compressive strength of in-place strength.

ACI 228.1R, (2003) also pointed out that measured strength of field-cured cylinders may significantly differ from in-place strengths because it is difficult and often impossible, to have identical bleeding, consolidation, and curing condition for concrete in cylinder and concrete in the structure.

2.8.4. Core Test (ASTM C 42/C 42M)

This test is used to determine the compressive, splitting tensile, and flexural strength of in-place concrete. Generally, test specimens are obtained when doubt exists about the in-place concrete quality due either to low strength test results during construction or signs of distress in the structure. Another use of this method is to provide strength information on older structures (ASTM C 42/C 42M, 2004).

Drilled cores provide representative samples of in-place concrete. Several factors, however, contribute to the uncertainty of measured core strength as being truly representative of the in-place strength. These factors include, among others, the presence of moisture gradients resulting from water-cooled drilling, undefined damage introduced by the core removal process, and differences in size and L/D value compared with standard molded specimens (Celik & Nicholas, 2006).

To resolve those problems, ACI 214.4R presents the equation by which the in-place strength of the concrete at the location from which a core test specimen was extracted can be computed. The equation is:

$$f_c = F_{l/d} F_{dia} F_{mc} F_d f_{core} \qquad 2\text{-}2$$

Where f_c is the equivalent in-place strength; f_{core} is the core strength; and strength correction factors$F_{l/d}$,F_{dia}, and F_{mc} account for the effects of the length-to-diameter ratio, diameter, and moisture condition of the core, respectively. Factor F_d accounts for the effect of damage sustained during drilling including microcracking and undulations at the drilled surface and cutting through coarse-aggregate particles that may subsequently pop out during testing.

In addition, core tested parallel to the casting direction may have about 8 % higher strengths than that tested perpendicular to the casting direction (Neville, 2013).

2.9. Research Gap Identification

Based on the above literature review the following gaps are identified and the researcher attempted to fill those gaps in the current study:

- Most researches and books discuss the effect of curing on compressive strength of concrete. However, there is a gap in research to quantify the effect of curing in order to estimate the compressive strength of in-place concrete.
- There are many types of research which dealt with the effectiveness of curing methods, the effect of curing delay on strength of concrete, moisture movement in curing affected zone and so on. However, there is a gap in the research so far to quantify the influence of curing for in-place compressive strength estimation purpose.
- Most researchers study the effect of curing on the compressive strength of concrete by using standardly sized specimens mostly 150mm cube. As discussed in section 2.6 and 2.7.3 curing affects only the surface of concrete up to 25mm. Studying with this big sized specimen means studying both curing affected zone and curing unaffected zone simultaneously in a single specimen. However, there is no research which separately studies the effect of curing on compressive strength of curing affected zone and unaffected zone.
- Type and size of concrete structures have its own influence on strength of in-place concrete regarding the proportion of curing affected zone with unaffected zone but the researcher found no research which deals with it.

Generally, the gap of research which is identified to be examined in this study is that there are no method or model provided to relate the compressive strength result of standard cured concrete specimen with strength of in-place concrete in order to estimate strength of in-place concrete taking curing duration, curing method, size and type of structure in to consideration.

3. Methodology

For the purpose of this research the following methodology was followed:

1. **Literature review:** Reviewing related literature was carried out critically in order to identify the limitations of existing methods in estimating the compressive strength of in-place concrete. Literature is used also to understand the basics about concrete, curing and compressive strength. And then they were taken as a secondary data source and necessary and reasonable assumption were taken like the thickness of curing affected zone to be 25mm.
2. **Development of theoretical equations:** Theoretical formulae were attempted to be developed to estimate in-place strength of concrete for beams (Rectangular, T-Section, and L-Section), slabs (solid and ribbed), and columns (rectangular and circular) putting type and size of structure, curing method, curing duration, and compressive strength result in to consideration.

   The formulae are developed based on the assumption that the compressive strength of curing affected zone and curing unaffected zone are different. So that, both zones together withstand the coming stress as reinforcement and concrete do in reinforced concrete structures.

   By representing both zones, specimens were prepared and compressive strength of both curing affected zone and curing unaffected zone were measured separately. Then, relationships were made between compressive strength of curing affected zone with standard cured (Laboratory cured) concrete sample and compressive strength of curing unaffected zone with standard cured concrete sample. Using size, type of structure and the relationship made, the equivalent compressive strength of in-place concrete can be estimated.

   In these formulae, curing methods of water spraying, covering with wet burlap and plastic by applying water twice a day, and without any curing (air curing) were studied with curing durations of 3, 7, and 14 days because these methods and durations are used most frequently in Ethiopia (Molla, 2017; Walelign, 2014). The water-cement ratio of 0.4, 0.5 and, 0.6 are selected because many of the concrete grade used are in a range from 20MPa to 35MPa (Walelign, 2014). Therefore, according to ACI 211.4, the water-cement ratio to produce such grade of concrete is in the range between 0.4 and 0.6.

2.1. Preparation of test specimen:

2.1.1. Representation of curing affected zone:

As briefly discussed in section 2.6, curing affects only the surface layer of concrete up to a depth of 20 - 30mm so that the size of specimens to represent this zone shall be small enough. As a result, instead of using the common 150mm sized cubes, the researcher takes the thickness of curing affected zone to be 25mm and with this smaller sized 50mm cubes were used so that 100% of its volume will be affected by curing (see Table 4 in chapter 2).

Smaller specimens are good representatives to observe the effect of curing on the compressive strength of concrete than larger specimens because a significant amount of its volume is affected by curing than large specimens. Neville (2013) also pointed out that the loss of strength due to inadequate curing is more pronounced in smaller specimen than in larger specimen. Cather (1994) also report that strength tests on 100 mm or 150 mm cubes or 150 mm diameter cylinders, is not sufficiently sensitive to curing to be useful unless small samples are used. Soroka & Baum, (1994) compares the influence of specimen size on the effect of different curing conditions on concrete compressive strength using 70, 120, 200 and 250mm cubes. In 70mm cube specimens, the variation of compressive strength due to different curing conditions was very significant compared to the strength variation in other size specimens (see Figure 9 in chapter 2).

Walelign (2014) report that compressive strength is a bulk property, but curing affects the top 20-30mm depth from the surface of concrete which is curing affected zone (CAZ). The depth of standard concrete molds is 100 or 150 mm (cubic) and 250 or 300 mm (cylindrical). Compared to the total depth of the mold, the CAZ is small, so that the compressive strength of concrete may not be a good indicator to observe curing effectiveness. Rater, the researcher used mortar cubes of 50mm size for compressive strength test to assess the effectiveness of curing. The problems with mortar specimens are first, mortar cannot represent concrete, mortar is just mortar not concrete and second the researcher cites Mather (1987), (who stated that, if the effect of curing influences the strength development only to a depth of 25 mm below the surface of the concrete, one should perhaps use strength specimens having a maximum dimension of 50mm.) in order to justify the size of specimen the researcher used but just because the size of mortar cube and Mather's (1987) recommendation about size of specimen accidentally matches, it doesn't mean mortar must be the material to be casted.

For this research, a small 50mm cube specimens were used. For each water to cement ratio, curing duration, and curing method, the required number of specimens are indicated in Table 8.

Table 8 - The required number of specimens to investigate curing affected zone

w/c	**Methods of curing**	**Curing durations**			**Total**
		3 days	**7 days**	**14 days**	
0.4	Water spraying twice a day	3	3	3	9
	Covering with wet burlap	3	3	3	9
	Covering with plastic	3	3	3	9
	Laboratory curing (to 28th day)				3
	Air curing (to 28th day)				3
				Subtotal	33
0.5	Water spraying twice a day	3	3	3	9
	Covering with wet burlap	3	3	3	9
	Covering with plastic	3	3	3	9
	Laboratory curing (to 28th day)				3
	Air curing (to 28th day)				3
				Subtotal	33
0.6	Water spraying twice a day	3	3	3	9
	Covering with wet burlap	3	3	3	9

Covering with plastic	3	3	3	9
Laboratory curing (to 28th day)				3
Air curing (to 28th day)				3
			Subtotal	33
			Grand total	99

2.1.2. Representation of curing unaffected zone:

Curing unaffected zone is the heart of concrete element on which curing treatment and ambient condition have no effect on strength development. As a result, in order to represent curing unaffected zone, 150mm cube concrete specimens were cast and sealed completely with plastic (because plastics are good in retaining water than burlap) to prevent moisture entry or loss and stored in a laboratory in order to eliminate the effect of ambient condition.

Concrete specimens which are sealed against water entry or loss and stored in a condition where ambient environment have no effect and are a good representative of curing affected zone. For water to cement ration below 0.4, the only cause for drying of sealed specimen is self-desiccation, which is the internal drying of concrete due to consumption of water by hydration (ACI 308R, 2001) but for lean mixes, the water added is more than needed for hydration reaction so that the sealed specimens can cure itself.

Taylor (2014) also pointed out that if the w/c ratio is less than a critical value of about 0.42 in a sealed system, there will be insufficient water to hydrate all the cement, and the fully hardened paste will consist of cement gel, empty capillaries, and hydrated cement. If the w/c ratio is greater than this critical value, all of the cement can hydrate, and the capillary voids will be partially filled with water. As a result, sealed concrete specimens are good representatives to study curing unaffected zone.

For mixes with three different water to cement ratio, a total of 9 specimens with the size of 150mm cube were prepared as listed in Table 9.

Table 9 - The required number of specimens for curing unaffected zone

W/c	The required number of specimens
0.4	3
0.5	3
0.6	3
Total	9

2.1.3. Preparation of control specimens:

Nine 150mm cube concrete specimens were prepared, which means three specimens for each water to cement ratio.

2.2. Preparation of formwork, concrete making materials, and concrete:

2.2.1. Wooden mold preparation:

Wooden mold with an internal dimension of 50mm was prepared. One mold set can handle six specimens as shown in Figure 14. (Dimensions are in mm)

Figure 14 – Specimen mold layout

A total of 10 molds were prepared so that they could be used to cast 60 specimens at a time. These molds were used three times to cast three mixes with 0.4, 0.5 and 0.6 water to cement ratios. Even though the required specimen at a time is 33 for one water to cement ratio, the researcher cast 60 specimens in order to make sure that if in case some specimens are happed to be defective, the reserves will be substituted. In the final outcome, 26 specimens were rejected due to the fact that either they are defective or unnecessary.

Before casting the specimens, wooden molds were saturated with water and the surface wiped to make them at SSD condition, so that the formwork may not take or supply moisture from or to the concrete specimens. A releasing agent was used in order to dismantle easily molds from the concrete specimen.

2.2.1. Aggregates:

Crushed coarse aggregate and natural fine aggregate (sand) were used. The maximum aggregate size was 19mm. The tests carried out on aggregates and the detailed result is given in Annex 1 and Annex 2. The summarized results are tabulated below.

Table 10 – Test result of aggregates

Tests	Fine aggregate	Coarse aggregate
Moisture content	3.09%	1.11%
Effective absorption	6.05%	1.07%
Specific gravity	2.63	2.83
Bulk unit weight	1566.83Kg/m^3	1640Kg/m^3
Silt content	9.76% (Before washing) 3.95% (After washing)	-
Fineness modules	2.95	-
Sieve analysis	See Annex 1	See Annex 2

2.3. Cement:

Since OPC cement type is the most widely used for all structural members (Molla, 2017; Walelign, 2014), Messebo OPC (42.5N) cement was used because Messebo was the only available OPC cement in the market of Bahir Dar at the time of purchase.

2.2.2. Water:

Bahir Dar City's tap water was used for making and curing concrete specimens.

2.2.3. Mix Design

The mix design was done according to ACI 211.1-91 (Reapproved 2002) due to the fact that almost all organizations in Bahir Dar those involved in mix design use ACI method (Molla, 2017). For detailed mix design calculation, see 0. Since the mold size is small, the concrete must be workable enough. As a result, a higher slump of 100-120mm was obtained. The amount of material used and the slump obtained are listed in Table 11.

Table 11 - The amount of material used and the slump obtained

Materials	**Water to cement ratio**		
	0.4	**0.5**	**0.6**
Water (Kg/m^3)	255.50	260.50	227.00
Cement (Kg /m^3)	512.50	410.00	341.50
Fine aggregate (Kg/ m^3)	619.00	696.00	790.50
Coarse aggregate (Kg/ m^3)	981.50	981.50	981.50
Slump obtained (mm)	105	120	110

2.2.4. Concrete:

Slump test was conducted as per ASTM C 143/C 143M (Test Method for Slump of Hydraulic Cement Concrete).

Concrete specimens which represent curing affected zone were cast in the prepared wooden molds. ASTM C 31/C 31M require that the minimum specimen dimension to be at least three times the maximum size aggregate but, in this research, the maximum aggregate size is 19mm which means three times the maximum aggregate size (19x3 = 57mm) is a little bit more than the dimension of mold (50mm). In such a situation, there are two types of methods, the large-sized aggregate may be removed either by handpicking or by wet sieving as stated in ASTM C172. (Neville, 2013; Celik & Nicholas, 2006). The researcher used handpicking of larger-sized aggregates.

After 24hrs, the specimens were demolded and in order to create approximately similar curing condition with the in-place concrete, specimens were exposed to the specified curing durations as noted in Table 8. For plastic covering, the actual practice in the construction site the application of water is twice a day by opening the plastic cover at the top (Molla, 2017). Similarly, the specimens which represent the curing affected zone for plastic covering curing method was covered with plastic and water was applied by opening the plastic twice a day and it was resealed after the application.

The daily average temperature was 23.2 C°, the average daily relative humidity was 52.6%, the average daily wind speed was 0.9m/s and the average daily rainfall was 3.6mm (Western Amhara Metrology Service Center, 2018). The detailed daily weather condition data is as tabulated in Annex 8.

At the end of the 28th day, the specimens were saturated for a minimum of 4hrs before testing to ensure uniform moisture condition from specimen to specimen (Soroka & Baum, 1994). Then, the compressive strength test was conducted.

Curing unaffected zone is the core of concrete body by which curing treatment and ambient condition have no effect. As a result, specimens were covered completely with plastic to prevent moisture gain or loss and stored in the laboratory in order to eliminate the effect of ambient condition. Before the time of testing, the specimens were saturated to ensure uniform moisture condition from specimen to specimen. Then, the compressive strength test was conducted at the age of 28 days. ASTM C 192/C 192M Standard Practice for Making and Curing Concrete Test Specimens in the Laboratory was used.

3. Data recording: Formats were prepared and all data were recorded for the anticipated analysis.
4. Data analysis: all recorded data were analyzed and coefficients were determined.
5. Conclusion and recommendation: finally, conclusions were drawn, recommendations were given and areas future researches were pointed out.

4. Results and Discussions

4.1. Development of Theoretical Equations

The following abbreviations are used to develop the formulae:

f_a – Compressive strength of curing affected zone
f_u – Compressive strength of curing unaffected zone
f_c – Compressive strength of standard cured concrete specimen at 28th day
f_e – Equivalent in-place concrete compressive strength
F – Axial load carrying capacity of cthe ross-section
A – Gross cross-sectional area
a – Cross-sectional area of curing affected zone
u – Cross-sectional area of curing unaffected zone
z – Depth of curing affected zone
β – Coefficient of curing (the ratio of f_a tof_c)
δ – Coefficient of water to cement ratio (the ratio of f_u to f_c)

4.1.1. Columns

A. Rectangular Plane column

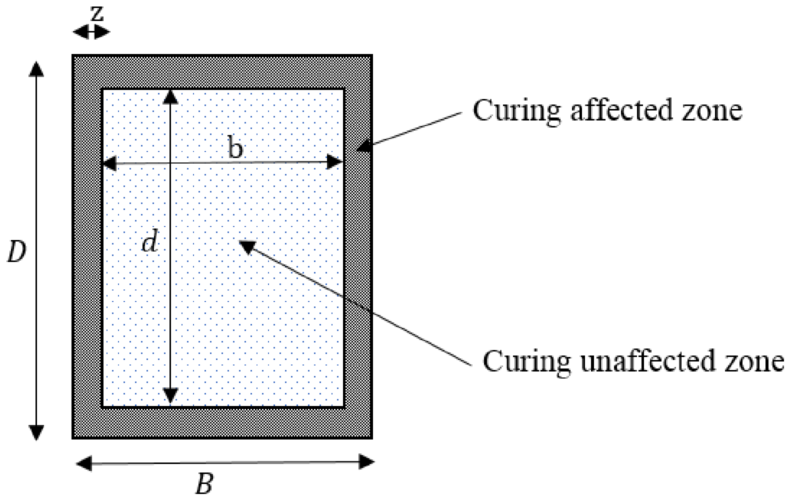

Figure 15. A cross-sectional view of a rectangular column

Calculation of cross-sectional areas:

$$A = BD \tag{4-1}$$

$$\mathrm{u} = \mathrm{bd} \qquad \text{Where:} \quad \mathrm{b} = \mathrm{B} - 2\mathrm{z}$$

$$d = D - 2z$$

$$\mathrm{u} = (\mathrm{B} - 2\mathrm{z})(\mathrm{D} - 2\mathrm{z})$$

$$= \mathrm{BD} - 2\mathrm{zB} - 2\mathrm{zD} + 4z^2 \tag{4-2}$$

$$\mathrm{a} = \mathrm{A} - \mathrm{u} \tag{4-3}$$

Substituting equations 4-1 and 4-2 into equation 4-3

$$\mathrm{a} = \mathrm{BD} - (\mathrm{BD} - 2\mathrm{zB} - 2\mathrm{zD} + 4z^2)$$

$$= 2\mathrm{zB} + 2\mathrm{zD} - 4z^2 \tag{4-4}$$

Calculation for equivalent in-place compressive strength: the equivalent in-place compressive strength can be obtained by dividing the axial load carrying capacity of the section (F) divided by cross-sectional area (A). Therefore, mathematically:

$$f_e = \frac{F}{A} \tag{4-5}$$

The axial load carrying capacity (F) of a section can be obtained by multiplying compressive strength of curing affected zone by the cross-sectional area of curing affected zone plus the compressive strength of curing unaffected zone multiplied by the cross-sectional area of curing unaffected zone. Therefore:

$$F = \mathrm{a}f_a + uf_u \tag{4-6}$$

Now, substituting equations 4-6 into equation 4-5

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \tag{4-7}$$

Then, substituting equation 4-1, 4-2, and 4-4 into equations 4-7

$$According, \quad f_e = \frac{(2\mathrm{zB} + 2\mathrm{zD} - 4z^2)f_a + (\mathrm{BD} - 2\mathrm{zB} - 2\mathrm{zD} + 4z^2)f_u}{BD} \tag{4-8}$$

The compressive strength of curing affected zone (f_a) depends on water to cement ratio, curing method and curing duration. But the compressive strength of curing unaffected zone (f_u) depends on water to cement ratio only (keeping other strength affecting factors in to constant other than curing method and duration). With that in mind, the following relationships are made:

$$f_a = \beta f_c \tag{4-9}$$

$$f_u = \delta f_c \tag{4-10}$$

Where:

β = coefficient of curing which can be determined by dividing compressive strength of curing affected zone to the compressive strength of standard cured concrete specimen at 28th day. This coefficient varies depending on water to cement ratio, curing method, and curing duration. See section 4.2.1 to see the values of β.

δ = coefficient of water to cement ratio which is obtained by dividing compressive strength of curing unaffected zone to compressive strength of standard cured concrete specimen. See section 4.2.2 to see the values of δ.

Now, substituting equations 4-9 and 4-10 into equation 4-8.

$$f_e = \frac{(2\mathrm{zB} + 2\mathrm{zD} - 4z^2)\beta f_c + (\mathrm{BD} - 2\mathrm{zB} - 2\mathrm{zD} + 4z^2)\delta f_c}{BD}$$

$$= \frac{f_c}{BD}(2\beta zB + 2\beta zD - 4\beta z^2 + \delta BD - 2\delta zB - 2\delta zD + 4\delta z^2)$$

$$= \frac{f_c}{BD}(2\beta zB - 2\delta zB + 2\beta zD - 2\delta zD - 4\beta z^2 + 4\delta z^2 + \delta BD)$$

$$= \frac{f_c}{BD}(2zB(\beta - \delta) + 2zD(\beta - \delta) - 4z^2(\beta - \delta) + \delta BD)$$

$$= \frac{f_c}{BD}(\beta - \delta)(2zB + 2zD - 4z^2) + \delta f_c$$

$$f_e = 2zf_c(\beta - \delta)\left(\frac{D + B - 2Z}{BD}\right) + \delta f_c \tag{4-11}$$

B. Circular Plane Column

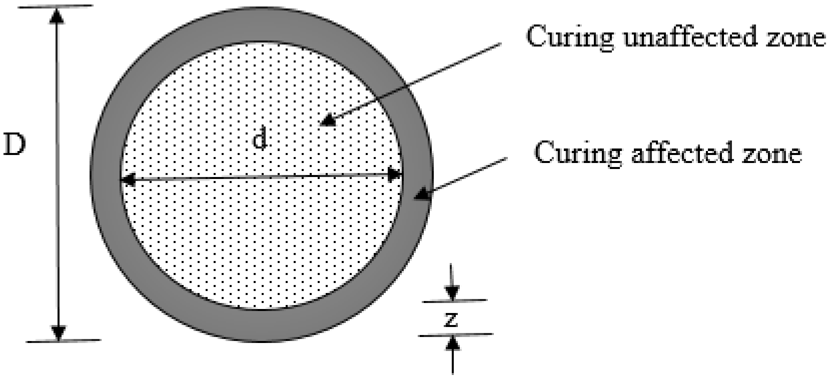

Figure 16 – Cross-sectional view of a circular column

Calculation of cross-sectional areas:

$$A = \frac{\pi D^2}{4} \qquad 4\text{-}12$$

$$u = \frac{\pi d^2}{4}$$

Where: $d = D - 2z$

$$u = \frac{\pi (D - 2z)^2}{4}$$

$$u = \frac{\pi (D^2 - 4zD + 4z^2)}{4}$$

$$u = \frac{\pi D^2 - 4\pi z(D - z)}{4}$$

$$u = \frac{\pi D^2}{4} - \pi z(D - z) \qquad 4\text{-}13$$

$$a = A - u \qquad 4\text{-}14$$

Substituting equations 4-12 and 4-13 into equation 4-14

$$\mathrm{a} = \frac{\pi D^2}{4} - \left[\frac{\pi D^2}{4} - \pi z(\mathrm{D} - \mathrm{z})\right]$$

$$\mathrm{a} = \frac{\pi D^2}{4} - \frac{\pi D^2}{4} + \pi z(\mathrm{D} - \mathrm{z})$$

$$\mathrm{a} \ = \pi z(\mathrm{D} - \mathrm{z}) \qquad 4\text{-}15$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \qquad 4\text{-}16$$

$$F = \ \mathrm{a}f_a + uf_u \qquad 4\text{-}17$$

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \qquad 4\text{-}18$$

Now, substituting equations 4-13 and 4-15 into equation 4-3

$$f_e = \frac{f_a\big(\pi z(\mathrm{D} - \mathrm{z})\big) + f_u(\frac{\pi D^2}{4} - \pi z(\mathrm{D} - \mathrm{z}))}{\frac{\pi D^2}{4}}$$

$$f_e = \frac{\beta f_c\big(\pi z(\mathrm{D} - \mathrm{z})\big) + \delta f_c(\frac{\pi D^2}{4} - \pi z(\mathrm{D} - \mathrm{z}))}{\frac{\pi D^2}{4}}$$

$$f_e = \pi f_c \left[\frac{(\beta z(\mathrm{D} - \mathrm{z}) - \delta z(\mathrm{D} - \mathrm{z}) + \delta \frac{D^2}{4})}{\frac{\pi D^2}{4}}\right]$$

$$f_e = 4\mathrm{z}f_c \left[\frac{(\beta - \delta)(\mathrm{D} - \mathrm{z})}{D^2}\right] + \delta f_c \qquad 4\text{-}19$$

4.1.2. Beams

While calculating the equivalent compressive strength of in-place concrete of all types of beams, the tensile strength of concrete is neglected; due to the fact that all the incoming tensile stresses are resisted by the reinforcement bars.

A. Rectangular Beam

I. Singly reinforced rectangular beam

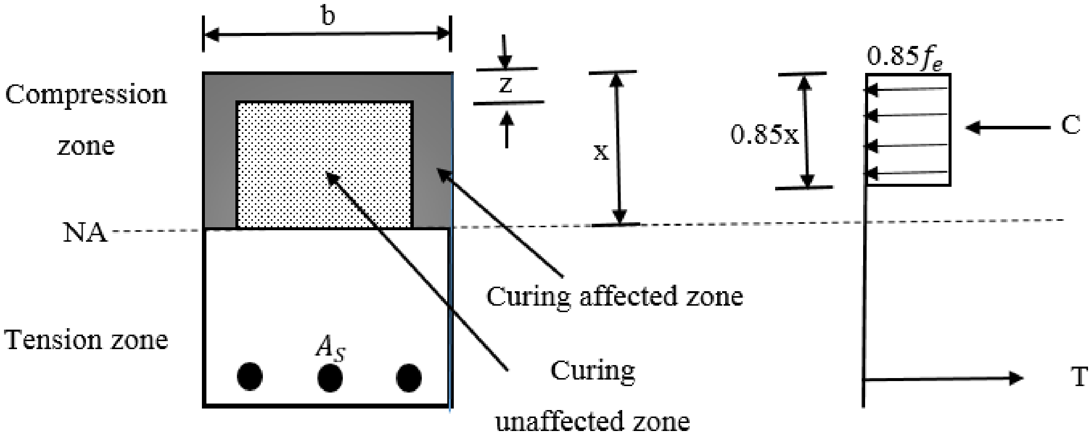

Figure 17 – Cross-sectional view of the rectangular singly reinforced beam

Calculation of cross-sectional areas:

$$A = bx \tag{4-20}$$

$$u = (b - 2z)(x - z)$$

$$= bx - bz - 2zx + 2z^2 \tag{4-21}$$

$$a = A - u$$

$$a = bz + 2zx - 2z^2 \tag{4-22}$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \tag{4-23}$$

$$F = af_a + uf_u \tag{4-24}$$

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \qquad 4\text{-}25$$

$$f_e = \frac{f_a(\mathrm{bz} + 2\mathrm{zx} - 2z^2) + f_u(bx - bz - 2zx + 2z^2)}{bx} \qquad 4\text{-}26$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C = T \qquad 4\text{-}27$$

$$C = 0.85xbf_e \qquad 4\text{-}28$$

$$T = A_s f_s \qquad 4\text{-}29$$

Now, equating equations 4-28 to 4-29:

$$0.85xbf_e = A_s f_s$$

$$x = \frac{A_s f_s}{0.85bf_e}$$

Let, $w = \frac{A_s f_s}{0.85}$ so that the value of x will be:

$$x = \frac{w}{bf_e} \qquad 4\text{-}30$$

Substituting equation 4-30 into eqation 4-26:

$$f_e = \frac{f_a(\mathrm{bz} + \frac{2\mathrm{zw}}{bf_e} - 2z^2) + f_u(\frac{\mathrm{bw}}{bf_e} - bz - \frac{2\mathrm{zw}}{bf_e} + 2z^2)}{\frac{\mathrm{bw}}{bf_e}}$$

$$f_a = \beta f_c \qquad 4\text{-}31$$

$$f_u = \delta f_c \qquad 4\text{-}32$$

$$f_e = f_e\left[\frac{\beta f_c(\mathrm{bz}+\frac{2\mathrm{zw}}{bf_e}-2z^2)+\delta f_c(\frac{\mathrm{w}}{f_e}-bz-\frac{2\mathrm{zw}}{bf_e}+2z^2)}{\mathrm{w}}\right]$$

$$\frac{w}{f_c} = \beta(\mathrm{bz}+\frac{2\mathrm{zw}}{bf_e}-2z^2)+\delta(\frac{\mathrm{w}}{f_e}-bz-\frac{2\mathrm{zw}}{bf_e}+2z^2)$$

$$\frac{w}{f_c}=\frac{2\beta\mathrm{zw}}{bf_e}+\frac{\delta\mathrm{w}}{f_e}-\frac{2\delta\mathrm{zw}}{bf_e}+\beta\mathrm{bz}-\delta bz-2\beta z^2++2\delta z^2$$

$$\frac{w}{f_c}=\frac{\mathrm{w}}{f_e}(\frac{2\beta\mathrm{z}}{b}+\delta-\frac{2\delta\mathrm{z}}{b})+\mathrm{bz}(\beta-\delta)-2z^2(\delta-\beta)$$

$$\frac{w}{f_c}-\mathrm{bz}(\beta-\delta)-2z^2(\delta-\beta)=\frac{\mathrm{w}}{f_e}(\frac{2\beta\mathrm{z}}{b}+\delta-\frac{2\delta\mathrm{z}}{b})$$

$$f_e=\frac{\mathrm{w}(\frac{2\beta z}{b}+\delta-\frac{2\delta\mathrm{z}}{b})}{\frac{w}{f_c}-\mathrm{bz}(\beta-\delta)-2z^2(\delta-\beta)}$$

$$f_e=\frac{\mathrm{w}(\frac{2\beta z}{b}+\delta-\frac{2\delta\mathrm{z}}{b})}{\frac{w}{f_c}-\mathrm{bz}(\beta-\delta)-2z^2(\delta-\beta)}$$

$$f_e=\frac{\mathrm{w}(\frac{2\beta z+\delta b-2\delta\mathrm{z}}{b})}{\frac{w-\mathrm{bz}f_c((\beta-\delta)-2z^2(\delta-\beta))}{f_c}}$$

$$f_e=\frac{\mathrm{w}f_c(2\beta z+\delta b-2\delta\mathrm{z})}{wb-f_c\mathrm{b}(\mathrm{bz}(\beta-\delta)-2z^2(\delta-\beta))}$$

$$f_e=\frac{\mathrm{w}f_c(2\beta z+\delta b-2\delta\mathrm{z})}{wb-f_c\mathrm{b}(\mathrm{bz}(\beta-\delta)+2z^2(\beta-\delta))}$$

$$f_e=\frac{\mathrm{w}f_c(2\beta z+\delta b-2\delta\mathrm{z})}{wb-f_c\mathrm{b}((\beta-\delta)(\mathrm{bz}+2z^2)}$$

$$f_e=\frac{\mathrm{w}f_c(2\beta z+\delta b-2\delta\mathrm{z})}{wb-f_c\mathrm{zb}((\beta-\delta)(\mathrm{b}+2z)} \tag{4-33}$$

II. Double reinforced rectangular beam

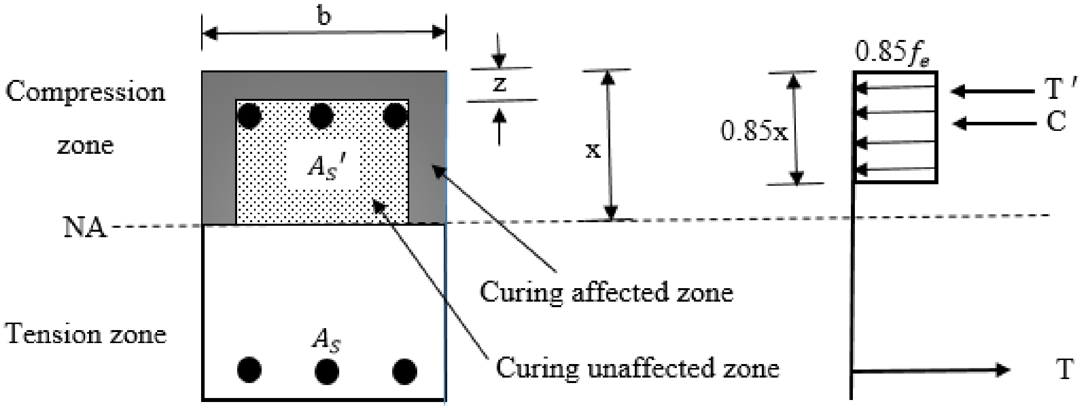

Figure 18 - Cross-sectional view of the rectangular doubly reinforced beam

Calculation of cross-sectional areas:

$$A = bx \quad \text{4-34}$$

$$u = bx - bz - 2zx + 2z^2$$

$$a = bz + 2zx - 2z^2 \quad \text{4-35}$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \quad \text{4-36}$$

$$F = af_a + uf_u \quad \text{4-37}$$

$$f_e = \frac{af_a + uf_u}{A} \quad \text{4-38}$$

$$f_e = \frac{f_a(\mathrm{bz} + 2\mathrm{zx} - 2z^2) + f_u(bx - bz - 2zx + 2z^2)}{bx} \quad \text{4-39}$$

In order to determine the value of x, the internal forces (i.e.C, T' and T) should be at equilibrium. Therefore:

$$C + T' = T \quad \text{4-40}$$

$$C + T' = 0.85xbf_e + A_{s'}f_s \quad \text{4-41}$$

$$T = A_sf_s \quad \text{4-42}$$

Now, equating equation 4-41 and equation 4-42:

$$0.85xbf_e + A_{s\prime}f_s = A_s f_s$$

$$x = \frac{A_s f_s - A_{s\prime} f_s}{0.85bf_e}$$

Let, $v = \frac{A_s f_s - A_{s\prime} f_s}{0.85}$ so that the value of x will be:

$$x = \frac{v}{bf_e} \tag{4-43}$$

Now, substitute equation 4-43 into equation 4-39:

$$f_e = \frac{f_a(\mathrm{bz} + \frac{2\mathrm{z}v}{bf_e} - 2z^2) + f_u(\frac{\mathrm{b}v}{bf_e} - bz - \frac{2\mathrm{z}v}{bf_e} + 2z^2)}{\frac{\mathrm{b}v}{bf_e}}$$

$$f_e = f_e\left[\frac{\beta f_c(\mathrm{bz} + \frac{2\mathrm{z}v}{bf_e} - 2z^2) + \delta f_c(\frac{v}{f_e} - bz - \frac{2\mathrm{z}v}{bf_e} + 2z^2)}{v}\right]$$

$$\frac{v}{f_c} = \beta(\mathrm{bz} + \frac{2\mathrm{z}v}{bf_e} - 2z^2) + \delta(\frac{v}{f_e} - bz - \frac{2\mathrm{z}v}{bf_e} + 2z^2)$$

$$\frac{v}{f_c} = \frac{2\beta\mathrm{z}v}{bf_e} + \frac{\delta v}{f_e} - \frac{2\delta\mathrm{z}v}{bf_e} + \beta\mathrm{bz} - \delta bz - 2\beta z^2 + +2\delta z^2$$

$$\frac{v}{f_c} = \frac{v}{f_e}(\frac{2\beta\mathrm{z}}{b} + \delta - \frac{2\delta\mathrm{z}}{b}) + \mathrm{bz}(\beta - \delta) - 2z^2(\delta - \beta)$$

$$f_e = \frac{v(\frac{2\beta z + \delta b - 2\delta\mathrm{z}}{b})}{\frac{v - \mathrm{bz}f_c((\beta - \delta) - 2z^2(\delta - \beta))}{f_c}}$$

$$f_e = \frac{vf_c(2\beta z + \delta b - 2\delta\mathrm{z})}{vb - f_c\mathrm{b}\big(\mathrm{bz}(\beta - \delta) + 2z^2(\beta - \delta)\big)}$$

$$f_e = \frac{vf_c(2\beta z + \delta b - 2\delta\mathrm{z})}{vb - f_c\mathrm{zb}((\beta - \delta)(\mathrm{b} + 2z)} \tag{4-44}$$

B. T-Section Beam

I. Singly reinforced T-Section beam when neutral axis lies within the web

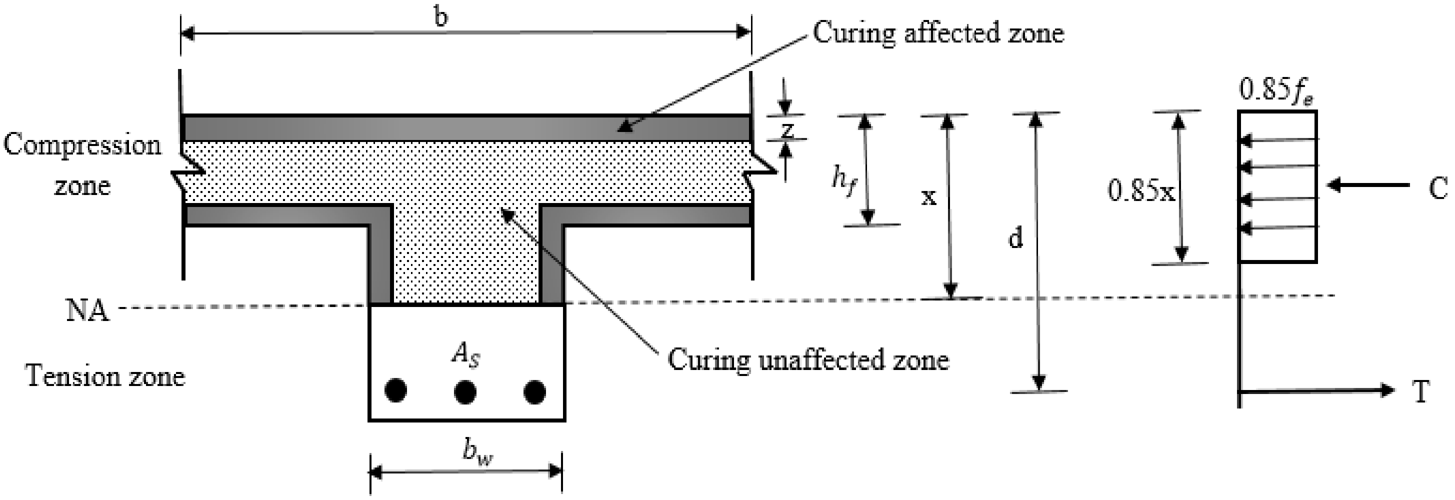

Figure 19 - Cross-sectional view of T-section singly reinforced beam (NA within the web)

Calculation of cross-sectional areas:

$$A = bh_f + b_w(x - h_f) \qquad 4\text{-}45$$

$$= h_f(b - b_w) + xb_w$$

$$\mathrm{u} = \mathrm{b}(h_f - 2z) + (b_w - 2z)(x - h_f + z)$$

$$= \mathrm{b}(h_f - 2z) + (z - h_f)(b_w - 2z) + x(b_w - 2z) \qquad 4\text{-}46$$

$$a = bz + z(b - b_w + 2z) + 2z(x - h_f)$$

$$= z(2b - b_w + 2z - 2h_f) + 2zx \qquad 4\text{-}47$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \qquad 4\text{-}48$$

$$F = \mathrm{a}f_a + uf_u \qquad 4\text{-}49$$

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \qquad 4\text{-}50$$

Substituting equations 4-45, 4-46 and 4-47 into equation 4-50: 4-51

$$f_e = \frac{f_a[z(2b - b_w + 2z - 2h_f) + 2zx] + f_u[\mathrm{b}(h_f - 2z) + (z - h_f)(b_w - 2z) + x(b_w - 2z)]}{h_f(b - b_w) + xb_w}$$

$$= \frac{\beta f_c[z(2b - b_w + 2z - 2h_f) + 2zx] + zx) + \delta f_c[\mathrm{b}(h_f - 2z) + (z - h_f)(b_w - 2z) + x(b_w - 2z)]}{h_f(b - b_w) + xb_w}$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C = T \qquad 4\text{-}52$$

$$C = 0.85Af_e$$

$$C = 0.85(h_f(b - b_w) + xb_w)f_e$$

$$C = 0.85h_f(b - b_w)f_e + 0.85xb_wf_e \qquad 4\text{-}53$$

$$T = A_sf_s \qquad 4\text{-}54$$

Substituting equations 4-53 and 4-54 into equation 4-52:

$$0.85h_f(b - b_w)f_e + 0.85xb_wf_e = A_sf_s$$

$$x = \frac{A_sf_s - 0.85h_f(b - b_w)f_e}{0.85b_wf_e}$$

$$x = \frac{A_sf_s}{0.85b_wf_e} - \frac{h_f(b - b_w)}{b_w}$$

Let, $w = \frac{A_sf_s}{0.85}$ and $= \frac{h_f(b-b_w)}{b_w}$; so that the value of x will be:

$$x = \frac{w}{b_wf_e} - l \qquad 4\text{-}55$$

Now, substituting equation 4-55 into equation 4-51:

$$f_e = \frac{\beta f_c\left[b(2b - b_w + 2z - 2h_f) + 2z(\frac{w}{b_wf_e} - l)\right] + \delta f_c\left[\mathrm{b}(h_f - 2z) + (z - h_f)(b_w - 2z) + (\frac{w}{b_wf_e} - l)(b_w - 2z)\right]}{h_f(b - b_w) + (\frac{w}{b_wf_e} - l)b_w}$$

$$= \frac{\beta f_c \left[b\left(2b - b_w + 2z - 2h_f\right) + \frac{2zw}{b_w f_e} - zl\right] + \delta f_c \left[\mathrm{b}\left(h_f - 2z\right) + \left(z - h_f\right)(b_w - 2z) + \frac{w}{f_e}\left(1 - \frac{2z}{b_w}\right) - l(b_w - 2z)\right]}{h_f(b - b_w) + \frac{w}{f_e} - lb_w}$$

$$f_e h_f(b - b_w) + f_e \frac{w}{f_e} - f_e b_w l - \beta f_c \frac{2zw}{b_w f_e} - \delta f_c \frac{w}{f_e}\left(1 - \frac{2z}{b_w}\right) = \beta f_c\left[z\left(2b - b_w + 2z - 2h_f\right) - 2zl\right] + \delta f_c\left[\mathrm{b}\left(h_f - 2z\right) + \left(z - h_f\right)(b_w - 2z) - l(b_w - 2z)\right] \quad \text{4-51'}$$

$$let, \quad m = \beta f_c\left[z\left(2b - b_w + 2z - 2h_f\right) - 2zl\right] + \delta f_c\left[\mathrm{b}\left(h_f - 2z\right) + \left(z - h_f\right)(b_w - 2z) - l(b_w - 2z)\right]$$

$$f_e h_f(b - b_w) + w - f_e b_w l - \frac{f_c w}{f_e}\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = m$$

$$\frac{{f_e}^2 h_f(b - b_w) + w f_e - {f_e}^2 b_w l - f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{f_e} = m$$

$${f_e}^2 h_f(b - b_w) + w f_e - {f_e}^2 b_w l - f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = m f_e$$

$${f_e}^2 h_f(b - b_w) - {f_e}^2 b_w l + w f_e - m f_e - f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = 0$$

$$(h_f(b - b_w) - b_w l){f_e}^2 + (w - m) f_e - f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = 0$$

$$Substituting, \qquad l = \frac{h_f(b - b_w)}{b_w}\ , from\ earlier\ equation$$

$$(h_f(b - b_w) - b_w\left(\frac{h_f(b - b_w)}{b_w}\right)){f_e}^2 + (w - m) f_e - f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = 0$$

$$(h_f(b - b_w) - h_f(b - b_w)){f_e}^2 + (w - m) f_e - f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = 0$$

$$(w - m) f_e = f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]$$

$$f_e = \frac{f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{(w - m)} \quad \text{4-56}$$

II. Singly reinforced T-Section beam when neutral axis lies within the flange

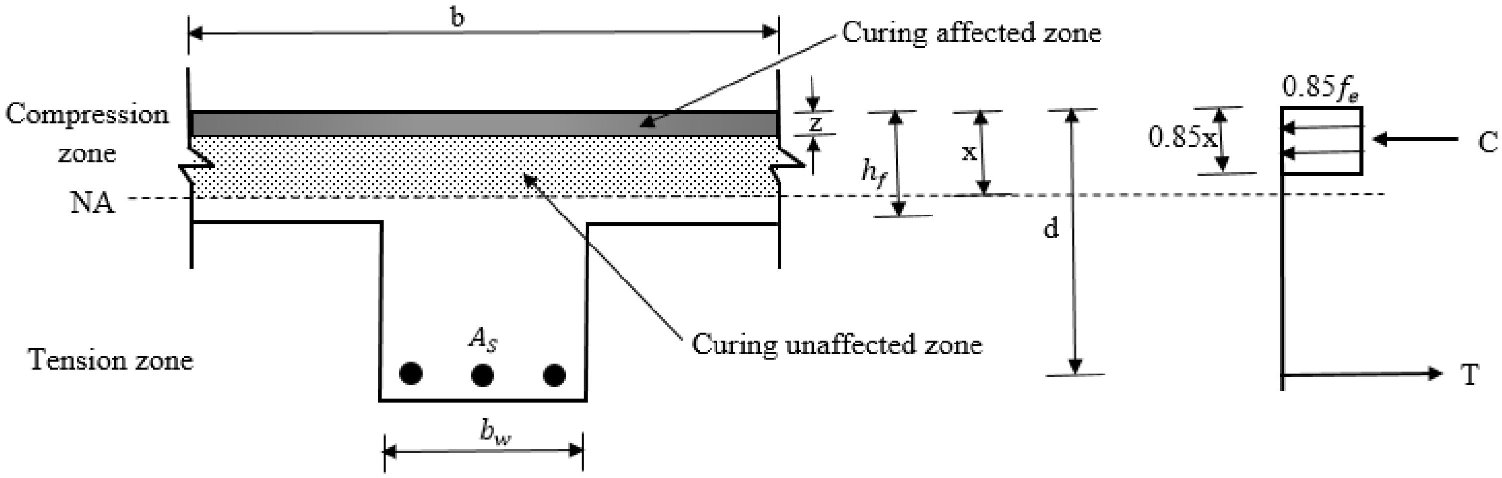

Figure 20 - Cross-sectional view of T-section singly reinforced beam (NA within flange)

Calculation of cross-sectional areas:

$$A = bx \tag{4-57}$$

$$\mathrm{u} = b(x - z) \tag{4-58}$$

$$\mathrm{a} = bz \tag{4-59}$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \tag{4-60}$$

$$F = \mathrm{a}f_a + uf_u \tag{4-61}$$

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \tag{4-62}$$

Substituting equations 4-57, 4-58 and 4-59 into equation 4-62:

$$f_e = \frac{f_a(bz) + f_u(b(x - z))}{bx} \tag{4-63}$$

$$f_e = \frac{bzf_a + xbf_u - bzf_u}{bx}$$

$$f_e = \frac{bz(f_a - f_u) + xbf_u}{bx}$$

$$f_e = \frac{zf_c(\beta - \delta)}{x} + \delta f_c \tag{4-64}$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C = T \qquad 4\text{-}65$$

$$C = 0.85xbf_e \qquad 4\text{-}66$$

$$T = A_s f_s \qquad 4\text{-}67$$

substituting equations 4-66 and 4-67 in to equation 4-65:

$$0.85xbf_e = A_s f_s$$

$$x = \frac{A_s f_s}{0.85bf_e}$$

Let $w = \frac{A_s f_s}{0.85}$ so that the value of x will be:

$$x = \frac{w}{bf_e} \qquad 4\text{-}68$$

Substituting equation 4-68 into equation 4-64:

$$f_e = \frac{zf_c bf_e(\beta - \delta)}{w} + \delta f_c$$

$$f_e - \frac{zf_c bf_e(\beta - \delta)}{w} = \delta f_c$$

$$f_e\left[1 - \frac{zf_c b(\beta - \delta)}{w}\right] = \delta f_c$$

$$f_e = \left[\frac{\delta f_c}{1 - \frac{zf_c b(\beta - \delta)}{w}}\right]$$

$$f_e = \left[\frac{\delta w f_c}{w - zbf_c(\beta - \delta)}\right] \qquad 4\text{-}69$$

III. Double reinforced T-Section beam when neutral axis lies within the web

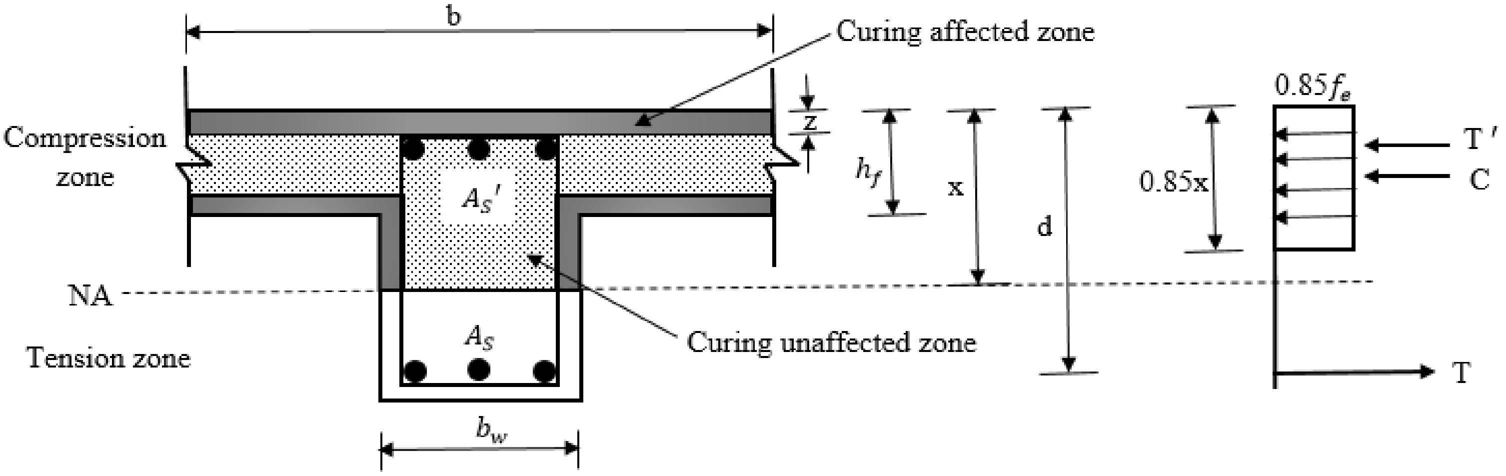

Figure 21 - Cross-sectional view of T-section doubly reinforced beam (NA within web)

In order to determine the value of x, the internal forces (i.e. C, T' and T) should be at equilibrium. Therefore:

$$C + T' = T \tag{4-70}$$

$$C = 0.85Af_e \tag{4-71}$$

$$T' = A_{s'}f_s \tag{4-72}$$

$$C = 0.85\big(h_f(b - b_w) + xb_w\big)f_e$$

$$C = 0.85h_f(b - b_w)f_e + 0.85xb_wf_e$$

$$T = A_sf_s \tag{4-73}$$

Substituting equations 4-71, 4-72 and 4-73 into 4-70:

$$0.85h_f(b - b_w)f_e + 0.85xb_wf_e + A_{s'}f_s = A_sf_s$$

$$x = \frac{A_sf_s - A_{s'}f_s - 0.85h_f(b - b_w)f_e}{0.85b_wf_e}$$

$$x = \frac{A_sf_s - A_{s'}f_s}{0.85b_wf_e} - \frac{h_f(b - b_w)}{b_w}$$

Let, $v = \dfrac{A_sf_s - A_{s'}f_s}{0.85}$ and $l = \dfrac{h_f(b - b_w)}{b_w}$ (from equation 4 – 55)

$$x = \frac{v}{b_w f_e} - l \tag{4-74}$$

Then substituting the value of x into equation 4-51’

$$f_e h_f(b - b_w) + f_e \frac{v}{f_e} - f_e b_w l - \beta f_c \frac{2zv}{b_w f_e} - \delta f_c \frac{v}{f_e}\left(1 - \frac{2z}{b_w}\right)$$
$$= \beta f_c(z\big(2b - b_w + 2z - h_f\big) - zl)) + \delta f_c(\mathrm{b}\big(h_f - 2z\big)$$
$$+ \big(z - h_f\big)(b_w - 2z) - l(b_w - 2z))$$

$$let \quad m = \beta f_c(z\big(2b - b_w + 2z - 2h_f\big) - 2zl)) + \delta f_c(\mathrm{b}\big(h_f - 2z\big)$$
$$+ \big(z - h_f\big)(b_w - 2z) - l(b_w - 2z))\ (as\ in\ the\ previeus)$$

Then, $f_e h_f(b - b_w) + v + f_e b_w l - \frac{f_c v}{f_e}(\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right) = m$

$$\frac{{f_e}^2 h_f(b - b_w) + v f_e - {f_e}^2 b_w l - f_c v(\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)}{f_e} = m$$

$${f_e}^2 h_f(b - b_w) + v f_e - {f_e}^2 b_w l - f_c v(\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right) = m f_e$$

$${f_e}^2 h_f(b - b_w) - {f_e}^2 b_w l + v f_e - m f_e - f_c v(\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right) = 0$$

$$(h_f(b - b_w) - b_w l){f_e}^2 + (v - m) f_e - f_c v(\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right) = 0$$

$$Substituting, l = \frac{h_f(b - b_w)}{b_w}\ (from\ the\ previous)$$

$$(h_f(b - b_w) - b_w\left(\frac{h_f(b - b_w)}{b_w}\right)){f_e}^2 + (v - m) f_e - f_c v\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right] = 0$$

$$(v - m) f_e = f_c v\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]$$

$$f_e = \frac{f_c v\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{(v - m)} \tag{4-75}$$

IV. Double reinforced T-Section beam when neutral axis lies within the flange

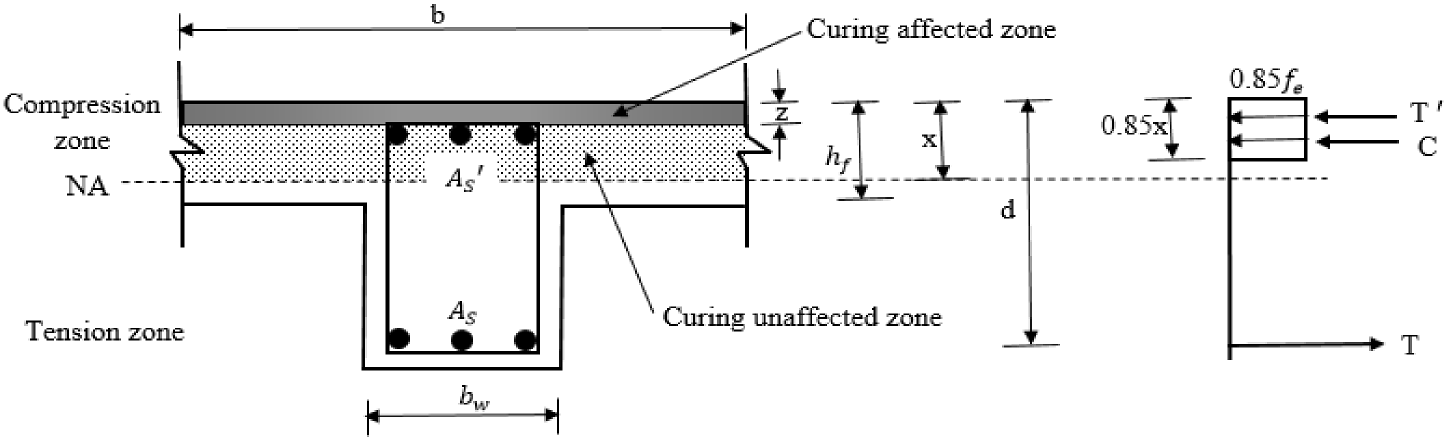

Figure 22 - Cross-sectional view of T-section doubly reinforced beam (NA within flange)

$$f_e = \frac{zf_c(\beta - \delta)}{x} + \delta f_c \; (From\ equation\ 4-64)$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C + T' = T \tag{4-76}$$

$$C + T' = 0.85xbf_e + A_{S'}f_s \tag{4-77}$$

$$T = A_s f_s \tag{4-78}$$

Now, substituting equations 4-77 and 4-78 into equations 4-76:

$$0.85xbf_e = A_s f_s - A_{S'}f_s$$

$$x = \frac{A_s f_s - A_{S'}f_s}{0.85bf_e}$$

As in the previous, let $v = \frac{A_s f_s - A_{S'}f_s}{0.85}$, so that the value of x will be:

$$x = \frac{n}{bf_e} \tag{4-79}$$

Substituting equations 4-79 into equation 4-64 and simplifying:

$$f_e = \frac{zf_c bf_e(\beta - \delta)}{v} + \delta f_c$$

$$f_e = \left[\frac{\delta v f_c}{v - zbf_c(\beta - \delta)}\right] \tag{4-80}$$

C. L-Section Beam

I. Singly reinforced L-Section beam when neutral axis lies within the web

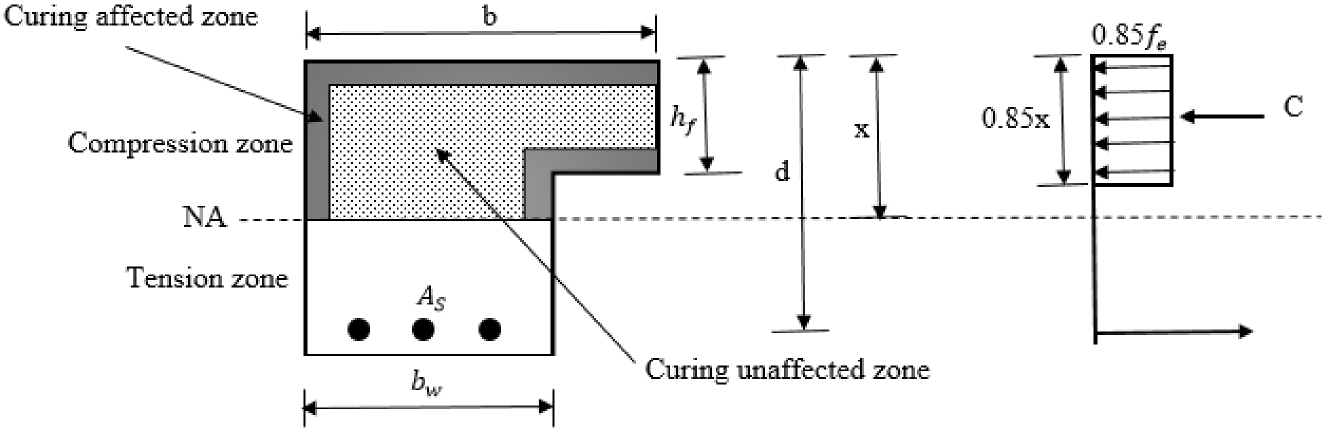

Figure 23 - Cross-sectional view of L-section singly reinforced beam (NA within web)

Calculation of cross-sectional areas:

$$A = bh_f + b_w(x - h_f) \qquad \text{4-81}$$

$$= h_f(b - b_w) + xb_w$$

$$\mathrm{u} = (b_w - z)(x - z) + (h_f - 2z)(b - b_w + z)$$

$$= (\mathrm{b} - 2\mathrm{z})(h_f - 2\mathrm{z}) + (\mathrm{z} - h_f)(b_w - 2z) + \mathrm{x}(b_w - 2z) \qquad \text{4-82}$$

$$a = bz + z(x - z) + z(b - b_w + z) + z(x - h_f)$$

$$= z(2b - b_w - h_f) + 2zx \qquad \text{4-83}$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \qquad \text{4-84}$$

$$F = \mathrm{a}f_a + uf_u \qquad \text{4-85}$$

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \qquad \text{4-86}$$

Substituting equations 4-81, 4-82 and 4-83 into equation 4-86.

$$f_e = \frac{f_a(z(2b - b_w - h_f) + 2zx) + f_u((\mathrm{b} - \mathrm{z})(h_f - 2\mathrm{z}) + (\mathrm{z} - h_f)(b_w - 2z) + \mathrm{x}(b_w - 2z))}{h_f(b - b_w) + xb_w} \qquad \text{4-87}$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C = T \tag{4-88}$$

$$C = 0.85Af_e$$

$$Now, inserting\ A\ = h_f(b - b_w) + xb_w\ in\ to\ the\ previous\ equation:$$

$$C = 0.85\big(h_f(b - b_w) + xb_w\big)f_e$$

$$C = 0.85h_f(b - b_w)f_e + 0.85xb_wf_e \tag{4-89}$$

$$T = A_sf_s \tag{4-90}$$

Substituting equations 4-89 and 4-90 into 4-88:

$$0.85h_f(b - b_w)f_e + 0.85Xb_wf_e = A_sf_s$$

$$x = \frac{A_sf_s - 0.85h_f(b - b_w)f_e}{0.85b_wf_e}$$

$$x = \frac{A_sf_s}{0.85b_wf_e} - \frac{h_f(b - b_w)}{b_w}$$

$$\text{Let } w = \frac{A_sf_s}{0.85}\ \ and\ l = \frac{h_f(b - b_w)}{b_w}; (as\ previeous)$$

$$\text{so that,} \qquad = \frac{w}{b_wf_e} - l \tag{4-91}$$

Inserting the expression of x and equations, 4-81, 4-82, and 4-83 into equation 4-86

$$f_e = \frac{f_a\left[z\big(2b - b_w - h_f\big) + 2z(\frac{w}{b_wf_e} - l)\right] + f_u\left[(\mathrm{b} - \mathrm{z})\big(h_f - 2\mathrm{z}\big) + \big(\mathrm{z} - h_f\big)(b_w - 2z) + (\frac{w}{b_wf_e} - l)(b_w - 2z)\right]}{h_f(b - b_w) + (\frac{w}{b_wf_e} - l)b_w}$$

$$\begin{aligned} f_e\left[h_f(b - b_w) + (\frac{w}{b_wf_e} - l)b_w\right] \\ = f_a\left[z\big(2b - b_w - h_f\big) + 2z(\frac{w}{b_wf_e} - l)\right] \\ + f_u\left[(\mathrm{b} - \mathrm{z})\big(h_f - 2\mathrm{z}\big) + \big(\mathrm{z} - h_f\big)(b_w - 2z) + (\frac{w}{b_wf_e} - l)(b_w - 2z)\right] \end{aligned}$$

$$f_e h_f(b-b_w) + f_e b_w\left(\frac{w}{b_w f_e}\right) - l b_w f_e - \frac{2zwf_a}{b_w f_e} - \frac{(b_w-2z)wf_u}{b_w f_e}$$

$$= f_a\big[z\big(2b-b_w-h_f\big) - 2zl)\big] + f_u\big[(\mathrm{b}-\mathrm{z})\big(h_f-2\mathrm{z}\big) + \big(\mathrm{z}-h_f\big)(b_w-2z) - l(b_w-2z)\big]$$

$$let \quad n = f_a\big[z\big(2b-b_w-h_f\big) - 2zl\big] + f_u\big[(\mathrm{b}-\mathrm{z})\big(h_f-2\mathrm{z}\big) + \big(\mathrm{z}-h_f\big)(b_w-2z) - l(b_w-2z)\big]$$

$$= \beta f_c\big[z\big(2b-b_w-h_f\big) - 2zl\big] + \delta f_c\big[(\mathrm{b}-\mathrm{z})\big(h_f-2\mathrm{z}\big) + \big(\mathrm{z}-h_f\big)(b_w-2z) - l(b_w-2z)\big]$$

$$f_e h_f(b-b_w) + f_e b_w\left(\frac{w}{b_w f_e}\right) - l b_w f_e - \frac{2zwf_a}{b_w f_e} - \frac{(b_w-2z)wf_u}{b_w f_e} = n$$

$$f_e h_f(b-b_w) + w - l b_w f_e - \frac{2zwf_a}{b_w f_e} - \frac{(b_w-2z)wf_u}{b_w f_e} = n$$

$$\frac{h_f(b-b_w){f_e}^2 + wf_e - lb_w{f_e}^2 - \frac{2zwf_a}{b_w} - \frac{(b_w-2z)wf_u}{b_w}}{f_e} = n$$

$$h_f(b-b_w){f_e}^2 + wf_e - lb_w{f_e}^2 - \frac{2zwf_a}{b_w} - \frac{(b_w-2z)wf_u}{b_w} = nf_e$$

$$h_f(b-b_w){f_e}^2 - lb_w{f_e}^2 + wf_e - nf_e - \frac{2zwf_a}{b_w} - \frac{(b_w-2z)wf_u}{b_w} = 0$$

$$(h_f(b-b_w) - lb_w){f_e}^2 + (w-n)f_e - \frac{wf_c}{b_w}(2\beta z + \delta(b_w-2z)) = 0$$

Substituting, $l = \frac{h_f(b-b_w)}{b_w}$ $(from\ the\ previous)$

$$(h_f(b-b_w) - \left(\frac{h_f(b-b_w)}{b_w}\right)b_w){f_e}^2 + (w-n)f_e - \frac{wf_c}{b_w}(2\beta z + \delta(b_w-2z)) = 0$$

$$(w-n)f_e = \frac{wf_c}{b_w}(2\beta z + \delta(b_w-2z))$$

$$f_e = \frac{\frac{wf_c}{b_w}(2\beta z + \delta(b_w-2z))}{w-n} \qquad 4\text{-}92$$

II. Singly reinforced L-Section beam when neutral axis lies within the flange

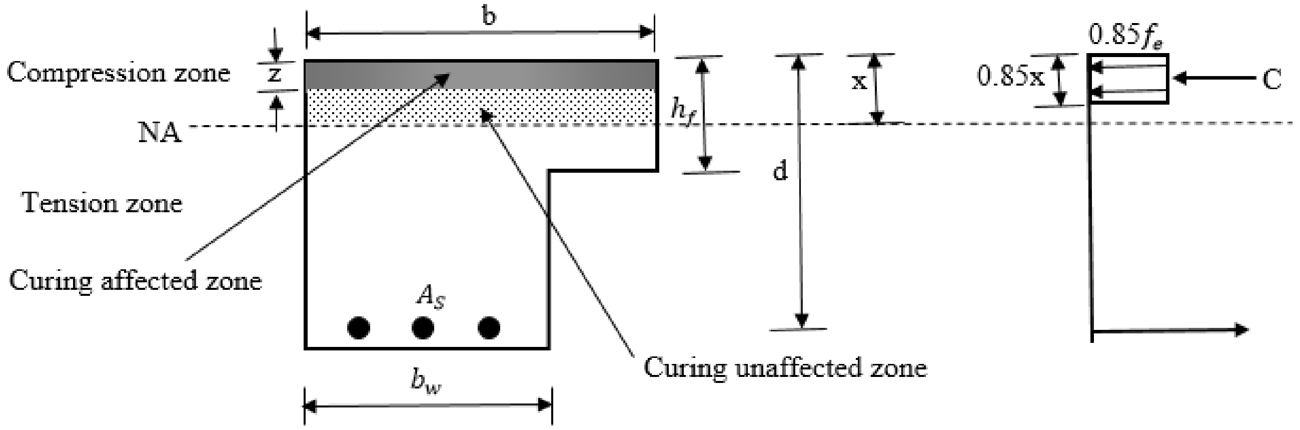

Figure 24 - Cross-sectional view of L-section singly reinforced beam (NA within flange)

Calculation of cross-sectional areas:

$$A = bx \tag{4-93}$$

$$\mathrm{u} = b(x - z) \tag{4-94}$$

$$\mathrm{a} = bz \tag{4-95}$$

Calculation for equivalent in-place compressive strength:

$$f_e = \frac{F}{A} \tag{4-96}$$

$$F = \mathrm{a}f_a + uf_u \tag{4-97}$$

Substituting equation 4-97 into equation 4-96

$$f_e = \frac{\mathrm{a}f_a + uf_u}{A} \tag{4-98}$$

Substituting equations 4-93, 4-94, 49-95 into equation 7-98

$$f_e = \frac{f_a(bz) + f_u(b(x - z))}{bx} \tag{4-99}$$

$$f_e = \frac{bzf_a + xbf_u - bzf_u)}{bx}$$

$$f_e = \frac{bz(f_a - f_u) + bzf_u)}{bx}$$

$$f_e = \frac{zf_c(\beta - \delta)}{x} + \delta f_c \tag{4-100}$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C = T \tag{4-101}$$

$$C = 0.85xbf_e \tag{4-102}$$

$$T = A_s f_s \tag{4-103}$$

Substituting equations 4-102 and 4-103 into 4-101:

$$0.85xbf_e = A_s f_s$$

$$x = \frac{A_s f_s}{0.85bf_e}$$

Let $w = \dfrac{A_s f_s}{0.85}$ so that the value of x will be:

$$x = \frac{w}{bf_e} \tag{4-104}$$

Now, substituting the expression of x into equation 4-100:

$$f_e = \frac{zf_c bf_e(\beta - \delta)}{w} + \delta f_c$$

$$f_e - \frac{zf_c bf_e(\beta - \delta)}{w} = \delta f_c$$

$$f_e\left[1 - \frac{zf_c b(\beta - \delta)}{w}\right] = \delta f_c$$

$$f_e = \left[\frac{\delta f_c}{1 - \frac{zf_c b(\beta - \delta)}{w}}\right]$$

$$f_e = \left[\frac{\delta w f_c}{w - zbf_c(\beta - \delta)}\right] \tag{4-105}$$

III. Double reinforced L-Section beam when neutral axis lies within the web

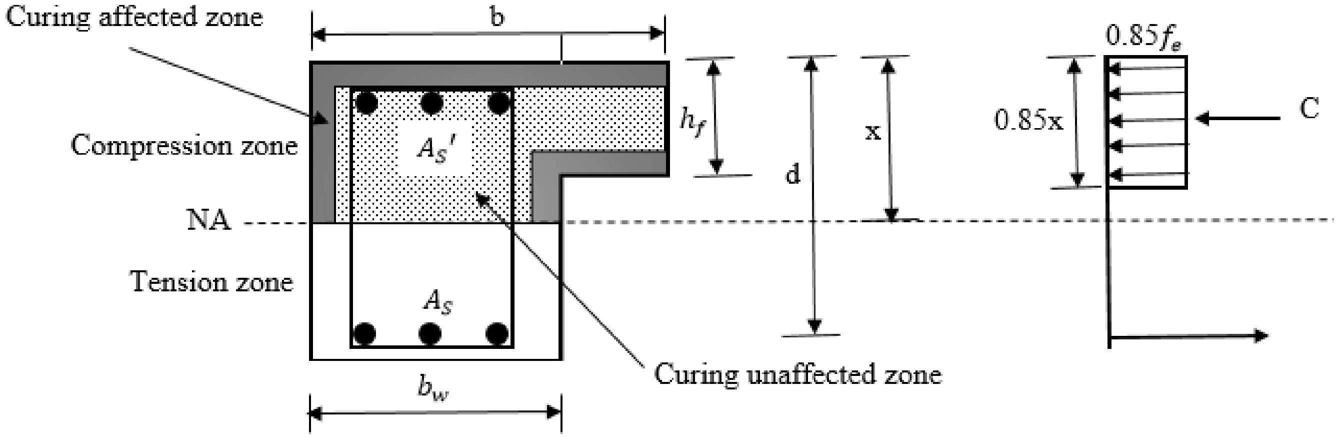

Figure 25 - Cross-sectional view of L-section doubly reinforced beam (NA within web)

In order to determine the value of x, the internal forces (i.e. C, T'and T) should be at equilibrium. Therefore:

$$C + T' = T \qquad 4\text{-}106$$

$$C = 0.85Af_e$$

$$C = 0.85\left(h_f(b - b_w) + xb_w\right)f_e$$

$$C = 0.85h_f(b - b_w)f_e + 0.85Xb_wf_e \qquad 4\text{-}107$$

$$T = A_sf_s \qquad 4\text{-}108$$

$$T' = A_{s'}f_s \qquad 4\text{-}109$$

Substituting equations 4-107, 4-108 and 4-109 into 4-106:

$$0.85h_f(b - b_w)f_e + 0.85Xb_wf_e + A_{s'}f_s = A_sf_s$$

$$x = \frac{A_sf_s - A_{s'}f_s - 0.85h_f(b - b_w)f_e}{0.85b_wf_e}$$

$$x = \frac{A_sf_s - A_{s'}f_s}{0.85b_wf_e} - \frac{h_f(b - b_w)}{b_w}$$

Let $v = \dfrac{A_sf_s - A_{s'}f_s}{0.85}$ $and\ l = \dfrac{h_f(b - b_w)}{b_w}$ (as in the previous)

$$so\ that, \qquad x = \frac{v}{b_w f_e} - l \tag{4-110}$$

Substituting the expression of x into equation 4-87:

$$f_e = \frac{f_a\left[z(2b - b_w - h_f) + 2z(\frac{v}{b_w f_e} - l)\right] + f_u\left[(\mathrm{b} - \mathrm{z})(h_f - 2\mathrm{z}) + (\mathrm{z} - h_f)(b_w - 2z) + (\frac{v}{b_w f_e} - l)(b_w - 2z)\right]}{h_f(b - b_w) + (\frac{v}{b_w f_e} - l)b_w}$$

$$\begin{aligned} f_e\left[h_f(b - b_w) + \left(\frac{v}{b_w f_e} - l\right)b_w\right] \\ &= f_a\left[z(2b - b_w - h_f) + 2z\left(\frac{v}{b_w f_e} - l\right)\right] \\ &+ f_u\left[(\mathrm{b} - \mathrm{z})(h_f - 2\mathrm{z}) + (\mathrm{z} - h_f)(b_w - 2z) + \left(\frac{v}{b_w f_e} - l\right)(b_w - 2z)\right] \end{aligned}$$

$$let \quad n = \ f_a[z(2b - b_w - h_f) - 2zl)] + f_u[(\mathrm{b} - \mathrm{z})(h_f - 2\mathrm{z}) + (\mathrm{z} - h_f)(b_w - 2z) - l(b_w - 2z)]$$

$$\begin{aligned} &= \ \beta f_c[z(2b - b_w - h_f) + -2zl] \\ &\qquad + \delta f_c[(\mathrm{b} - \mathrm{z})(h_f - 2\mathrm{z}) + (\mathrm{z} - h_f)(b_w - 2z) - l(b_w - 2z)] \end{aligned}$$

$$f_e h_f(b - b_w) + f_e b_w \frac{v}{b_w f_e} - 2z\beta f_c \frac{v}{b_w f_e} - (b_w - 2z)\delta f_c \frac{v}{b_w f_e} = n$$

$$(h_f(b - b_w) - lb_w){f_e}^2 + (w - n)f_e - \frac{vf_c}{b_w}(2\beta z + \delta(b_w - 2z)) = 0$$

$$substituting, l = \frac{h_f(b - b_w)}{b_w} \ (from\ the\ previous\ equation)$$

$$(h_f(b - b_w) - \left(\frac{h_f(b - b_w)}{b_w}\right)b_w){f_e}^2 + (v - n)f_e - \frac{vf_c}{b_w}(2\beta z + \delta(b_w - 2z)) = 0$$

$$(v - n)f_e = \frac{vf_c}{b_w}(2\beta z + \delta(b_w - 2z))$$

$$f_e = \frac{\frac{vf_c}{b_w}(2\beta z + \delta(b_w - 2z))}{v - n} \tag{4-111}$$

IV. Double reinforced L-Section beam when neutral axis lies within the flange

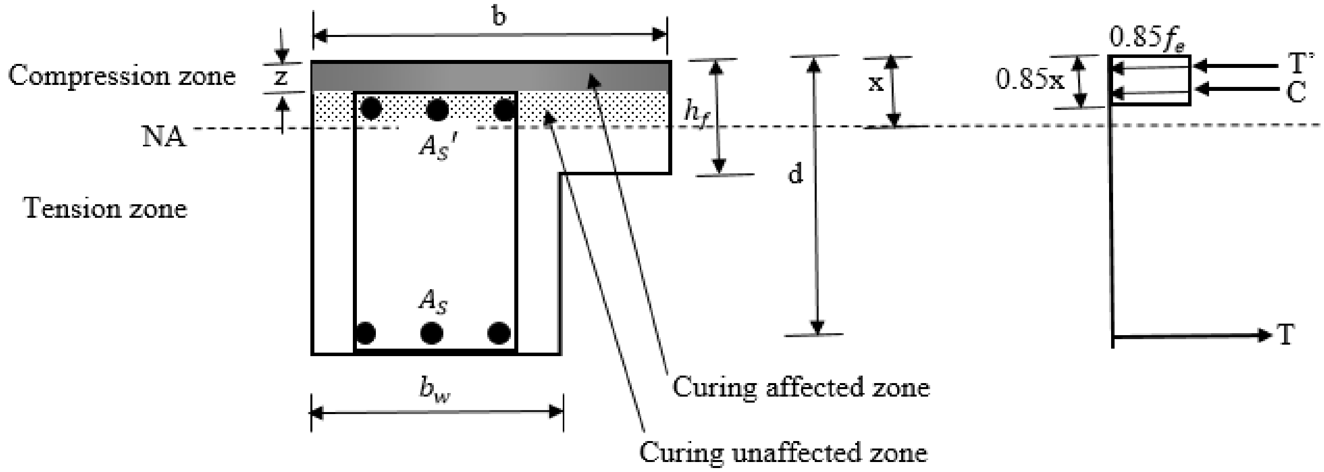

Figure 26 - Cross-sectional view of L-section doubly reinforced beam (NA within flange)

$$f_e = \frac{zf_c(\beta - \delta)}{x} + \delta f_c \qquad \text{4-112}$$

In order to determine the value of x, the internal forces (i.e. C and T) should be at equilibrium. Therefore:

$$C + T' = T \qquad \text{4-113}$$

$$0.85xbf_e + A_{s'}f_s = A_sf_s$$

$$x = \frac{A_sf_s - A_{s'}f_s}{0.85bf_e}$$

Let $v = \frac{A_sf_s - A_{s'}f_s}{0.85}$ so that the value of x will be:

$$x = \frac{v}{bf_e} \qquad \text{4-114}$$

Now, substituting equation 4-113 into equation 4-111:

$$f_e = \frac{zf_cbf_e(\beta - \delta)}{v} + \delta f_c \qquad \text{4-115}$$

$$f_e = \left[\frac{\delta v f_c}{v - zbf_c(\beta - \delta)}\right]$$

1.1.3. Slabs

A. Ribbed slab

All the steps of the ribbed slab are similar to singly reinforced T-Section beams.

I. When NA lies on the web

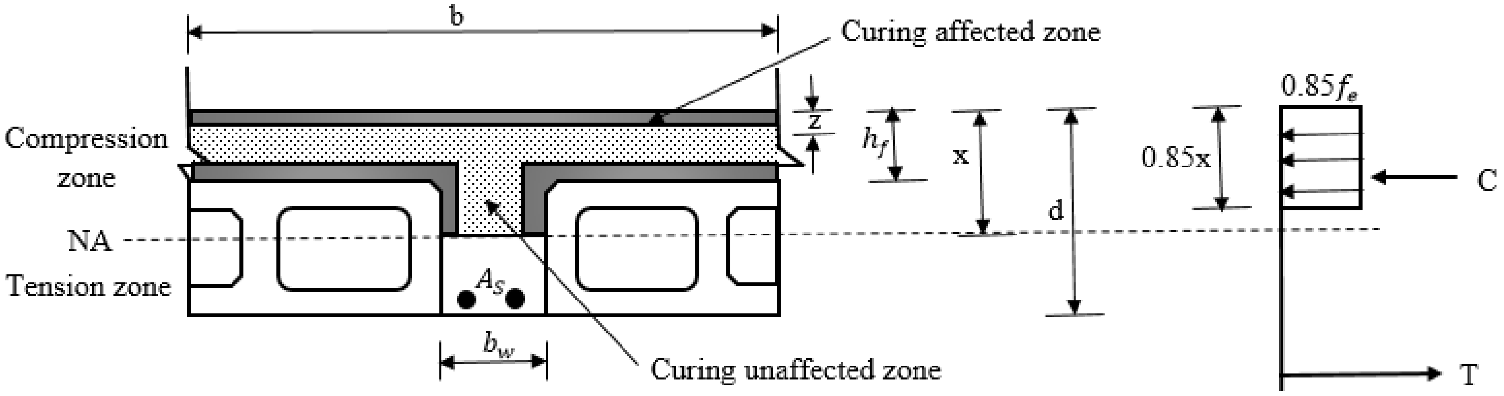

Figure 27 - Cross-sectional view of ribbed slab (NA on web)

$$f_e = \frac{f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{w - m} \qquad 4\text{-}116$$

II. When NA lies on the web

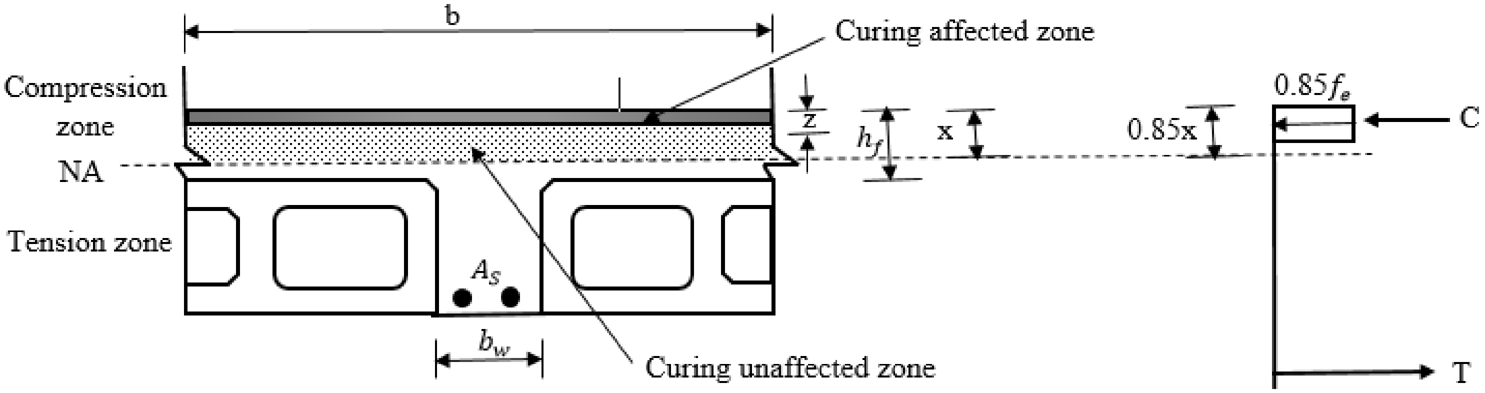

Figure 28 - Cross-sectional view of ribbed slab (NA on flange)

$$f_e = \left[\frac{\delta w f_c}{w - z b f_c(\beta - \delta)}\right] \qquad 4\text{-}117$$

B. Solid slab

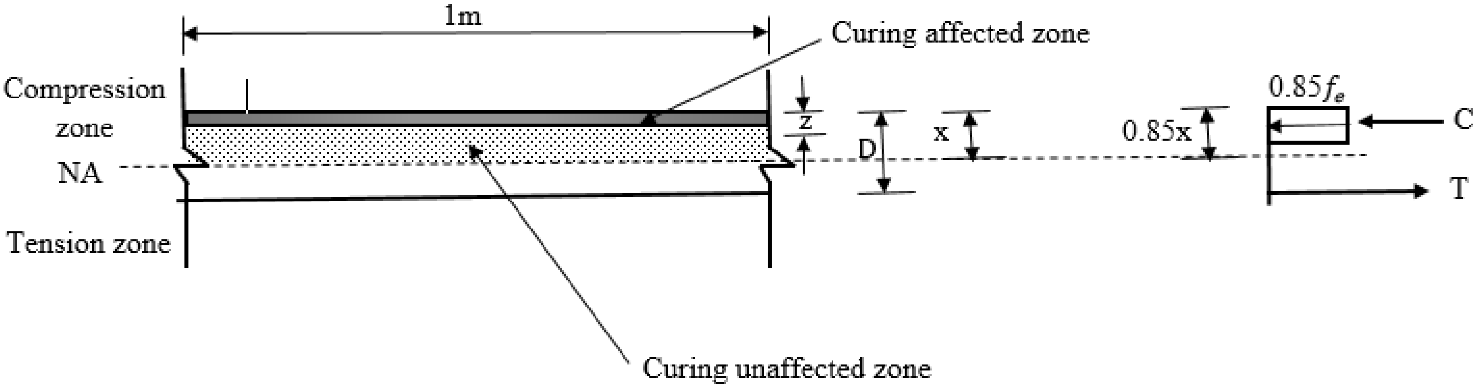

Figure 29 - Cross-sectional view of solid slab

$$A = 1 * x \tag{4-118}$$

$$\mathrm{u} = 1 * (x - z) \tag{4-119}$$

$$\mathrm{a} = 1 * z \tag{4-120}$$

Substituting equations 4-118, 4-119 and 4-120 into equation 4-86:

$$f_e = \frac{f_a(z) + f_u((x - z))}{x} \tag{4-121}$$

$$f_e = \frac{zf_a + xf_u - zf_u}{x}$$

$$f_e = \frac{zf_c(\beta - \delta)}{x} + \delta f_c \tag{4-122}$$

$$x = \frac{A_s f_s}{0.85 b f_e} \qquad (from\ equation\ 4 - 104)$$

Let, $w = \dfrac{A_s f_s}{0.85}$ so that the value of x will be:

$$x = \frac{w}{bf_e} \tag{4-123}$$

Substituting expression of x into equation 1-122 and simplifying:

$$f_e = \frac{zf_c b f_e(\beta - \delta)}{w} + \delta f_c$$

$$f_e = \left[\frac{\delta w f_c}{w - zbf_c(\beta - \delta)}\right] \tag{4-124}$$

4.2. Experimental Results and Discussions

In this section, the value of β (the compressive strength ratio of curing affected zone to standard cured specimen) and the value of δ (the compressive strength ratio of curing unaffected zone to standard cured specimen) are presented. Mathematically;

$$\beta = \frac{Compressive\ strength\ of\ curing\ affected\ zone\ (f_a)}{Compressive\ strength\ of\ standard\ cured\ specimen\ (f_c)}$$

$$\delta = \frac{Compressive\ strength\ of\ curing\ unaffected\ zone\ (f_u)}{Compressive\ strength\ of\ standard\ cured\ specimen\ (f_c)}$$

The compressive strength result of each concrete specimen for each curing duration and curing method are tabulated in Annex 5, Annex 6 and Annex 7.

4.2.1. Curing Affected Zone

4.2.1.1. For w/c = 0.4

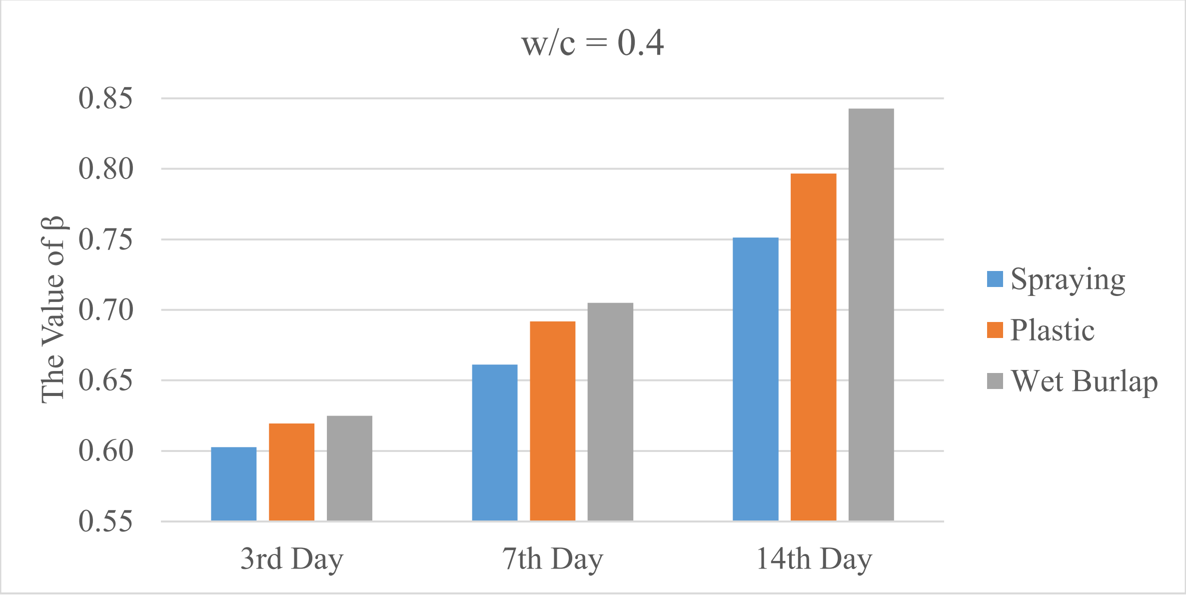

Figure 30 - Value of β for w/c = 0.4

Keeping the water to cement ratio and curing duration constant, the wet burlap curing method and plastic covering did not show a significant difference at 3rd day but at the 14th day the wet burlap curing shows a better strength than plastic. From all curing methods that are assessed, air curing method is the least effective method and water spraying is the second least effective method of curing for all curing durations.

4.2.1.2. For w/c = 0.5

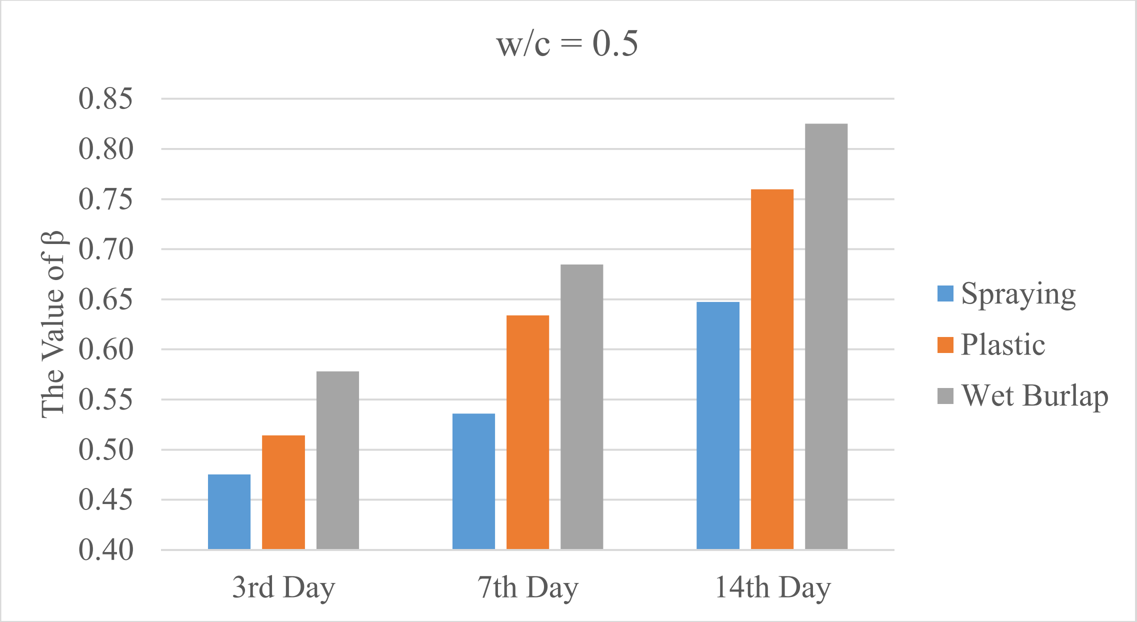

Figure 31 - Value of β for w/c = 0.5

4.2.1.3. For w/c = 0.6

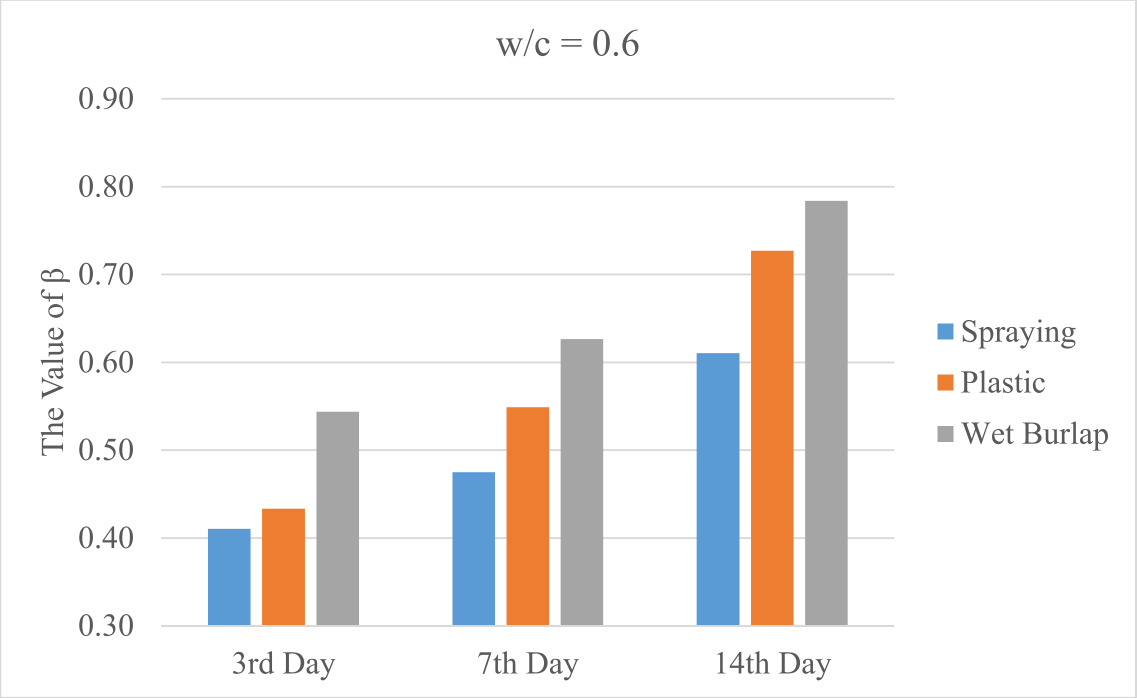

Figure 32 - Value of β for w/c = 0.6

4.3. Comparison within Different Water to Cement Ratio

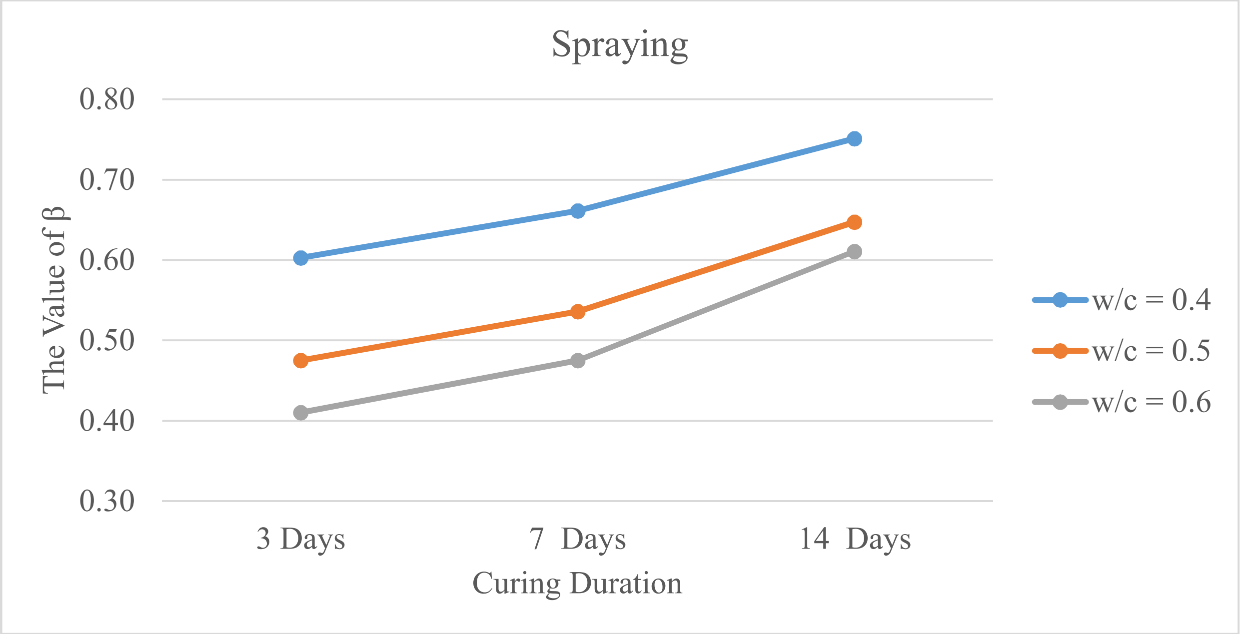

Figure 33 - Value of β for different water to cement ratio and water spraying

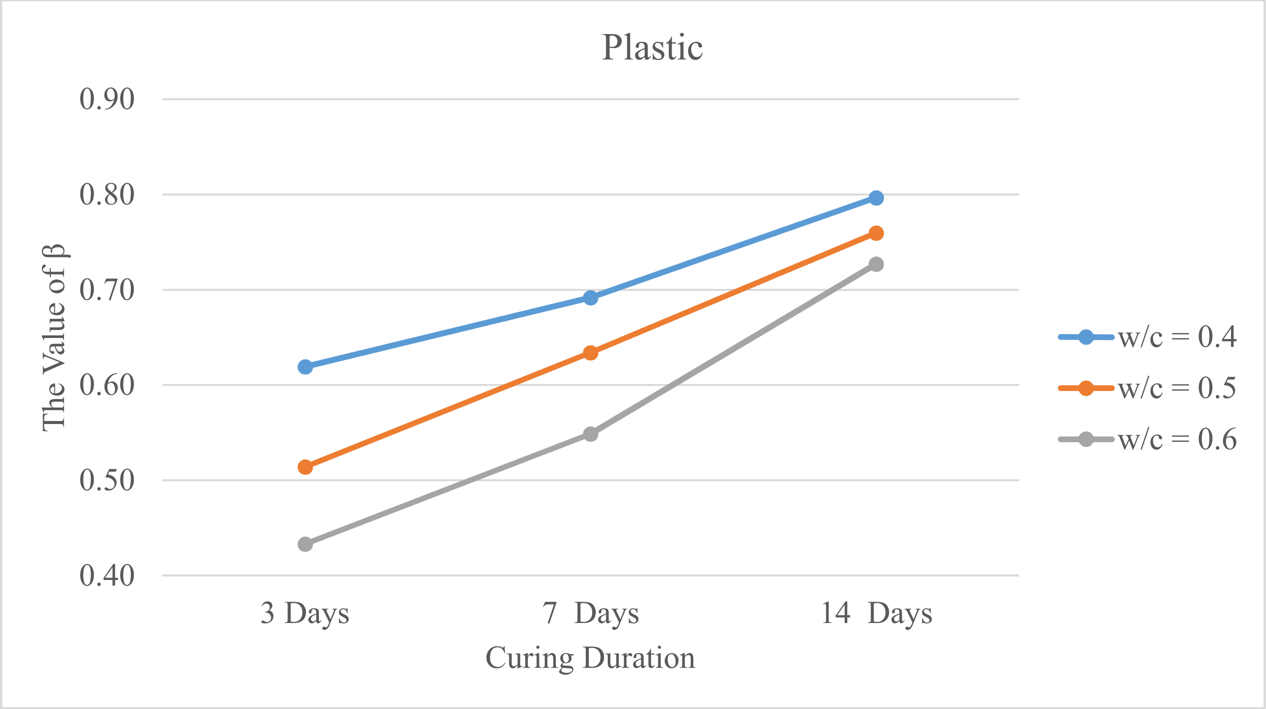

Figure 34 - Value of β for different water to cement ratio and plastic covering

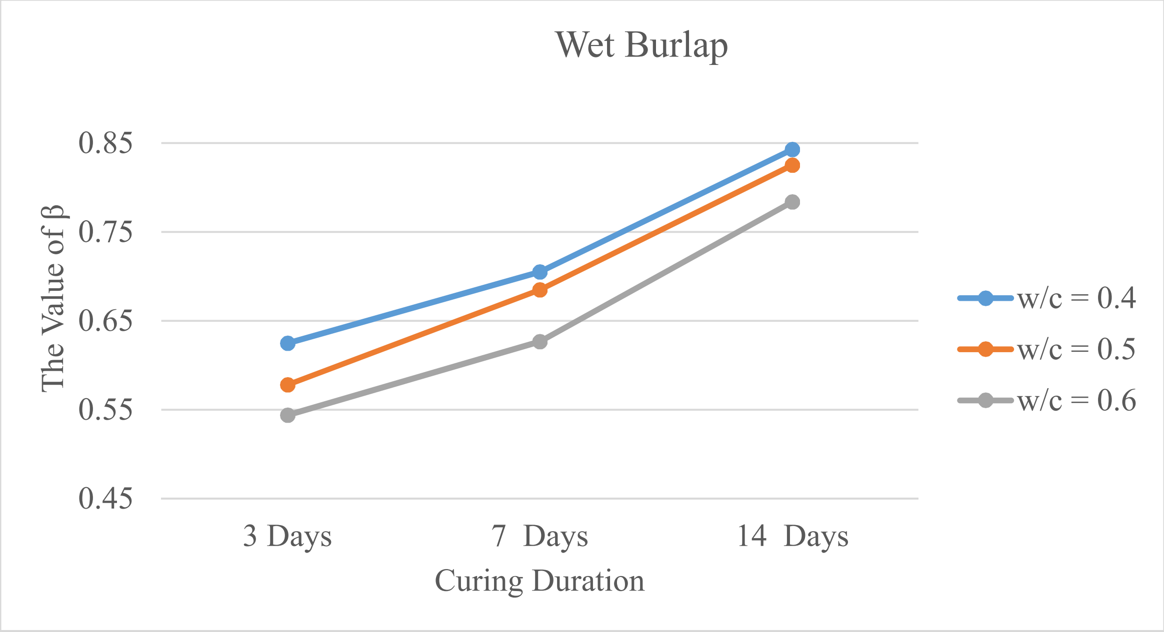

Figure 35 - Value of β for different water to cement ratio and wet burlap covering

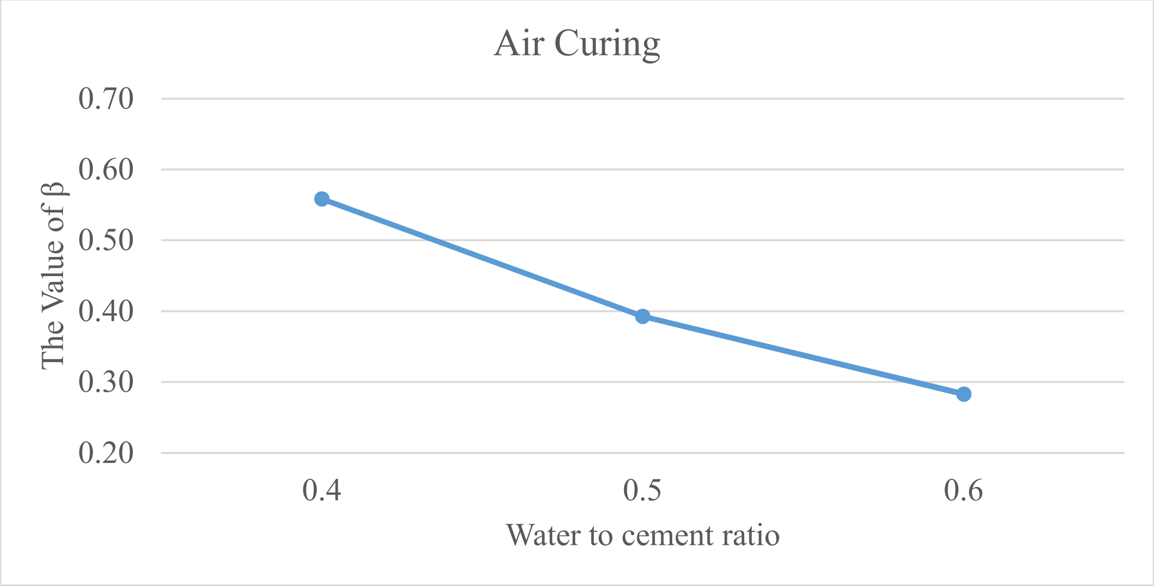

Figure 36 - Value of β for different water to cement ratio for air curing

In all the curing method and curing durations, concrete mix with lower water to cement ratio shows the greater value of strength ratio because of the fact that as water to cement ratio decreases moisture cannot escape easily due to its small pore structure. But the ratio values are relatively small in the case of concrete mix with high water to cement ratio because moisture can escape easily due to its large pore structure.

4.2.2. Curing Unaffected Zone

As discussed in section 2.7.1 in detail, concrete with lean mix especially for water-cement ratio greater than 0.4, the inside section of concrete is self-curing because the mix contains more water than required for hydration reaction. As a result, the compressive strength of the interior part of concrete (curing unaffected zone) is approximately similar to the standard cured specimen as expected. As shown in Figure 37, the compressive strength ratio of curing unaffected zone to standard cured specimen concrete mix with 0.6 and 0.5 water to cement ratio is almost close to 1.0 but as the water to cement ratio decreases to 0.4 the ratio also decreases substantially to 0.78 due to the fact that self-desiccation cause lack of water for hydration reaction at the interior part of concrete.

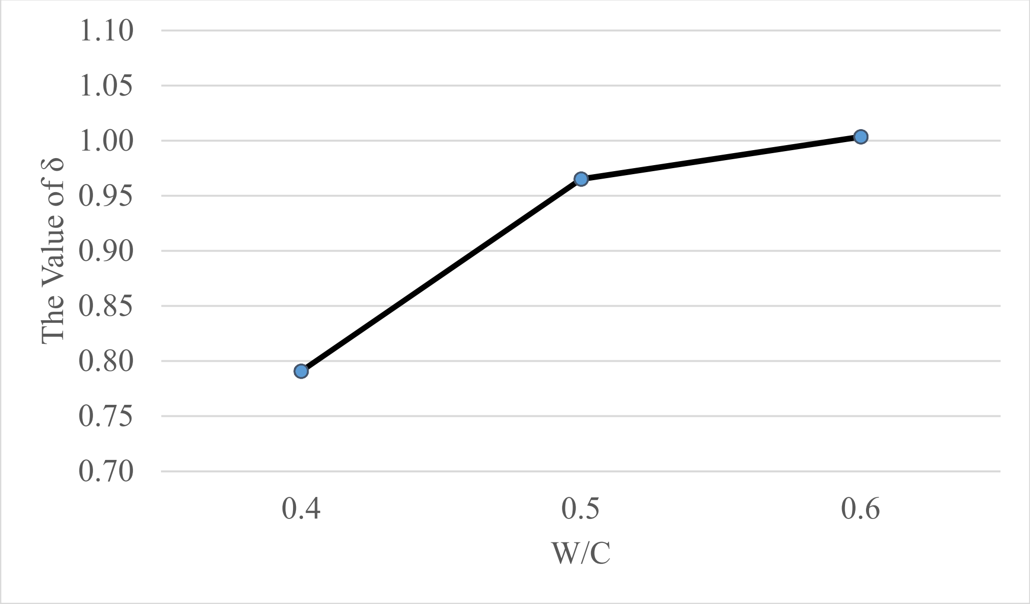

Figure 37 - The Value of δ for different water to cement ratio.

4.2.3. Numerical Examples

In this sub-section, two numerical examples are presented by combining the general theoretical equations developed earlier and values of coefficients which are determined experimentally for selected water to cement ratio, curing methods and curing durations. Numerical examples for all the formulas are computed and presented in Annex 9.

4.2.3.1.Rectangular Plane Column

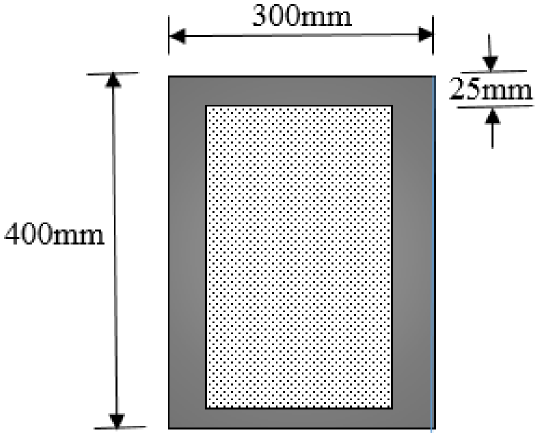

Figure 38 – Example of a rectangular plane column

- Type of structure - Rectangular Plane Column
- Assume:
    - Curing Method – Wet Burlap
    - Curing Duration – 7 days
    - Compressive Strength – 30MPa
    - Water to Cement Ratio – 0.45

Having w/c, curing method, and duration, the value of β is 0.7 (from Fig-35) and δ is 0.875 (from Fig-37).

Therefore;

$$f_e \ = 2zf_c(\beta - \delta)\left(\frac{D + B - 2Z}{BD}\right) + \delta f_c$$

$$f_e \ = 2 * 25 * 30 * (0.7 - 0.875)\left(\frac{400 + 300 - 2 * 25}{300 * 400}\right) + 0.875 * 30$$

$$f_e \ = 24.61MPa$$

4.2.3.2.Singly Reinforced Rectangular Beam

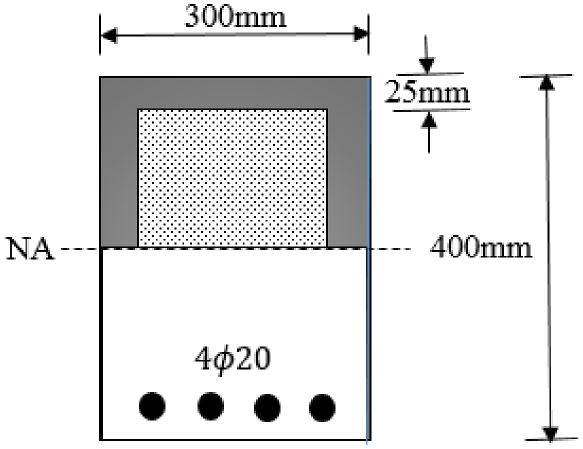

Figure 39 – Example of singly reinforced rectangular beam

- Type of structure – Singly Reinforced Rectangular Beam
- Assume:
  - Curing Method – Wet Burlap
  - Curing Duration – 14 days
  - Compressive Strength of concrete – 30MPa
  - Compressive Strength of Steel – 400MPa
  - Water to Cement Ratio – 0.45

Having w/c, curing method, and duration, the value of β is 0.82 (from Fig-35) and δ is 0.875 (from Fig-37).

Therefore;

$$w = \frac{A_s f_s}{0.85} = \frac{\pi * 20^2 * 400}{4 * 0.85} = 594{,}058.823N$$

$$f_e = \frac{\mathrm{w} f_c(2\beta z + \delta b - 2\delta \mathrm{z})}{wb - f_c \mathrm{zb}((\beta - \delta)(\mathrm{b} + 2z)}$$

$$f_e = \frac{594{,}058.823 * 30(2 * 0.82 * 25 + 0.875 * 25 - 2 * 0.875 * 25)}{594{,}058.823 * 300 - 30 * 25 * 300(0.82 - 0.875)(300 + 2 * 25)}$$

$$f_e = 25.356MPa$$

5. Conclusions and Recommendations

5.2. Conclusions

The influence of curing practice on compressive strength of in-place concrete was studied. Mathematical formulae are developed and supported by experimental findings. Based on the findings of the study the following conclusions are drawn:

- Most of the existing in-situ concrete compressive strength determination methods have limitations in taking into account factors that significantly affect test results. Maturity method considers only temperature history of in-place concrete, core testing damage the specimen while drilling, curing test specimen on the field underestimates the strength because a significant volume of specimen is affected by curing compared with in-place concrete structures, and in the case of cast in place cylinder, the actual loading and the testing direction is different.
- Equations are developed based on the assumption that compressive strength of curing affected zone and curing unaffected zone is different and both zones respond jointly for the coming stress as reinforcement bar and concrete do in reinforced concrete. The values of coefficients from the expressions are determined experimentally by representing different curing conditions and curing duration with 0.4, 0.5, and 0.6 water to cement ratio.
- For all water to cement ratios assessed in this research, wet burlap covering curing method shows a better β value than all curing methods followed by the plastic covering curing method because burlap can soak up and hold the applied curing water for long period of time and supply the water to specimen continuously. Water spraying curing method shows the second last β value next to air curing because the sprayed water over the surface of concrete may be lost soon due to wind, evaporation and so on.
- Mix with high water to cement ratio shows the higher δ value because it consists of more water than required to finish its hydration reaction under a sealed system. The mix with low water to cement ratio shows lower δ value due to the fact that its hydration reaction runs out of water in the sealed system.

5.3. Recommendations

Based on the finding of this study, the following recommendations are forwarded:

- In order to compensate the strength lost in in-place concrete due to curing, type and size of structure, organizations or persons who involve in mix design shall predict using the developed formulas so that they can include it in their mix design calculation.

- Relying on compressive strength test alone does not ensure the strength of in-place concrete. The researcher recommends that the formulas developed in this research shall be used so that it is possible to take strength affecting factors into account. As a result, both contractor and engineers fell confidence in their concrete work.

- Bulk data are required to make the coefficients more accurate so that researchers shall work more to develop the values of coefficients for different locations, water to cement ratio, and cement type. The researcher recommends that the formals shall be used as compliance criteria of concrete work.

- Contractors may want to remove formworks or terminate curing earlier like at the age of 3, 7 or other days to proceed to next work. In this situation, the in-place strength shall be estimated. However, the formulas developed in this research work only at the age of 28th day. Therefore, this research must be continued to be developed for different ages of concrete so that one can be able to estimate the strength of concrete at the age of any day before arriving at 28th day.

- Wet burlap curing method is the most effective method for all water to cement ratio and for all curing duration. Therefore, the researcher suggests that wet burlap curing method shall be used taking cost and other factors into consideration.
- For a lean mix, the wet burlap and plastic curing method did not show a significant difference at 3rd day and the cost of plastic covering is around three times cheaper than burlap (Molla, 2017). Therefore, the researcher recommends that plastic covering curing method is more suitable for lean mix with shorter curing duration.

5.4. Recommendation for Future Study

In order to make complete the proposed model, the following recommendations are forwarded for future studies:

- This research was done by assuming the thickness of curing affected zone to be 25mm for all water to cement ration, curing method and curing duration. But the thickness of curing affected zone vary depending on many factors. As a result, some models shall be developed to estimate the thickness of curing affected zone taking many determining factors into account.
- One model cannot be completed within a go of one research. As a result, other researchers shall compare and contrast the output of the formulas with other methods like rebound hammer tests output to make sure that the formulae are reliable and compatible.
- For vertically erected concrete structures like column and shear wall, the pressure of fresh concrete at the bottom cause better compaction. As a result, the bottom section is stronger than the top. In the future, it is better to study the strength variation throughout the height of the structure and include it in the formulae developed in this research.
- The formulae for both circular and rectangular columns are developed for only short column with zero eccentricity (concentrically loaded short column) but column are not always loaded with zero eccentricity. Therefore, other formulae shall be developed for eccentrically loaded short and slender columns.
- Since some formulae are very long and complex, it will be difficult to remember and work on it. Therefore, computer software or smartphone application shall be developed so that anybody can use it easily.
- The research was carried out for only a specified duration of the year (from May to June); as a result, next researchers are encouraged to assess the effect of other or all seasons of the year based on compressive strength of concrete.

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Annex

Annex 1. Testes and results for fine aggregate

The moisture content of FA		
Wt. of sample	1000	g
Wt. of OD sample	970	g
Moisture content	3.09%	

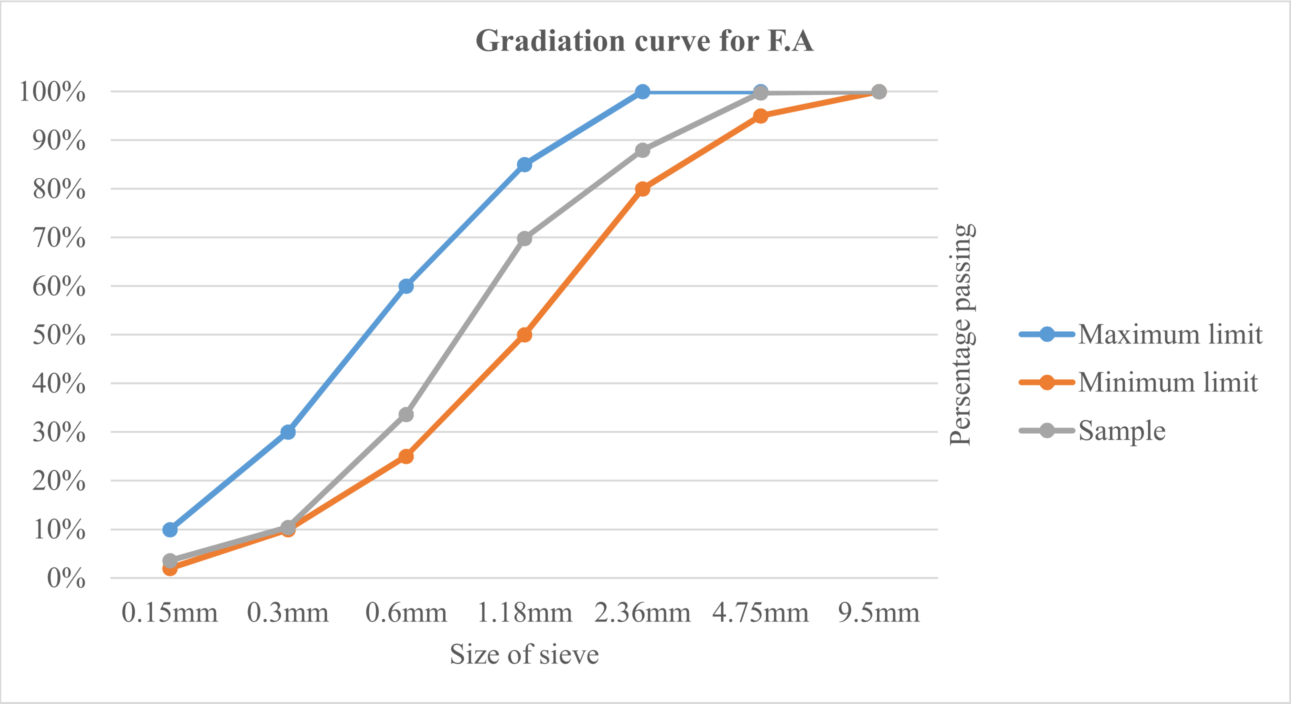
the specific gravity of FA		
Wt. of pycnometer + water + sample (C)	2090.5	g
Wt. of pycnometer + water (B)	1772	g
Wt. of OD sample(A)	458	g
Specific gravity	2.52342	
Specific gravity at SSD	2.75482	
Apparent Specific gravity	3.28315	
Absorption	9.17%	
Effective Absorption	6.05%	

Bulk unit weight of FA		
Wt. of mold	2972	g
Wt. of mold + sample	7672.5	g
Volume of mold	0.003	m^3
Bulk unit weight	1566.83	Kg/m^3

Silt content		
Before washing		
Height of silt	8	mm
Height of sand	82	mm
Silt content	9.76%	
After washing		
Height of silt	3	mm
Height of sand	76	mm
Silt content	3.95%	

Sieve Analysis for Fine Aggregate

Sieve Size	Wt. of Sieve(g)	Wt. of Sieve + Sample(g)	Wt. of Sample(g)	Retained (%)	Cumulative (%)	Passing (%)	Specification (%)
9.5mm	695	695	0	0.00%	0.00%	100.00%	100
4.75mm	724.5	737	12.5	0.29%	0.29%	99.71%	95-100
2.36mm	732	1242	510	11.77%	12.06%	87.94%	80-100
1.18mm	612	1396.5	784.5	18.10%	30.16%	69.84%	50-85
0.6mm	611	2179	1568	36.18%	66.34%	33.66%	25-60
0.3mm	582.5	1590	1007.5	23.25%	89.59%	10.41%	10.- 30
0.15mm	574.5	870	295.5	6.82%	96.41%	3.59%	2 - 10
pan	556.5	712	155.5	3.59%	100.00%	0.00%	
			4333.5	100%	FM = 2.95		

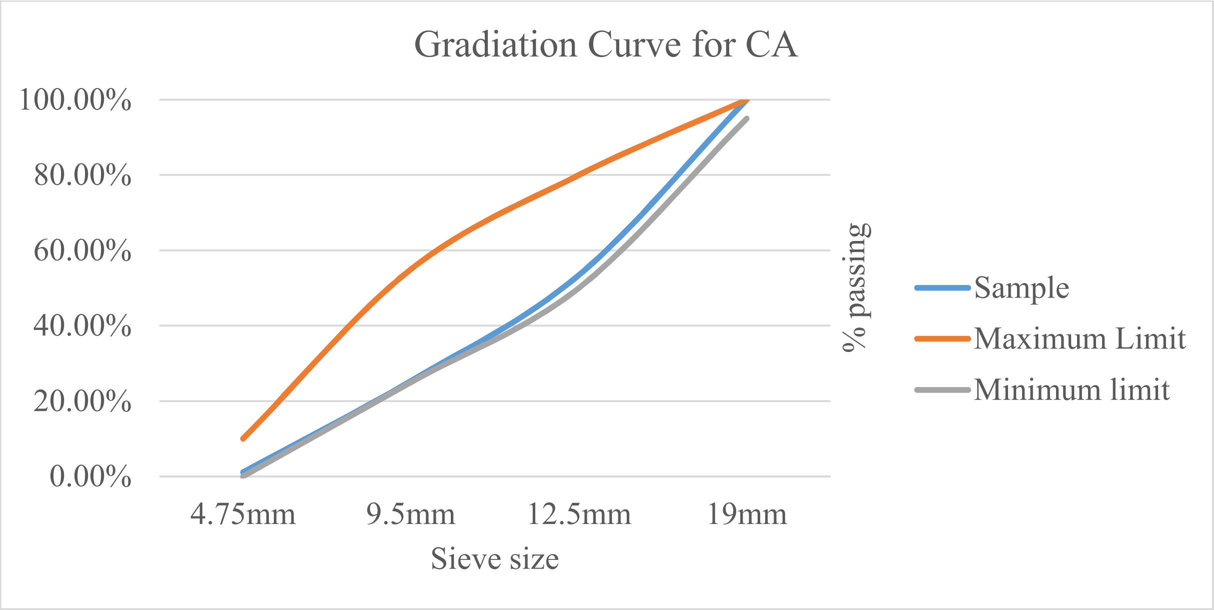


Annex 2. Testes and results for coarse aggregate

The moisture content of CA		
Wt. of sample	2000	g
Wt. of OD sample	1978	g
Moisture content	1.11%	
the specific gravity of CA		
Wt. at SSD in the air (B)	2860	g
Wt. at SSD in water (C)	1869.5	g
Wt. at OD in air (A)	2799	g
Specific gravity	2.8258	
Specific gravity at SSD	2.8874	
Apparent Specific gravity	3.0113	
Absorption	2.18%	
Effective Absorption	1.07%	
Bulk unit weight of CA		
Wt. of mold	5011.5	g
Wt. of mold + sample	16492	g
Volume of mold	0.007	m^3
Bulk unit weight	1640	Kg/m^3

Sieve analysis for coarse aggregate

Sieve Size	Wt. of Sieve (g)	Wt. of Sieve + Sample (g)	Wt. of Sample (g)	Retained (%)	Cumulative (%)	Passing (%)	Specification (%)
19mm	718.5	718.5	0	0.00%	0.00%	100.00%	95-100
12.5mm	674	3030	2356	46.58%	46.58%	53.42%	
9.5mm	695	2114	1419	28.05%	74.63%	25.37%	25-55
4.75mm	723.5	1951.5	1228	24.28%	98.90%	1.10%	0-10
pan	556.5	612	55.5	1.10%	100.00%	0.00%	



Annex 3. Mix design calculation as per ACI 211.1-91 (Reapproved 2002)

Annex 3.1. For w/c = 0.4

step 1: Slump	75 – 100	Mm
step 2: Nominal maximum aggregate size	19	Mm
step 3: Estimated water content	205	Kg/m^3
Air content	2	%
Step 4: Target Mean Strength	-	MPa
Step 5: Water/cement ratio	0.4	
Step 6: Cement content	512.5	Kg/m^3
Step 7: Estimation of coarse aggregate		
Fineness modulus of sand	2.95	
Maximum aggregate size	19	Mm

The density of dry rodded CA	1640	Kg/m^3
Coarse aggregate	992.2	Kg/m^3

Step 8: Estimation of fine aggregate

Volume of water	0.205	m^3
Volume of cement	0.162698	m^3
Volume of CA	0.351116	m^3
Volume of Air	0.02	m^3
Volume of FA	0.261185	m^3
Mass of FA	659.08	Kg/m^3

Step 9: Moisture adjustment

Effective absorption of FA	6.08%	
Effective absorption of CA	1.07%	
Amount of water required for mix	255.6436	Kg/m^3
Amount of CA required for mix	981.6121	Kg/m^3
Amount of FA required for mix	619.0238	Kg/m^3

The estimated batch for trial	0.04	m^3
Water	10.0	Kg
Cement	20.5	Kg
FA	24.76095	Kg
CA	39.26448	Kg

Annex 3.2. For w/c = 0.5

step 1: Slump	75 – 100	mm
step 2: Nominal maximum aggregate size	19	mm
step 3: Estimated water content	205	Kg/m^3
Air content	2	%
Step 4: Target Mean Strength		MPa
Step 5: Water/cement ratio	0.5	
Step 6: Cement content	410	Kg/m^3
Step 7: Estimation of coarse aggregate		
Fineness modules of sand	2.95	
Maximum aggregate size	19	mm
Density of dry rodded CA	1640	Kg/m^3
Coarse aggregate	992.2	Kg/m^3
Step 8: Estimation of fine aggregate		
Volume of water	0.205	m^3
Volume of cement	0.130159	m^3
Volume of CA	0.351116	m^3
Volume of Air	0.02	m^3
Volume of FA	0.293725	m^3
Mass of FA	741.191	Kg/m^3
Step 9: Moisture adjustment		
Effective absorption of FA	6.08%	

Effective absorption of CA	1.07%	
Amount of water required for mix	260.6339	Kg/m^3
Amount of CA required for mix	981.6121	Kg/m^3
Amount of FA required for mix	696.1446	Kg/m^3

The estimated batch for trial	0.04	m^3
Water	10.5	Kg
Cement	16.5	Kg
F.A	28.0	Kg
C.A	39.0	Kg

Annex 3.3. For w/c = 0.6

step 1: Slump	75 – 100	mm
step 2: Nominal maximum aggregate size	19	mm
step 3: Estimated water content	205	Kg/m^3
Air content	2	%
Step 4: Target Mean Strength		MPa
Step 5: Water/cement ratio	0.6	
Step 6: Cement content	341.6667	Kg/m^3
Step 7: Estimation of coarse aggregate		
Fineness modulus of sand	2.95	
Maximum aggregate size	19	mm
The density of dry rodded CA	1640	Kg/m^3

Coarse aggregate	992.2	Kg/m^3

Step 8: Estimation of fine aggregate

Volume of water	0.205	m^3
Volume of cement	0.108466	m^3
Volume of CA	0.351116	m^3
Volume of Air	0.002	m^3
Volume of FA	0.333418	m^3
Mass of FA	841.353	Kg/m^3

Step 9: Moisture adjustment

Effective absorption of FA	6.08%	
Effective absorption of CA	1.07%	
Amount of water required for mix	266.7213	Kg/m^3
Amount of CA required for mix	981.6121	Kg/m^3
Amount of FA required for mix	790.2195	Kg/m^3

The estimated batch for trial	0.04	m^3
Water	10.5	Kg
Cement	13.5	Kg
F.A	31.5	Kg
C.A	39.0	Kg

Annex 5. Compressive Strength Result for w/c = 0.4

A) Curing Affected Zone

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m³)
Water Spraying	3rd Day	1	330.0	43.6	-1	42.6	17.0	26.40
		2	315.5	48.5	-1	47.5	19.0	25.24
		3	331.0	43.9	-1	42.9	17.2	26.48
						Average	17.73	26.04
	7th Day	1	340.5	51.6	-1	50.6	20.2	27.24
		2	345.5	45.2	-1	44.2	17.7	27.64
		3	313.0	52.1	-1	51.1	20.4	25.04
						Average	19.45	26.64
	14th Day	1	343.5	58.8	-1	57.8	23.1	27.48
		2	327.5	56.5	-1	55.5	22.2	26.20
		3	358.0	53.5	-1	52.5	21.0	28.64
						Average	22.11	27.44

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m³)
Plastic Covering	3rd Day	1	337.0	41.1	-1	40.1	16.0	26.96
		2	348.5	50.7	-1	49.7	19.9	27.88
		3	339.5	47.9	-1	46.9	18.8	27.16
						Average	18.23	27.33
	7th Day	1	355.0	51.9	-1	50.9	20.4	28.40
		2	331.5	56.8	-1	55.8	22.3	26.52
		3	350.0	47	-1	46	18.4	28.00
						Average	20.36	27.64
	14th Day	1	349.0	63.1	-1	62.1	24.8	27.92
		2	356.5	54.5	-1	53.5	21.4	28.52
		3	320.0	61.2	-1	60.2	24.1	25.60
						Average	23.44	27.35

	Curing Duration	**Specimen No.**	**Weight (g)**	**Load Reading (KN)**	**Calibration Factor (KN)**	**Corrected Load (KN)**	**Compressive Strength (MPa)**	**Density (KN/m^3)**
Wet Burlap Covering	3rd Day	1	332.0	41.7	-1	40.7	16.3	26.56
		2	320.0	42.6	-1	41.6	16.6	25.60
		3	342.0	56.6	-1	55.6	22.2	27.36
						Average	18.39	26.51
	7th Day	1	322.5	50	-1	49	19.6	25.80
		2	344.7	51.7	-1	50.7	20.3	27.58
		3	325.0	56.9	-1	55.9	22.4	26.00
						Average	20.75	26.46
	14th Day	1	331.5	59.6	-1	58.6	23.4	26.52
		2	352.0	67.4	-1	66.4	26.6	28.16
		3	346.5	62	-1	61	24.4	27.72
						Average	24.80	27.47

	Curing Duration	**Specimen No.**	**Weight (g)**	**Load Reading (KN)**	**Calibration Factor (KN)**	**Corrected Load (KN)**	**Compressive Strength (MPa)**	**Density (KN/m^3)**
Standard Curing	28th Day	1	332.5	73.1	-1	72.1	28.8	26.60
		2	320.0	73.5	-1	72.5	29.0	25.60
		3	337.0	77.1	-1	76.1	30.4	26.96
						Average	29.43	26.39

	Curing Duration	**Specimen No.**	**Weight (g)**	**Load Reading (KN)**	**Calibration Factor (KN)**	**Corrected Load (KN)**	**Compressive Strength (MPa)**	**Density (KN/m^3)**
Air Curing	28th Day	1	346.5	34.5	-1	33.5	13.4	27.72
		2	343.0	47.5	-1	46.5	18.6	27.44
		3	340.5	44.3	-1	43.3	17.3	27.24
						Average	16.44	27.47

B) Curing Unaffected Zone

Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
28th Day	1	8223.0	642.6	-25	617.6	27.45	24.36
	2	8297.5	620.0	-25	595.0	26.44	24.59
	3	8366.0	594.4	-25	569.4	25.31	24.79
					Average	26.40	24.58

C) Control Specimen Result

Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
28th Day	1	8294.0	806.0	-25	781.0	34.71	24.57
	2	8151.5	777.8	-25	752.77	33.46	24.15
	3	8305.0	745.0	-25	720	32.00	24.61
					Average	33.39	24.44

Annex 6. Compressive Strength Result for w/c = 0.5

A) Curing Affected Zone

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Water Spraying	3rd Day	1	342.5	19.1	-1	18.1	7.2	27.40
		2	336.5	26.9	-1	25.9	10.4	26.92
		3	325.0	29.1	-1	28.1	11.2	26.00
						Average	9.6	26.77
	7th Day	1	351.1	26.7	-1	25.7	10.3	28.09
		2	338.5	27.6	-1	26.6	10.6	27.08
		3	332.5	30.0	-1	29.0	11.6	26.60
						Average	10.8	27.26
	14th Day	1	348.5	33.1	-1	32.1	12.8	27.88
		2	338.5	34.5	-1	33.5	13.4	27.08
		3	334.5	33.6	-1	32.6	13.0	26.76
						Average	13.1	27.24

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Plastic Covering	3rd Day	1	332.0	18.5	-1	17.5	7.0	26.56
		2	345.0	31.1	-1	30.1	12.0	27.60
		3	337.5	31.4	-1	30.4	12.2	27.00
						Average	10.4	27.05
	7th Day	1	333.5	34.5	-1	33.5	13.4	26.68
		2	331.0	31.4	-1	30.4	12.2	26.48
		3	349.0	33.3	-1	32.3	12.9	27.92
						Average	12.8	27.03
	14th Day	1	337.0	42.0	-1	41.0	16.4	26.96
		2	351.5	40.7	-1	39.7	15.9	28.12
		3	320.5	35.6	-1	34.6	13.8	25.64
						Average	15.4	26.91

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Wet Burlap Covering	3rd Day	1	354.0	30.0	-1	29.0	11.6	28.32
		2	325.0	27.5	-1	26.5	10.6	26.00
		3	322.5	33.2	-1	32.2	12.9	25.80
						Average	11.7	26.71
	7th Day	1	334.5	38.0	-1	37.0	14.8	26.76
		2	313.0	29.4	-1	28.4	11.4	25.04
		3	337.5	39.5	-1	38.5	15.4	27.00
						Average	13.9	26.27
	14th Day	1	343.0	45.6	-1	44.6	17.8	27.44
		2	330.0	40.6	-1	39.6	15.8	26.40
		3	350.0	42.0	-1	41.0	16.4	28.00
						Average	16.7	27.28

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Standard Curing	28th Day	1	320.0	54.8	-1	53.8	21.5	25.60
		2	342.5	50.7	-1	49.7	19.9	27.40
		3	334.5	49.2	-1	48.2	19.3	26.76
						Average	20.2	26.59

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Air Curing	28th Day	1	347.5	21.3	-1	20.3	8.1	27.80
		2	336.5	24.0	-1	23.0	9.2	26.92
		3	349.5	17.3	-1	16.3	6.5	27.96
						Average	7.9	27.56

B) Curing Unaffected Zone

Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
28th Day	1	8223.0	428.4	-13	415.4	18.5	24.36
	2	8297.5	451.4	-13	438.4	19.5	24.59
	3	8366.0	471.0	-13	458.0	20.4	24.79
					Average	19.4	24.58

C) Control Specimen

Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
28th Day	1	8294.0	476.2	-13	463.2	20.6	24.57
	2	8151.5	456.6	-13	443.6	19.7	24.15
	3	8305.0	465.6	-13	452.6	20.1	24.61
					Average	20.1	24.44

Annex 7. Compressive Strength Result for w/c = 0.6

A) Curing Affected Zone

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Water Spraying	3rd Day	1	293.5	17.4	-1	16.4	6.6	23.48
		2	326.5	13.1	-1	12.1	4.8	26.12
		3	328.5	13.7	-1	12.7	5.1	26.28
						Average	5.5	25.29
	7th Day	1	313.5	15.5	-1	14.5	5.8	25.08
		2	305.5	17.7	-1	16.7	6.7	24.44
		3	307.0	17.5	-1	16.5	6.6	24.56
						Average	6.4	24.69
	14th Day	1	307.5	24.4	-1	23.4	9.4	24.60
		2	303.0	20.6	-1	19.6	7.8	24.24
		3	330.0	19.3	-1	18.3	7.3	26.40
						Average	8.2	25.08

	Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
Plastic Covering	3rd Day	1	312.5	16.2	-1	15.2	6.1	25.00
		2	320.0	14.8	-1	13.8	5.5	25.60
		3	320.5	15.5	-1	14.5	5.8	25.64
						Average	5.8	25.41
	7th Day	1	346.0	17.1	-1	16.1	6.4	27.68
		2	306.5	19.9	-1	18.9	7.6	24.52
		3	306.0	21.1	-1	20.1	8.0	24.48
						Average	7.3	25.56
	14th Day	1	305.5	24.9	-1	23.9	9.6	24.44
		2	297.5	27.6	-1	26.6	10.6	23.80
		3	351.0	23.5	-1	22.5	9.0	28.08
						Average	9.7	25.44

	Curing Duration	**Specimen No.**	**Weight (g)**	**Load Reading (KN)**	**Calibration Factor (KN)**	**Corrected Load (KN)**	**Compressive Strength (MPa)**	**Density (KN/m^3)**
Wet Burlap Covering	3rd Day	1	303.0	19.2	-1	18.2	7.3	24.24
		2	308.5	20.3	-1	19.3	7.7	24.68
		3	318.5	18.1	-1	17.1	6.8	25.48
						Average	7.3	24.80
	7th Day	1	312.5	20.4	-1	19.4	7.8	25.00
		2	302.0	23.7	-1	22.7	9.1	24.16
		3	306.0	21.8	-1	20.8	8.3	24.48
						Average	8.4	24.55
	14th Day	1	304.0	28.8	-1	27.8	11.1	24.32
		2	346.5	24.4	-1	23.4	9.4	27.72
		3	324.5	28.5	-1	27.5	11.0	25.96
						Average	10.5	26.00

	Curing Duration	**Specimen No.**	**Weight (g)**	**Load Reading (KN)**	**Calibration Factor (KN)**	**Corrected Load (KN)**	**Compressive Strength (MPa)**	**Density (KN/m^3)**
Standard Curing	28th Day	1	297.5	36.0	-1	35.0	14.0	23.80
		2	305.0	30.0	-1	29.0	11.6	24.40
		3	296.0	37.4	-1	36.4	14.6	23.68
						Average	13.4	23.96

	Curing Duration	**Specimen No.**	**Weight (g)**	**Load Reading (KN)**	**Calibration Factor (KN)**	**Corrected Load (KN)**	**Compressive Strength (MPa)**	**Density (KN/m^3)**
Air Curing	28th Day	1	314.5	12.5	-1	11.5	4.6	25.16
		2	344.0	9.1	-1	8.1	3.2	27.52
		3	316.5	9.8	-1	8.8	3.5	25.32
						Average	3.8	26.00

B) Curing Unaffected Zone

Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
28th Day	1	8211.5	301.0	-9	292.0	13.0	24.33
	2	8206.5	292.1	-9	283.1	12.6	24.32
	3	8124.5	306.8	-9	297.8	13.2	24.07
					Average	12.9	24.24

C) Curing Unaffected Zone

Curing Duration	Specimen No.	Weight (g)	Load Reading (KN)	Calibration Factor (KN)	Corrected Load (KN)	Compressive Strength (MPa)	Density (KN/m^3)
28th Day	1	8239.0	296.0	-9	287.0	12.8	24.41
	2	8142.0	316.0	-9	307.0	13.6	24.12
	3	8095.0	285.0	-9	276.0	12.3	23.99
					Average	12.9	24.17

Annex 8. Weather Condition Data

(Data from Western Amhara Metrology Service Center, Bahir Dar, Ethiopia, 2018)

	T° Max (C°)	T° Min (C°)	T° Average (C°)	Rainfall (mm)	Wind Speed (m/s)	Sunshine (hrs.)	Relative Humidity (%)					
Time	-	-		-	-	-	6:00	9:00	12:00	15:00	18:00	RH Average
9-May	30.5	9.2	20.7	0.0	0.80	10.8	67	28	20	23	23	32
10-May	32.0	9.4	20.8	0.0	0.70	10.9	67	30	25	19	17	32
11-May	31.5	10.0	22.8	0.0	0.78	11.1	62	34	24	18	27	33
12-May	32.5	13.0	23.1	0.0	0.84	11.4	42	36	32	17	43	34
13-May	33.0	13.2	22.5	0.0	0.94	11.6	62	34	32	25	24	35
14-May	31.0	14.0	31.0	0.0	1.17	10.6	76	39	39	34	56	49
15-May	31.0	na	21.8	8.9	0.84	5.2	74	58	48	36	46	52
16-May	29.5	14.0	23.6	0.0	1.02	8.7	80	66	52	45	46	58
17-May	30.5	16.6	23.7	0.0	1.03	8.7	68	76	48	31	45	54
18-May	31.0	16.4	23.3	0.0	0.95	9.6	71	62	45	35	48	52
19-May	31.5	15.0	23.8	0.0	0.94	10.8	79	56	39	36	33	49
20-May	31.3	16.2	23.8	0.0	0.67	10.6	66	54	31	27	27	41
21-May	32.0	15.6	22.3	0.2	0.79	10.1	62	49	32	28	43	43
22-May	32.2	12.4	24.0	0.0	0.89	8.8	87	52	33	26	31	46
23-May	31.5	16.4	22.7	0.0	1.00	10.1	52	43	35	29	36	39
24-May	30.6	14.8	22.6	0.0	1.34	10.0	66	55	49	46	43	52
25-May	29.3	15.9	21.9	0.0	1.06	9.5	69	56	45	37	47	51
26-May	30.2	13.6	31.6	0.0	1.06	10.8	80	48	42	32	38	48
27-May	31.6	na	23.2	0.0	0.93	10.4	60	47	36	30	23	39
28-May	29.5	16.8	23.5	0.0	0.74	4.1	66	46	38	40	41	46
29-May	31.0	16.0	22.0	0.0	0.84	7.3	78	58	46	30	34	49

30-May	28.5	15.4	28.5	0.0	0.91	1.4	80	81	52	49	48	62
31-May	28.5	na	21.3	33.0	1.25	10.0	80	72	56	40	50	60
1-Jun	28.0	14.6	24.8	0.0	0.52	2.7	86	84	70	45	54	68
2-Jun	32.2	17.4	22.6	0.0	0.83	9.5	84	64	52	40	46	57
3-Jun	29.2	16.0	22.6	0.0	1.06	7.4	79	74	44	40	49	57
4-Jun	29.0	16.2	22.7	0.0	0.75	5.1	84	81	54	37	56	62
5-Jun	28.6	16.8	21.1	15.9	0.60	7.0	88	70	54	46	80	68
6-Jun	26.0	16.2	21.4	0.0	0.43	2.8	96	80	72	62	64	75
7-Jun	26.0	16.8	21.0	15.1	0.80	2.8	90	84	72	78	76	80
8-Jun	26.6	15.4	22.1	6.5	0.58	5.8	91	84	64	58	80	75
9-Jun	28.6	15.6	21.2	17.6	1.03	6.3	88	84	58	47	56	67
10-Jun	27.6	14.8	21.3	21.6	0.93	7.2	92	88	66	56	54	71
Daily Average	**30.1**	**14.8**	**23.2**	**3.6**	**0.9**	**8.2**	**74.9**	**59.8**	**45.6**	**37.6**	**45.0**	**52.6**

Annex 9. Numerical Examples

1) Columns

A) Rectangular Plane Column

i Dimensions

Width	400	mm
Depth	300	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	7 Days

iv Calculation of in-place Compressive Strength

Value of β	0.7
Value of δ	0.875

$$f_e = 2zf_c(\beta - \delta)\left(\frac{D + B - 2Z}{BD}\right) + \delta f_c$$

$f_e =$ 24.61 MPa

B) Circular Plane Column

i Dimensions

Diameter	400	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	7 Days

iv Calculation of in-place Compressive Strength

Value of β	0.7
Value of δ	0.875

$$f_e = 4zf_c\left[\frac{(\beta - \delta)(\mathrm{D} - \mathrm{z})}{D^2}\right] + \delta f_c$$

$f_e =$ 25.02 MPa

2) Beam

A) Singly Reinforced Rectangular Beam

i Dimensions

Width	400	mm
Depth	300	mm
No of Bar	4	
Diameter of Bar	20	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	1256	mm^2
	591058.823	
Value of *W*	5	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \frac{wf_c(2\beta z + \delta b - 2\delta z)}{wb - f_c zb((\beta - \delta)(b + 2z)}$$

$f_e =$ 25.25 MPa

B) Doubly Reinforced Rectangular Beam

i Dimensions

Width	400	mm
Depth	300	mm
No of Bar for tension	4	
Diameter of Bar for tension	20	mm
No of Bar for compression	3	
Diameter of Bar for compression	20	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap	
Curing Duration	14 Days	

iv Calculation of in-place Compressive Strength

A_s	1256	mm^2
$A_{s'}$	942	mm^2
	147764.705	
Value of *v*	9	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \frac{vf_c(2\beta z + \delta b - 2\delta z)}{vb - f_c zb((\beta - \delta)(b + 2z)}$$

f_e = 23.14 MPa

C) Singly Reinforced T-Section Beam when NA lies within the web

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	300	mm
Total depth	400	mm
No of Bar	4	
Diameter of Bar	20	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap	
Curing Duration	14 Days	

iv Calculation of in-place Compressive Strength

A_s	1256	mm^2
	591058.823	
Value of *w*	5	N
Value of *l*	150	
Value of *m*	-14437.50	

Value of β	0.82	
Value of δ	0.875	

$$f_e = \frac{f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{(w - m)}$$

$f_e =$ 25.356 MPa

D) Singly Reinforced T-Section Beam when NA lies within the flange

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	250	mm
Total depth	400	mm
No of Bar	3	
Diameter of Bar	16	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	602.88	mm^2
	283708.235	
Value of *w*	3	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \left[\frac{\delta w f_c}{w - zbf_c(\beta - \delta)}\right]$$

$f_e =$ 24.144 MPa

E)	**Doubly Reinforced T-Section Beam when NA lies within the web**		
i	Dimensions		
	Width of flange	600	mm
	Depth of flange	150	mm
	Width of web	300	mm
	Total depth	500	mm
	No of Bar for Compression	6	
	Diameter of Bar for Compression	20	mm
	No of Bar for Tension	2	
	Diameter of Bar for Tension	20	mm
	Thickness of CAZ	25	mm
ii	Materials Property		
	Compressive strength of concrete	30	MPa
	Compressive strength of bar	400	MPa
	Water to cement ratio	0.45	
iii	Curing Parameters		
	Curing Method	Wet Burlap	
	Curing Duration	14 Days	
iv	Calculation of in-place Compressive Strength		
	A_s	1884	mm^2
	$A_{s'}$	628	mm^2
	Value of v	591058.8235	N
	Value of l	150	
	Value of m	-14437.5	
	Value of β	0.82	
	Value of δ	0.875	

$$f_e = \frac{f_c v\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{(v - m)}$$

	$f_e =$	25.36	MPa

F) Doubly Reinforced T-Section Beam when NA lies within the flange

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	250	mm
No of Bar for Compression	6	
Diameter of Bar for Compression	20	mm
No of Bar for Tension	2	
Diameter of Bar for Tension	20	mm
Diameter of Bar	20	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	1884	mm^2
$A_{s'}$	628	mm^2
Value of v	591058.8235	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \left[\frac{\delta v f_c}{v - zbf_c(\beta - \delta)}\right]$$

f_e =	25.19	MPa

G) Singly Reinforced L-Section Beam when NA lies within the web

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	300	mm
Total depth	500	mm
No of Bar	4	
Diameter of Bar	20	mm

	Thickness of CAZ	25	mm
ii	Materials Property		
	Compressive strength of concrete	30	MPa
	Compressive strength of bar	400	MPa
	Water to cement ratio	0.45	
iii	Curing Parameters		
	Curing Method	Wet Burlap	
	Curing Duration	14 Days	
iv	Calculation of in-place Compressive Strength		
	A_s	1256	mm^2
		591058.823	
	Value of *w*	5	N
	Value of *l*	150	
	Value of *n*	-18562.5	
	Value of β	0.82	
	Value of δ	0.875	

$$f_e = \frac{\frac{w f_c}{b_w}\left(2\beta z + \delta(b_w - 2z)\right)}{w - n}$$

$f_e =$ 25.18 MPa

H) Singly Reinforced L-Section Beam when NA lies within the flange

i	Dimensions		
	Width of flange	600	mm
	Depth of flange	150	mm
	Width of web	250	mm
	Total depth	400	mm
	No of Bar	3	
	Diameter of Bar	16	mm
	Thickness of CAZ	25	mm
ii	Materials Property		
	Compressive strength of concrete	30	MPa
	Compressive strength of bar	400	MPa
	Water to cement ratio	0.45	
iii	Curing Parameters		
	Curing Method	Wet Burlap	
	Curing Duration	14 Days	

iv Calculation of in-place Compressive Strength

A_s	602.88	mm^2
	283708.235	
Value of *w*	3	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \left[\frac{\delta w f_c}{w - zbf_c(\beta - \delta)}\right]$$

f_e =	24.144	MPa

I) Doubly Reinforced L-Section Beam when NA lies within the web

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	300	mm
Total depth	500	mm
No of Bar for Compression	6	
Diameter of Bar for Compression	20	mm
No of Bar for Tension	2	
Diameter of Bar for Tension	20	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	1884	mm^2
$A_{s'}$	628	mm^2
	1182117.64	
Value of *v*	7	N
Value of *l*	150	
Value of *n*	-18562.5	
Value of β	0.82	
Value of δ	0.875	

$$f_e = \frac{\frac{vf_c}{b_w}(2\beta z + \delta(b_w - 2z))}{v - n}$$

f_e =	25.57	MPa

J) Doubly Reinforced L-Section Beam when NA lies within the flange

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	250	mm
No̲ of Bar for Compression	6	
Diameter of Bar for Compression	20	mm
No̲ of Bar for Tension	2	
Diameter of Bar for Tension	20	mm
Diameter of Bar	20	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	1884	mm^2
$A_{s'}$	628	mm^2
Value of v	591058.823 5	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \left[\frac{\delta v f_c}{v - zbf_c(\beta - \delta)}\right]$$

f_e =	25.19	Mpa

3) Slabs

A) Solid Slab

i Dimensions

Width	1000	mm
Depth	150	mm
No of Bar	10	
Diameter of Bar	14	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	1538.6	mm^2
Value of W	724047.0588	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \left[\frac{\delta w f_c}{w - zbf_c(\beta - \delta)}\right]$$

$f_e =$ 24.84 MPa

B)	Ribbed Slab When NA lies within the web		
i	Dimensions		
	Width of flange	600	mm
	Depth of flange	150	mm
	Width of web	300	mm
	Total depth	400	mm
	No̲ of Bar	4	
	Diameter of Bar	20	mm
	Thickness of CAZ	25	mm
ii	Materials Property		
	Compressive strength of concrete	30	MPa
	Compressive strength of bar	400	MPa
	Water to cement ratio	0.45	
iii	Curing Parameters		
	Curing Method	Wet Burlap	
	Curing Duration	14 Days	
iv	Calculation of in-place Compressive Strength		
	A_s	1256	mm^2
		591058.823	
	Value of w	5	N
	Value of l	150	
	Value of m	-14437.50	
	Value of β	0.82	
	Value of δ	0.875	

$$f_e = \frac{f_c w\left[\frac{2\beta z}{b_w} + \delta\left(1 - \frac{2z}{b_w}\right)\right]}{(w - m)}$$

f_e = 25.356 MPa

C) Ribbed Slab When NA lies within the flange

i Dimensions

Width of flange	600	mm
Depth of flange	150	mm
Width of web	250	mm
Total depth	400	mm
N<u>o</u> of Bar	3	
Diameter of Bar	16	mm
Thickness of CAZ	25	mm

ii Materials Property

Compressive strength of concrete	30	MPa
Compressive strength of bar	400	MPa
Water to cement ratio	0.45	

iii Curing Parameters

Curing Method	Wet Burlap
Curing Duration	14 Days

iv Calculation of in-place Compressive Strength

A_s	602.88	mm^2
	283708.235	
Value of *w*	3	N
Value of β	0.82	
Value of δ	0.875	

$$f_e = \left[\frac{\delta w f_c}{w - z b f_c(\beta - \delta)}\right]$$

$f_e =$ 24.144 MPa